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Lectures Notes on General Equilibrium

These lecture notes are used for the second time this year (fall 2014). I would appreciate a lot if you could inform me about any errors that you find in the notes. Please also let me know if there are any parts that are difficult to follow.

1 Introduction

Investors in the technology sector should better keep themselves informed about the evolution of modern culture. The profit opportunities for companies producing computers, mobile phones and Internet networks are heavily influenced by the developments in the “contents sector”, including the movie and music industry. The CEOs of car and truck companies should better follow the political debate about nuclear energy. A higher price on electricity increases the cost of transportations by train, which, in turn, increases the demand for their produce.

Less obvious, perhaps, is that Swedish manufacturing firms should keep an eye on Norwegian oil. Even substantial new deposits may not affect the world market price of oil by much. But as Norwegians get ever richer, they consume ever more services, making it harder for Norwegian manufacturing firms to hire labor at competitive wages.

Most large cities are severely congested during rush hours. One reason is the difficulty to charge a price for using the streets. (Efforts to introduce congestion fees often provoke political opposition, possibly as a result of the risk that a congestion fee soon evolves into an instrument for taxation.) Subsidizing public transport may then be the way to go. Expressed differently, if one good (travel by car) for some reasons cannot be made bear its social costs, alternative goods (travel by buss) should perhaps not bear their social costs either.

In short, markets are interdependent. And decision makers need to understand how to analyze these interdependencies. The necessary analytical framework is the so-called theory of general equilibrium.

We only have two lectures on general equilibrium, however. So, to acquaint ourselves with this topic, we will use the simplest possible model, with a minimum of interaction between different markets. In fact, the only reason why the different markets are interrelated in the model is a resource constraint, in the form of a limited amount of labor. With a given amount of labor, more of one good necessarily means less of another. All other reasons why markets are interdependent are removed for the sake of simplicity. The different goods are neither substitutes nor complements. There are no economies or diseconomies of scope.

The first lecture focuses on the role of prices. We will see how prices coordinate firms and households to pursue production and consumption plans that all can be realized at the same time. In fact, the firms and the households only need to know the prices to make their choices. They do not need to know anything about other peoples' preferences or the production technologies of rival firms. Still, they can rely on other people and firms to buy what they plan to produce and to supply what they plan to consume. Even more remarkable, perhaps, is that the resulting allocation is efficient. Expressed differently, it is impossible to improve the welfare of someone without lowering the welfare of somebody else.

The second lecture is more applied. The focus here will be on the welfare effects of taxes. But as even this topic is enormous, I will zoom in on only one point. The primary reason why taxation is distortive is that governments cannot tax leisure. I will be careful, however, to show why we need general equilibrium theory to reach this conclusion.

2 A simple model of general equilibrium

Consider a country with many inhabitants and a large number of firms. The inhabitants supply production factors, which the firms use to produce a large number of consumer goods, which, in turn, are sold to the inhabitants. All goods and labor are exchanged on markets at prices that all firms and households take as given. There are N different consumer goods but, for simplicity, only one production factor, labor.

The economy is in a state of *general equilibrium* if the $N+1$ prices are such that all households and firms can realize their consumption and production plans in all markets at the same time. Other words that are often used for the same idea are Walrasian equilibrium and competitive equilibrium.

The first question that we need to address is whether it is even possible to find such a set of prices that coordinate all agents' plans. Expressed differently, is it possible for firms and households to decide how much to consume and how much to produce, knowing only the prices? Can they really rely on other people to buy what they plan to produce and to sell what they plan to consume?

2.1 How prices coordinate peoples' plans

2.1.1 Households

2.1.1.1 Demand for goods

There are M inhabitants and they all have identical Cobb-Douglas preferences over the N goods:

$$U = \prod_{i=1}^{N} q_i^{\alpha_i} .$$

For convenience we normalize the sum of the exponents in the utility function to one, i.e. $\sum \alpha_i = 1$. (This normalization is simply equivalent to a positive monotone transformation of the utility function.)

All people also have the same income I and the prices of the goods are denoted p_i. The budget constraint is therefore given by

$$\sum_{i=1}^{N} p_i \cdot q_i = I .$$

Each person maximizes his utility subject to this budget constraint. The Lagrangeian is given by

$$L = \prod_{i=1}^{n} q_i^{\alpha_i} + \lambda \cdot \left[I - \sum_{i=1}^{n} p_i \cdot q_i \right]$$

where λ is the Lagrange multiplier. All the first-order conditions for the individual goods can be written:

$$\frac{\partial L}{\partial q_i} = \alpha_i \cdot q_i^{\alpha_i - 1} \cdot \prod_{j \neq i} q_j^{\alpha_j} - \lambda \cdot p_i = 0$$

or equivalently

$$\frac{\partial L}{\partial q_i} = \alpha_i \cdot q_i^{-1} \cdot U - \lambda \cdot p_i = 0 .$$

That is

$$p_i \cdot q_i = \alpha_i \cdot \lambda^{-1} \cdot U .$$

Substituting these first-order conditions into the budget constraint gives

$$\sum_{i=1}^{N} p_i \cdot q_i = \lambda^{-1} \cdot U \cdot \sum_{i=1}^{N} \alpha_i ,$$

that is, $\lambda^{-1} \cdot U = I$, since the sum of alphas is equal to one. Substituting this expression back into the first-order conditions for the individual goods gives the individual demand function:

$$q_i = \alpha_i \cdot \frac{I}{p_i} .$$

Total market demand for each good is found by (horizontal) summation, i.e.

$$Q_i^D = \alpha_i \cdot \frac{I \cdot M}{p_i} ,$$

where $I \cdot M$ is total income in the country.

2.1.1.2 Supply of labor

The people in this country do not experience any utility or disutility of working. Therefore everyone simply works as much as they can as long as the wage is positive. We normalize the maximum time to be one time-unit. Thus, everyone supplies one time-unit of labor independent of the wage. The total supply of labor is therefore equal to the number of people, $L^S = M$, independent of the wage. The wage is denoted by w per time-unit.

Assuming that the labor market clears and that all people find a job, each person earns a labor income equal to w. (We will later verify that the labor market indeed clears.)

2.1.1.3 Income

All people in this country own the same number of shares in every firm. The sum of all profits in the country is given by Π. Therefore every person receives Π / M as dividends. Total income is thus equal to

$$I = w + \frac{\Pi}{M}$$

which means that total market demand for each good is given by

$$Q_i^D = \alpha_i \cdot \frac{w \cdot M + \Pi}{p_i}.$$

To see this simply substitute the expression for individual income into the demand function derived above.

2.1.2 Firms

2.1.2.1 Supply of goods

Firms produce all goods and they only use labor as a factor of production. The production function is given by

$$q_i = \frac{1}{\varphi_i} \cdot l_i$$

for all firms in market *i*. This means that it takes φ_i units of labor to produce one unit of good *i*. The marginal cost of producing good *i* is therefore constant and equal to $w \cdot \varphi_i$. Since there are no fixed costs, also the average cost is equal to $w \cdot \varphi_i$. In short, all technologies have constant returns to scale.

There is a large number of firms competing in every product market. Since all firms have constant returns to scale, it is not necessary to determine the exact number. The reason why we should think of the number of firms as large is simply that we wish to motivate the assumption that firms take prices as given. Total supply of goods in market *i* is denoted Q_i^S.

Notice that since all firms have constant returns to scale and since all firms are price-takers, the supply curve is perfectly elastic at the marginal cost, $w \cdot \varphi_i$. That is, as long as the price is equal to or higher than $w \cdot \varphi_i$ the firms are willing to supply any amount of goods that the consumers demand.

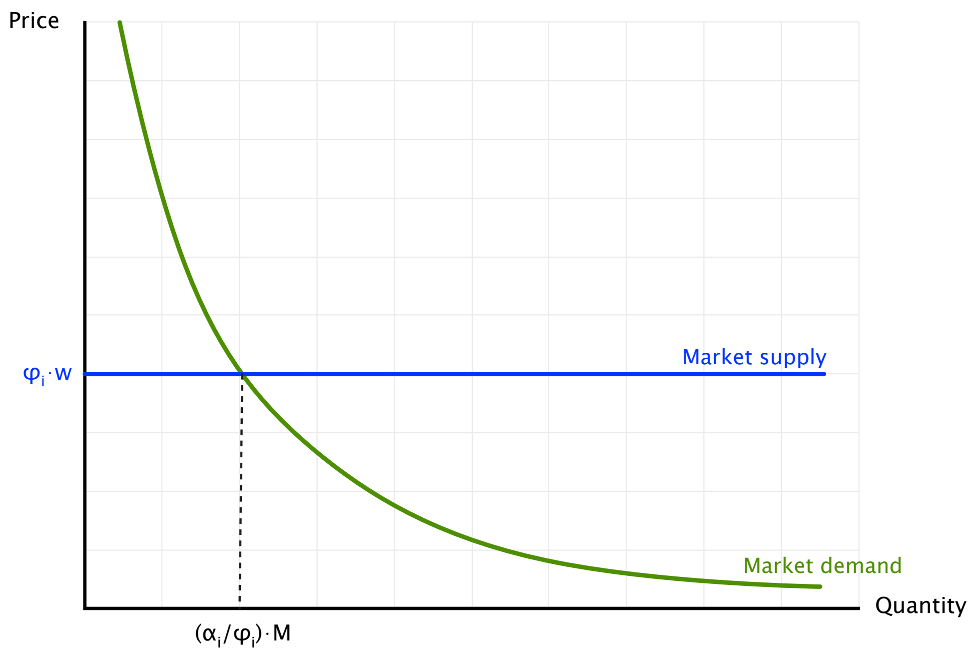
2.1.2.2 Demand for labor

Demand for labor will be discussed shortly.

2.1.3 Equilibrium

2.1.3.1 Goods markets

The equilibrium in product market *i* is found by equating demand and supply in this market. Since supply is perfectly elastic at the marginal cost which equals $w \cdot \varphi_i$, it follows that the equilibrium price must also equal this constant marginal cost, i.e. $p_i = w \cdot \varphi_i$ in every product market.



Since price is equal to marginal cost and since marginal cost is equal to average cost, all firms earn a zero profit, that is $\Pi = 0$. It follows that total quantity produced and consumed in market *i* is given by

$$Q_i^D = \frac{\alpha_i}{\varphi_i} \cdot M$$

and that every consumer buys

$$q_i = \frac{\alpha_i}{\varphi_i}$$

units of good *i*.

Be warned, however, that this simple result is the result of a simple model only. In the model, it is the supply side alone that determines the equilibrium price and the demand side that determines the equilibrium quantity (at the relevant price). The reason for this simple dichotomy is that the supply curve is perfectly elastic. In reality, supply may be very elastic in the long run (at least in some industries), but this is probably very rare in the short run. Moreover, any person's consumption of a good is determined by only two factors, namely how important the good is for utility (α_i) and the factor requirement φ_i to produce this good. In reality, the consumption of a certain good also depends on the characteristics of substitute and complementary goods. But, luckily, we do not need to consider all the complicated features of the real world for the issues that are to be discussed here.

2.1.3.2 Labor market (Walras law)

So far, the discussion has focused on the product markets and very little has been said about the labor market. The reason is the so-called Walras law. Walras law guarantees that if all product market clear, then also the labor market must clear. Thus, we actually do not need to analyze the labor market in detail.

To see this, recall that in order to produce Q_i^S units of output, the firms in market *i* need to hire $\varphi_i \cdot Q_i^S$ units of labor. That is, the demand for labor in market *i* is given by the total production times the factor requirement. Total demand for labor is consequently given by $L^D = \sum \varphi_i \cdot Q_i^S$.

Since there is a large number of product markets and since there are many firms in every market, all firms are price-takers also on the labor market.

Whenever all product market are in equilibrium

$$Q_i^S = Q_i^D = \frac{\alpha_i}{\varphi_i} \cdot M$$

and thus

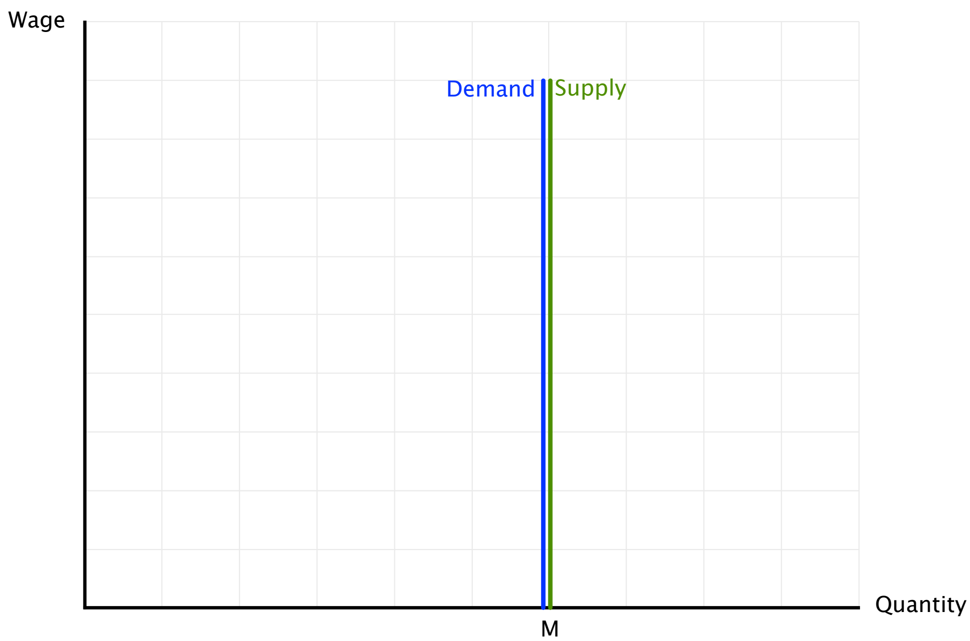
$$L^D = \sum \varphi_i \cdot \left\{ \frac{\alpha_i}{\varphi_i} \cdot M \right\} = M \cdot \sum \alpha_i = M$$

The firm's demand for labor is simply derived from the consumers' demand for goods, which in turn is constrained by their supply of labor. Recall that there are M people, all supplying one unit of labor.

Recall that there are M people, all supplying one unit of labor, i.e. $L^S = M$.

In other words, if all product markets are in equilibrium, the firms demand M units of labor, independent of the wage (since prices are adjusted in proportion to the wage anyway).

It follows that, if all product markets are in equilibrium, the labor market clears at any wage.



Expressed differently, any wage is an equilibrium wage.

This is an instance of the so-called Walras law, which states that if all markets except one in an economy are in equilibrium, then also the last market must be in equilibrium. Walras law is very general. It is a manifestation of the resource constraint in the economy and it applies independent of what the utility and production functions look like.

We will use Walras law here to omit the analysis of the labor market. We know that if we can find a set of prices that clears all the product markets, the labor market will be cleared as well. Thus, we do not need to study the labor market in detail.

2.1.3.3 About the price level

Notice that we have determined how much every person works and how much every person consumes of every good. That is, we have determined the allocation of production factors and goods in the economy. This is the primary purpose of the analysis. We have also determined that the price of each good is given by $p_i = w \cdot \varphi_i$. To determine the price in market *i*, we simply equated demand and supply in market *i*.

Notice that all prices depend on the wage. The higher the wage is, the higher every price must be. But since we have not determined what the wage is, we actually do not know what the prices are either. We only know what the relative prices are in equilibrium. For example, the price of good *i* relative to the wage is $p_i / w = \varphi_i$. And the price of good *i* in relation to good *j* is $p_i / p_j = \varphi_i / \varphi_j$. We have thus succeeded to determine the allocation of factors and goods in the economy, only knowing the relative prices. Thus, not knowing the price level does not matter so much, since the price level does not matter for the allocation anyway.

It is actually impossible to determine the price level in the economy, only using a standard micro model. The formal reason why we cannot determine the wage (and thus the price level) is related to Walras law. Walras law shows that demand and supply in different markets are interdependent. Here, the demand and supply of labor are equal whenever all product markets clear. We thus do not have an extra *independent* supply-equals-demand-equation that can be used to determine the final unknown, the wage. In fact, the demand for labor is always equal to the supply of labor, independent of the wage.

Is this indeterminacy a failure of the model? Actually not. For instance, it does not matter if we express prices in Euros or if we express them in Euro cents. As long as we divide all prices by 100, nothing important is changed.

But, it also depends on what we want to use the model for. To study inflation, we need to determine the price level. We then need to augment the model and think of money as a good. The price of money is by definition equal to one and all other prices in the economy can then be expressed in terms of money. Money is used as the "numeraire." Modeling the money market is far from trivial, however. We need to understand why and how people and firms demand money. Since part of the reason is that money creates utility indirectly by simplifying transactions, we need to work with a model that keeps track of how the money moves between agents as a result of all the transactions they take part in. We also need to understand how the government (and banks) supplies money. We will leave this to the macro economists.

2.1.4 A note on the simplifying assumptions and market interdependence

Cobb-Douglas preferences are easy to work with, but they are also rather special. The demand for a good depends only on the price of that good and not on the prices of other goods. Expressed differently, the consumers consider the different goods as neither substitutes (which have a positive cross-price elasticity of demand) nor complements (which have a negative cross-price elasticity of demand). The goods are independent. This means that one important reason why markets are interdependent is missing in the present model.

Another important simplification is that I have assumed that people do not experience any disutility of working. But this assumption will soon be removed

Also the production function is chosen for simplicity. By assuming that there is only one factor of production we do not need to think about the firms' cost-minimization problems. By assuming that there are constant returns to scale, we no not need to think about how many firms that enter each market. Production is equally efficient independent of the number of firms.

The assumption that there are no economies or diseconomies of scope simplifies the analysis, but it also eliminates important interactions that exist between markets in the real world.

The present model thus assumes that all consumers consider all consumers goods as neither substitutes nor complements and that every firm is active in only one product market. As a result the equilibrium price and quantity in every goods market only depend on the characteristics of that particular good (i.e. α_i and φ_i).

It seems that the product markets are completely independent. This is not true, however. The interaction takes place in the labor market. There is a fixed supply of labor and the firms in the different markets compete to acquire enough labor to produce their goods.

Consider, for example, what would happen if the quality of some good is increased for some reason. An increase in the quality of good *i* would be manifested in a higher α_i. The consumers would then demand more of good *i* and the firms would increase their production of this good. The firms in market *i* therefore also increase their demand for labor. All else equal, market *i*'s increased demand for labor would increase the price of labor (the wage) in relation to the price of the other goods. With a higher wage, the other markets buy less labor, thereby making room for market *i* to expand.[1]

2.2 How prices lead people to make efficient plans

We say that an allocation is *Pareto efficient* if it is impossible to improve the welfare of at least one individual without reducing the welfare of somebody else. To investigate if the market outcome is Pareto efficient we need to recall a few basic concepts.

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[1] When α_i is increased, the consumers actually also demand less of other goods. (Recall that the sum of alphas is normalized to one. So when one alpha is increased, the others must be reduced.) Therefore the firms in the other markets actually demand less labor. Taking all these effects into account implies that the price of labor (i.e. the wage) does not change in relation to the prices of the goods.

2.2.1 Efficient distribution of consumption

2.2.1.1 Marginal rate of substitution

Mr. Anderson enjoys apples and pears. If a small number of additional apples increase Mr. Anderson's happiness by 10 utiles each and additional pears increase it by only 5 utiles, then Mr. Anderson is willing to give up 2 pears to get an additional apple. To Mr. Anderson, the value of an apple is two pears.

Notice where the number 2 comes from. Mr. Anderson derives utility from consuming apples and pears. If he consumes q_{apple} units of apples and q_{pear} units of pears, his utility is $u = U^{Andersson}\left(q_{apple}, q_{pear}\right)$. If Mr. Anderson increases his consumption of apples by $\Delta q_{apple} > 0$ units, his utility is increased by $\Delta u = U_{apple}^{Anderson} \cdot \Delta q_{apple} > 0$, where $U_{apple}^{Anderson} > 0$ is Mr. Anderson's marginal utility of apples. The marginal utility of apples is the partial derivative of the utility function with respect to the quantity of apples. If Mr. Anderson decreases his consumption of pears by $\Delta q_{pear} < 0$ units, his utility is decreased by $\Delta u = U_{pear}^{Anderson} \cdot \Delta q_{pear} < 0$, where $U_{pear}^{Anderson} > 0$ is Mr. Anderson's marginal utility of pears. The marginal utility of pears is the partial derivative of the utility function with respect to the quantity of pears.

We assumed that Mr. Anderson's marginal utility of apples is given by $U_{apple}^{Anderson} = 10$ and his marginal utility of pears is given by $U_{pear}^{Anderson} = 5$. To find out how many pears Mr. Anderson is willing to give up in exchange for an apple, we simply divide the two marginal utilities, and multiply by minus one, i.e. $dq_{pear} = -\frac{U_{apple}^{Anderson}}{U_{pear}^{Anderson}} = -\frac{10}{5} = -2$. The minus signifies that if Mr. Anderson get to consume more one fruit, he is willing to give up some of the other.

More formally, the *marginal rate of substitution* between two goods i and j for a consumer h is defined as the number of units of good j that the consumer is willing to give up for an additional unit of good i. We denote this by MRS_{ij}^{h} and may think of it as the value of *i* in terms of *j*. (Beware that different authors switch the order of the indexes *i* and *j* however.)

To find the marginal rate of substitution, we must differentiate the consumer's utility function. Differentiating the utility function shows how the consumer's welfare is changed (Δu) if we increase the consumer's consumption of good i by $\Delta q_i > 0$ units and reduce the consumer's consumption of good j by $\Delta q_j < 0$ units at the same time. The total change in utility is given by

$$\Delta u = U_i^h \cdot \Delta q_i + U_j^h \cdot \Delta q_j ,$$

where U_i^h and U_j^h are consumer h's marginal utilities of the two goods.

To find out how many units of good *j* the consumer is willing to give up in exchange for an extra unit of good *i*, we need to find the $\Delta q_j < 0$ that keeps the person's utility constant, $\Delta u = 0$. Thus, we need to satisfy the equation

$$U_i^h \cdot \Delta q_i + U_j^h \cdot \Delta q_j = 0 .$$

Solving the equation gives:

$$\Delta q_j = -\frac{U_i^h}{U_j^h} \cdot \Delta q_i .$$

That is, if we increase the consumption of good *i* by one unit, $\Delta q_i = 1$, this consumer is willing to reduce consumption of good *j* by $\Delta q_j = -\frac{U_i^h}{U_j^h}$ units.

Expressed differently,

$$MRS_{ij}^h = -\frac{U_i^h}{U_j^h} .$$

The marginal rate of substitution is thus simply the relative marginal utilities of the two goods.

2.2.1.2 Efficient consumption

Two people, Mr. Anderson and Mr. Peterson, both plan to consume some amounts of apples and pears. Given their current plans, Mr. Anderson is willing to give up two pears for an apple while Mr. Peterson is willing to swap only one pear for an apple. That is $MRS_{apple,pear}^{Anderson} = -2$ while $MRS_{apple,pear}^{Peterson} = -1$. The value of apples in terms of pears is thus higher for Mr. Anderson than for Mr. Peterson.

So if Mr. Peterson would give Mr. Anderson one apple, Mr. Anderson would be willing to give two pears in return. But since Mr. Peterson would only require one pear, both could be better of by swapping fruits with each other.

The initial consumption plans were not Pareto efficient.

It would not be possible to improve the welfare of one person without hurting somebody else, using a similar reallocation of goods between consumers, if and only if $MRS_{ij}^{h} = MRS_{ij}^{g}$ for all pairs of people and all pairs of goods.

2.2.1.3 Does the market induce efficient consumption?

To maximize utility, any person must adjust his consumption of any two goods *i* and *j* so that the marginal utilities of the goods are proportional to their prices. For h this means that

$$U_i^h - \lambda^h \cdot p_i = 0 \qquad \text{and} \qquad U_j^h - \lambda^h \cdot p_j = 0 .$$

This is simply the first-order condition for utility maximization.

Dividing one of the conditions by the other reveals that *h*'s marginal rate of substitution between the two goods must be equal to (the negative of) the relative price, i.e.

$$MRS_{ij}^{h} \equiv -\frac{U_i^h}{U_j^h} = -\frac{p_i}{p_j} .$$

Since all consumers buy goods at the same prices, they will all make sure to satisfy the same condition. Therefore

$$MRS_{ij}^{h} = -\frac{p_i}{p_j} = MRS_{ij}^{g} .$$

In other words, given that people maximize utility and that they can buy goods that the same prices in the market, the market will induce an efficient allocation of goods between consumers. All consumers will hold the same value of *i* in terms of *j*.

2.2.2 Efficient activity levels

2.2.2.1 *Marginal rate of transformation*

If it takes ten units of labor to build a car and only two units of labor to build a moped, a country could produce one more car by producing five mopeds less. We say that the cost of a car is five mopeds.

More formally, the marginal rate of transformation between two goods i and j is defined as the number of units of good j that cannot be produced in order to produce an additional unit of good i. We denote this by MRT_{ij} and may think of it as the (opportunity) cost of *i* in terms of *j*.

Recall that it takes φ_i units of labor to produce one unit of good *i*. The total amount of labor used to produce the goods is therefore $L = \sum \varphi_i \cdot q_i$. If we wish to produce an additional unit of *i* without changing the total amount of labor used we need to reduce production of *j* by an amount satisfying:

$$\Delta L = \varphi_i \cdot \Delta q_i + \varphi_j \cdot \Delta q_j = 0,$$

that is by

$$\Delta q_j = -\frac{\varphi_i}{\varphi_j} \cdot \Delta q_i$$

units.

Expressed differently the marginal rate of transformation between the two goods is simply the relative factor requirements

$$MRT_{ij} = -\frac{\varphi_i}{\varphi_j}.$$

In more general models, with many factors of production, the marginal rate of transformation is the relative marginal cost of the two goods. This is also the case in the present model, since

$$MRT_{ij} = -\frac{w \cdot \varphi_i}{w \cdot \varphi_j},$$

where $w \cdot \varphi_i$ is the marginal cost of *i*.

2.2.2.2 Efficient activity level

Consider a person who plans to consume two goods *i* and *j*. This person would be willing to swap two units of good *j* for one unit of *i*. That is the value of *i* in terms of *j* is two, $MRS_{ij}^{h} = -2$.

The cost of *i* in terms of *j* is

$$MRT_{ij} = -\frac{\varphi_i}{\varphi_j}.$$

If the cost of *i* in terms of *j* is equal to one, it would be possible to increase the welfare of the consumer. To do so the production and consumption of *j* should be reduced by one unit. As a result it is possible to increase the production and consumption of *i* by one unit without breaking the resource constraint and without asking somebody else to give up any consumption. Since the consumer values a unit of *i* higher than a unit of *j*, this reallocation increases welfare of one person without reducing it for somebody else.

The initial consumption plans were not Pareto efficient.

It would not be possible to improve the welfare of one person without hurting somebody else, using a similar reallocation of production resources between the production of different goods, if and only if $MRS_{ij}^{h} = MRT_{ij}$ for all pairs of goods.

2.2.2.3 Does the market induce efficient activity levels?

When all firms are price takers they will guarantee that their marginal cost is equal to price in every market, i.e. $p_i = w \cdot \varphi_i$. Consequently

$$MRT_{ij} \equiv -\frac{w \cdot \varphi_i}{w \cdot \varphi_j} = -\frac{p_i}{p_j}.$$

Since the consumers' marginal rate of substitution is also equal to the same price ratio, it follows that the condition for efficiency is fulfilled

$$MRT_{ij} = -\frac{p_i}{p_j} = MRS_{ij}^{h}.$$

In other words, given that people maximize utility and that firms are profit-maximizing price takers, the market will induce efficient activity levels. The value of *i* in terms of *j* is equal to the cost of *i* in terms of *j*, at the margin.

2.3 Conclusion

The theory of general equilibrium reveals what role prices play in the economy.

- Prices coordinate firms and households to choose production and consumption plans that all can be realized at the same time. The firms and the households only need to know the prices to make their choices. They do not need to know anything about other peoples' preferences or the production technologies of rival firms. Still, they can rely on other people and firms to buy what they plan to produce and to sell what they plan to consume.
- Even more remarkable, perhaps, is that the resulting allocation is efficient. Expressed differently, it is impossible to improve the welfare of someone without lowering the welfare of somebody else.

In coordinating all firms and households and inducing an efficient allocation, prices also, at the same time, fill a third role. The prices determine how material welfare is distributed between different people.

3 Inequality, Redistribution and Tax-distortions

In Sweden, people with very low incomes earn about a fifth of those with very high incomes.[2] A major reason for income inequality is that people have different productivity. Many people think that it is fair that people keep what they produce. They are therefore not worried about the income differences. Other people are disturbed by the differences and wish to reduce them using various policies.

[2] I am comparing people in the 5th – 10th percentiles with people in the 90th – 95th percentiles during the years 2003 – 2005. See Consumption and Income Inequality in Sweden, by Daunfelt et al (available on the Internet).

And governments do in fact redistribute wealth from those with higher incomes to those with lower on a substantial scale, using taxes. It is therefore important to study what the effects of such policies are. This is the topic of this second lecture.

3.1 Inequality

3.1.1 Productivity and income

Assume that there are two types of people. Half the population can work very fast. They produce $(1+\delta)/\varphi_i$ units of good *i* per time unit. The other half only produces $(1-\delta)/\varphi_i$ units of good *i* per time unit. It is assumed that $0 \le \delta \le 1$. The larger is δ, the larger is the difference between the two groups. But average productivity in the economy is unaffected by δ, since there are equally many people of both types.

People earn a wage per hour that is proportional to their productivity, i.e. the hourly wage is proportional to how much they produce per hour. Expressed differently, w is the wage per "efficiency unit".[3] Since all people work full time, this means that the labor income of a person with high productivity is given by $I^H = w \cdot (1+\delta)$ and that the labor income of a person with low productivity is given by $I^L = w \cdot (1-\delta)$.

From now on we forget about firm profits since they will be zero anyway.

3.1.2 Demand and supply

To determine demand, simply plug in the new expressions for income into the demand function derived above. The high productive people demand

$$q_i^H = \alpha_i \cdot \frac{w \cdot (1+\delta)}{p_i}$$

and the low productive people demand

[3] If the two groups would earn the same wage, the firms would prefer to hire those with high productivity. To compete, the firms would start to bid up this wage. This process would only stop when wages correspond to productivity.

$$q_i^L = \alpha_i \cdot \frac{w \cdot (1-\delta)}{p_i}.$$

Market demand is given by

$$Q_i^D = \alpha_i \cdot \frac{w \cdot (1+\delta)}{p_i} \cdot \frac{M}{2} + \alpha_i \cdot \frac{w \cdot (1-\delta)}{p_i} \cdot \frac{M}{2} = \alpha_i \cdot \frac{w}{p_i} \cdot M ,$$

which is the same as above. (The reason why market demand is unchanged is that the average productivity is unchanged.)

Market supply remains the same as before in every product market. That is, the marginal cost of producing good *i* is constant and given by $w \cdot \varphi_i$. Market supply of good *i* is therefore perfectly elastic at this marginal cost.

3.1.3 Equilibrium: Efficiency and distribution

Since both market demand and supply are unaffected by the introduction of productivity differences, the equilibrium price remains equal to $p_i = w \cdot \varphi_i$ in market *i*.

The market is Pareto efficient. (To prove this, use the conditions: $MRS^h = MRS^g = MRT$). That is, it is impossible to improve the welfare of any person without reducing the welfare of somebody else.

The market outcome is, however, not equally favorable to all. The difference in productivity entails a difference in consumption. Those with high productivity get to consume

$$q_i^H = \frac{\alpha_i}{\varphi_i} \cdot (1+\delta)$$

while those with low productivity get to consume

$$q_i^L = \frac{\alpha_i}{\varphi_i} \cdot (1-\delta)$$

which is less. That is, people get to consume goods in proportion to their productivity.

3.2 Redistribution and taxation I

The government wishes to redistribute welfare and to do so the government imposes an ad valorem tax that all firms have to pay on all the goods they sell. That is, all goods are taxed with the same proportional tax rate t. All tax revenues T are distributed equally among all people with a low income.

3.2.1 Demand and supply

The high-productive people continue to receive an income of $I^H = w \cdot (1+\delta)$ while the low-productive people now receive $I^L = w \cdot (1-\delta) + \frac{T}{M/2}$, where the second term is the total tax revenues divided by the number of low-productive people. Total demand is therefore given by

$$Q_i^D = \alpha_i \cdot \frac{I^H}{p_i} \cdot \frac{M}{2} + \alpha_i \cdot \frac{I^L}{p_i} \cdot \frac{M}{2} = \alpha_i \cdot \frac{w \cdot M + T}{p_i}$$

As before, market demand is determined by total consumer income in the country, i.e. $w \cdot M + T$.

For simplicity, I will assume that the tax is proportional to the firms' marginal costs. (Collecting a tax in proportion to marginal costs is equivalent to collecting a tax as a percentage of price when all firms are price takers.) Recall that the marginal production cost is $w \cdot \varphi_i$. Now the firms also need to pay $w \cdot \varphi_i \cdot t$ per unit they sell in taxes. In effect, the tax means that the firms' perceived marginal cost is increased to $w \cdot \varphi_i \cdot (1+t)$. It follows that the market supply curve in all goods markets is shifted up in proportion to the tax rate.

3.2.2 Equilibrium

Since market supply is perfectly elastic at the constant marginal cost and since the marginal cost is increased in proportion to the tax rate, it follows that also the equilibrium price in every market must be increased by the same rate, i.e. $p_i = w \cdot \varphi_i \cdot (1+t)$.

Thus, even if it is the firms that formally pay the taxes, it is actually all people that bear the taxes in the form of higher prices on the goods they buy. (In a more general model, a commodity tax may reduce the firms' profits. But, then it is the owners of the firms that bear the taxes in the form of lower dividends. Only people can bear taxes. Firms cannot.)

Total tax revenues are given by $T = \sum_{i=1}^{N} w \cdot \varphi_i \cdot t \cdot Q_i$. To compute the equilibrium level of tax revenues, use the market demand function

$$Q_i^D = \alpha_i \cdot \frac{w \cdot M + T}{p_i}$$

and the equilibrium price $p_i = w \cdot \varphi_i \cdot (1+t)$ to rewrite the total tax revenues as

$$T = \frac{t}{1+t} \cdot [w \cdot M + T].$$

Notice that demand depends on T since all tax revenues are paid out as income-support to the low-productive people. As a result T appears also on the right hand side of the equation. (The tax revenues are increasing in the amount consumes in the market, which is increasing in the income support to the low-productive people.) To take care of this interdependence, we need to solve the equation for T . Solving this expression for total tax revenues gives

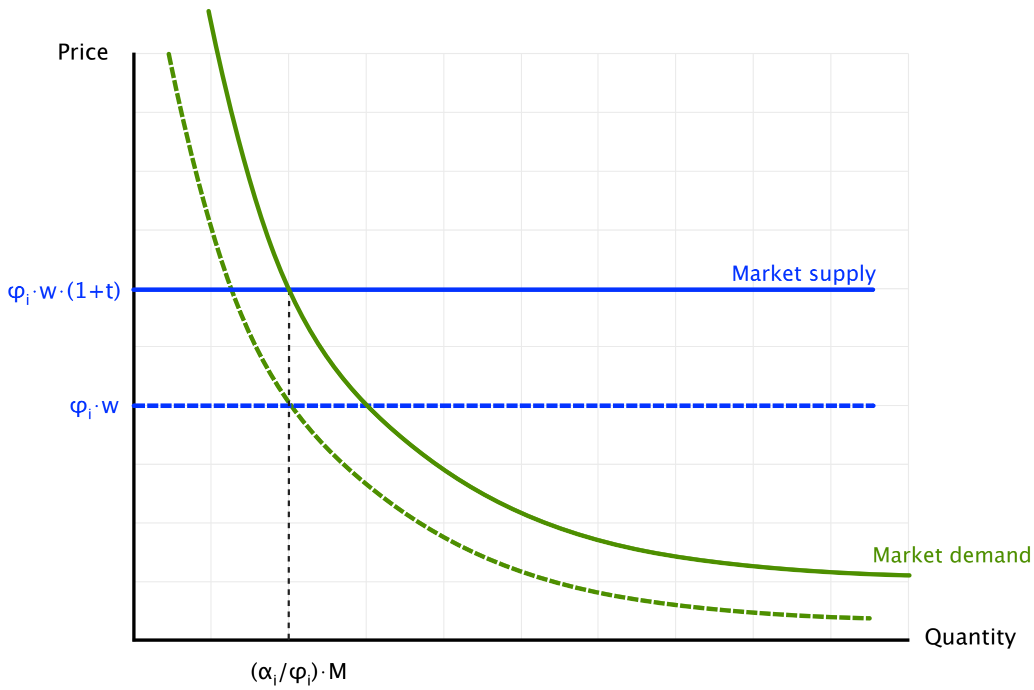
$$T = t \cdot w \cdot M .$$

This expression is interesting since it shows that the total tax revenues from a uniform commodity tax equals the total tax revenues that would have been the result of a uniform tax on income.

Now let us determine how much is consumed of each good in equilibrium. To do so, substitute the equilibrium price and the equilibrium total tax revenues back into the market demand functions. The result is:

$$Q_i^D = \frac{\alpha_i}{\varphi_i} \cdot M .$$

The remarkable feature of this expression is that total consumption of each good is independent of the tax. Expressed differently, total consumption of every good is the same as when there are no taxes. The equilibrium must therefore look as described by the figure below.



People with low productivity increase their demand for goods thanks to the income support that they receive from the government. This increase in demand is large enough to exactly offset the reduction in supply caused by the commodity tax. Even if prices (the prices of goods relative to the wage) are higher, total consumption is the same as before.

3.2.3 Efficiency

Despite the taxes, the market is efficient. To show this, we need to work with the conditions for efficiency stated in a previous section.

First notice that consumption is efficient. The reason whey all the $MRS_{ij}^{h} = MRS_{ij}^{g}$ conditions are fulfilled. The reason why all these conditions are fulfilled is that all consumers pay the same prices. To maximize their utilities they all adjust their consumption to equalize their marginal rates of substitution to (the negative of) the same relative prices.

Less obvious is that also production is efficient. But since prices are given by $p_i = w \cdot \varphi_i \cdot (1+t)$ it follows that

$$\frac{p_i}{p_j} = \frac{w \cdot \varphi_i \cdot (1+t)}{w \cdot \varphi_j \cdot (1+t)} = \frac{\varphi_i}{\varphi_j}.$$

And since the right hand side is identical to (minus) the marginal rate of transformation (i.e. $MRT_{ij} = -\varphi_i/\varphi_j$) it follows that also $MRT_{ij} = MRS_{ij}^g$.

The key point here is that the tax rate is the same on all goods.

If the tax rate would have been higher on good *i* than on *j*, the relative price would have been given by

$$\frac{p_i}{p_j} = \frac{w \cdot \varphi_i \cdot (1+t_i)}{w \cdot \varphi_j \cdot (1+t_j)} = \frac{\varphi_i \cdot (1+t_i)}{\varphi_j \cdot (1+t_j)} > -MRT_{ij},$$

meaning that the value of *i* in terms of *j* (MRS_{ij}) would have been larger than the cost of *i* in terms of *j* (MRT_{ij}). It would then have been possible to increase the welfare for all people by producing more of *i* and less of *j*. With different tax rates, the market would have been inefficient. We will return to this point soon.

3.2.4 Distribution

While total consumption is unaffected by the tax, the distribution of consumption between people with high and low productivity changes with the tax. A person with high productivity consumes

$$q_i^H = \frac{\alpha_i}{\varphi_i} \cdot \frac{1+\delta}{1+t}.$$

Clearly high productive people consume less of all goods in equilibrium, the higher the tax is. The reason is that their income remains constant, but the prices of all goods are increased.

A person with low productivity consumes

$$q_i^L = \frac{\alpha_i}{\varphi_i} \cdot \frac{1-\delta+2\cdot t}{1+t}.$$

Notice that the tax rate affects the low-productive people for two reasons. Exactly like people with high productivity, people with low productivity consume less as a result of a higher price on all goods. However, since the people with low productivity receive the tax revenues as income support, they will tend to consume more of all goods. The second effect dominates the first. Since total consumption of each good is unaffected by the tax and since the high-productive people consume less when the tax is increased, it follows that the low-productive people must consume more. (This can also be seen by taking the derivative of the expression above with respect to the tax rate.)

If the government imposes a sufficiently high tax rate, it can remove all inequality. The critical tax rate solves the following equation:

$$q_i^L = \frac{\alpha_i}{\varphi_i} \cdot \frac{1-\delta+2\cdot t}{1+t} = \frac{\alpha_i}{\varphi_i} \cdot \frac{1+\delta}{1+t} = q_i^H .$$

That is the tax rate must be set equal to $t=\delta$ to completely even out consumption between the two groups. Not surprisingly, the larger is the difference in productivity, the larger is the necessary tax to equalize consumption possibilities.

3.2.5 What is wrong with partial equilibrium analysis?

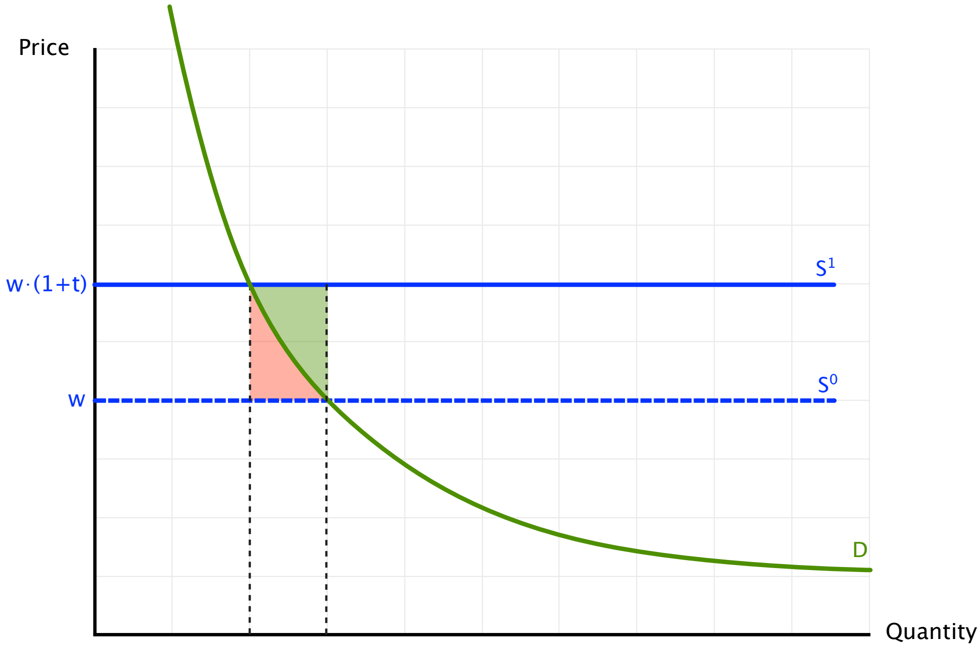
The striking conclusion from general equilibrium analysis is: If the government imposes a uniform tax on all goods, it can remove all inequality between people with different productivity, without reducing the efficiency of the markets. This conclusion stands in sharp contrast to the conclusion derived from partial equilibrium analysis.

Using partial equilibrium analysis, it appears that a tax on any individual good causes an inefficiency in that market. Imposing a tax on all goods should therefore cause inefficiencies in all markets. And the total inefficiency should simply be the sum of all inefficiencies. That conjecture is thus not correct, however.

So why is partial equilibrium analysis misleading?

Let us go through the standard partial equilibrium story again. To simplify the story assume that all goods require one unit of labor per unit of good, i.e. $\varphi_i = 1$.

Our goal is to assess the welfare effects of a tax on good *i*. At the outset there is no tax on *i*, but the government levies the same tax t on all other goods. Then, in market *i*, demand is illustrated by the D-curve and supply is illustrated by the S^0-curve in the following figure.



Now the government also levies the same tax on *i*. Supply is shifted up to S¹. The demand curve remains approximately constant. (Tax revenues are increased as are the transfers to those with low productivity. But if the number of goods is very large, only a very small share of the increased income is used in market *i*. Market demand is therefore approximately constant.)

Thus, the tax on good *i*, increases the equilibrium price and reduces the quantity produced and consumed of this good. The prices on all other markets remain constant at $p_j = w \cdot (1+t)$. It thus appears that the tax reduces welfare and that the loss is equal to the size of the red triangle in the figure. Not so.

The mistake here is that while the wage, w, reflects the firms' production costs, it does not reflect the *opportunity cost* of production to society. The social cost of producing a unit of *i* is not the labor used, but the loss of consumption of another good *j*.

The firms' production cost is equal to w for one unit of *i*. But producing an additional unit of *i* reduces production of *j* by one unit. And the consumers' valuation of this loss is equal to the price of that good, i.e. $p_j = w \cdot (1+t)$.

Expressed differently, the social opportunity cost is larger than the firms' production cost as a result of the tax on *j*.

The true opportunity cost of *i* is $p_j = w \cdot (1+t)$. Thus, when all other goods are taxed, it is actually the new supply curve that reflects the true opportunity cost of *i*, not the old. The reduced production and consumption of *i* therefore increases welfare and the gain is equal to the green triangle in the figure above.

The lesson is that when we assess welfare in partial equilibrium it is important to ask if the firms' production costs reflect the social opportunity cost of the resources they use. Do the prices of all inputs reflect their true value? If not, the analysis must correct for the difference.

3.3 Consumption vs. leisure

But we know that taxes cause distortions. And we actually already know the answer why. Taxation creates distortions because not all "goods" can be taxed the same way. To reinforce this point further, it is convenient to introduce a new "good" in the economy, namely leisure. People value leisure, i.e. the time not spent working, and it may loosely be viewed as just another good to be included in the utility function. But leisure is difficult to tax.

3.3.1 Demand

More formally, let l denote leisure and let preferences be given by the Cobb-Douglas function:

$$U = l^{\beta} \cdot \prod_{i=1}^{N} q_i^{\alpha} .$$

For simplicity, it is assumed that all the goods have the same exponent $\alpha_i = \alpha$ in the utility function. For convenience, we continue to normalize the exponents in the utility function by $\beta + N \cdot \alpha = 1$.

Ignoring profits (which are zero anyway), the income of high-productivity and low-productivity people are given by

$$I^H = (1-l)\cdot(1+\delta)\cdot w \qquad \text{and} \qquad I^L = (1-l)\cdot(1-\delta)\cdot w .$$

The first term $1-l$ is the amount of time that the person works. This is simply all available time (which is equal to one) minus leisure time l. The second term

$1 \pm \delta$ is the person's productivity, i.e. how much the person can produce in a time unit. Finally the wage w is the wage per "efficiency unit".

The budget constraint for a high-productive person is thus given by

$$\sum_{i=1}^{N} p_i \cdot q_i = (1-l) \cdot (1+\delta) \cdot w .$$

Each high-productive person maximizes his utility subject to this budget constraint. The Lagrangian is given by

$$L = l^{\beta} \cdot \prod_{i=1}^{N} q_i^{\alpha} + \lambda \cdot \left[w \cdot (1+\delta) \cdot (1-l) - \sum_{i=1}^{N} p_i \cdot q_i \right]$$

where λ is the Lagrange multiplier. All the first-order conditions for the individual goods can be written:

$$\frac{\partial L}{\partial q_i} = \alpha \cdot q_i^{-1} \cdot U - \lambda \cdot p_i = 0 \qquad \Leftrightarrow \qquad p_i \cdot q_i = \lambda^{-1} \cdot \alpha \cdot U .$$

The first-order condition for leisure is given by

$$\frac{\partial L}{\partial l} = \beta \cdot l^{-1} \cdot U - \lambda \cdot w \cdot (1+\delta) = 0 \qquad \Leftrightarrow \qquad w \cdot (1+\delta) \cdot l = \beta \cdot \lambda^{-1} \cdot U$$

Substituting these conditions into the budget constraint and solving for $\lambda^{-1} \cdot U$ yields $\lambda^{-1} \cdot U = (1+\delta) \cdot w$.

Substituting the Lagrange multiplier back into the first-order conditions for the individual goods shows that high-productive people's demand function is given by:

$$q_i^H = \alpha \cdot (1+\delta) \cdot \frac{w}{p_i} .$$

Substituting the Lagrange multiplier back into the first-order conditions for leisure gives the high-productivity people's demand for leisure:

$$l = \beta .$$

The supply of labor is simply the mirror image, i.e. $1 - \beta$.

The low-productive people's demand for goods and leisure can be derived exactly the same way. It is given by

$$q_i^L = \alpha \cdot (1-\delta) \cdot \frac{w}{p_i}$$

and

$$l = \beta .$$

Show this as an exercise!

Total market demand for good *i* is given by:

$$Q_i^D = \alpha \cdot \frac{w \cdot M}{p_i} .$$

3.3.1.1 What is the "price" of leisure?

Notice that those with high-productivity demand more goods than those with low-productivity, as a result of a higher income. In contrast, the two groups of people demand the same amount of leisure. Expressed differently, they work the same number of hours. One might have conjectured that those with high productivity should have demanded more leisure since leisure is just another "good."

The reason why they don't is that the people with high productivity also pay a higher price for leisure than those with low productivity. The "price" of leisure for someone with high productivity is $w \cdot (1+\delta)$ while the "price" for someone with low productivity is $w \cdot (1-\delta)$. Expressed differently: the opportunity cost of leisure is the income (or rather consumption) lost. Since people with high productivity have a higher income per hour, their opportunity cost of an hour of leisure is higher.

To see this, note that the budget constraints may be rewritten as

$$\sum_{i=1}^{N} p_i \cdot q_i + (1 \pm \delta) \cdot w \cdot l = (1 \pm \delta) \cdot w ,$$

where $(1 \pm \delta) \cdot w$ is the "virtual" income.

Thus, while leisure as just another good ($q_{N+1} = l$) and while we may think of $p_{N+1} = (1 \pm \delta) \cdot w$ as the price of this good, it is clear that the price of leisure depends on a person's productivity. (Formally, this means that we should think of leisure as different goods, depending on whose leisure we are talking about.)

3.3.2 Supply

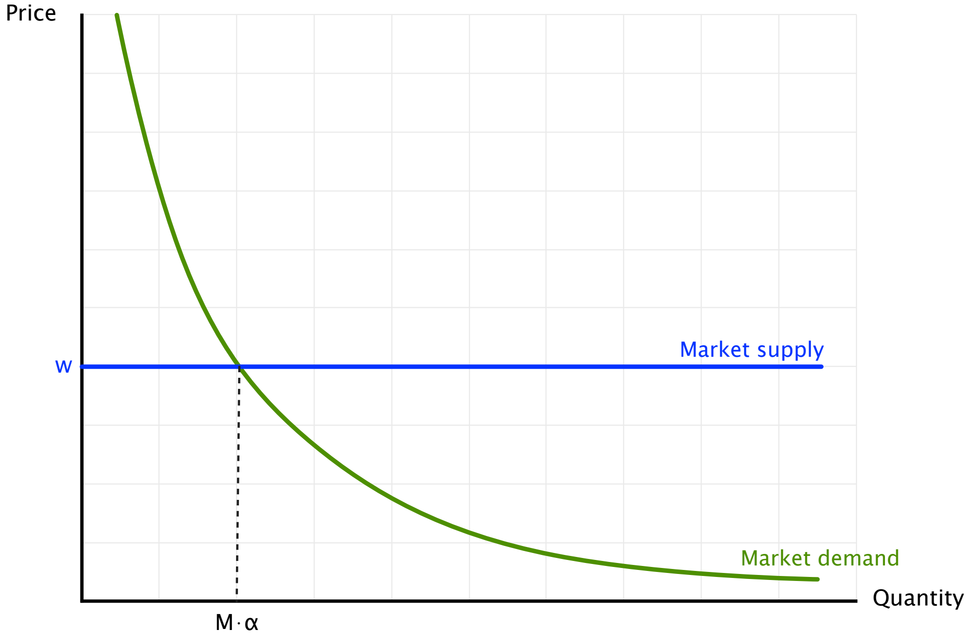
To simplify the computations we assume that all goods have the same factor requirement $\varphi_i = 1$.

To produce a unit of good *i* a firm can use $1/(1+\delta)$ time-units of high-productive labor. Since the wage for high-productive labor is $w \cdot (1+\delta)$ per time-unit, the marginal cost of producing a unit of good *i* using high-productive labor is $\frac{1}{1+\delta} \cdot \left[w \cdot (1+\delta) \right] = w$. A similar exercise shows that the marginal cost is the same if low-productive workers are used.

This means that the supply is perfectly elastic at the marginal cost, which is equal to the wage.

3.3.3 Equilibrium

The equilibrium in product market *i* is found by equating demand and supply in this market. Since supply is perfectly elastic at the marginal cost which equals w, it follows that the price must also equal this wage, i.e. $p_i = w$ in every product market.



Total consumption of any good is therefore $Q_i = M \cdot \alpha$.

Consumption is unevenly distributed between people depending on their productivity. High-productive people consume $q_i^H = \alpha \cdot (1+\delta)$ and low-productive people consume $q_i^L = \alpha \cdot (1-\delta)$.

3.3.3.1 Efficiency

The market is, however, efficient.

To prove this we need to work with the formal conditions for efficiency. First notice that production is efficient. For every person h and for all pairs of (market) goods *i* and *j*, $MRS_{ij}^h = MRT_{ij}$. Second note that consumption is efficient. For all pairs of people and for all pairs of (market) goods *i* and *j*, $MRS_{ij}^h = MRS_{ij}^g$.

Finally, note that all people also chose an efficient amount of leisure. Expressed differently, they strike the right balance between leisure and (market) goods. In order to maximize utility, any high productive person makes sure that $MRS_{i,leisure}^h \equiv -\frac{\partial U^h / \partial q_i}{\partial U^h / \partial l} = -\frac{p_i}{(1+\delta)\cdot w}$. Moreover, firms produce $1+\delta$ units of *i* per unit of time that a high productive person works. Therefore, the cost of a unit of *i* in terms of leisure is $MRT_{i,leisure}^{h} = -\frac{1}{1+\delta}$. Since firms are price takers $p_i = w$. It follows that the marginal rate of transformation is equal to the marginal rate of substitution.[4] Exercise: Show that also people with low productivity chose an efficient amount of leisure.

3.4 Redistribution and commodity taxation II

Again, the government aims to even out consumption possibilities. The government imposes a proportional tax t that all firms have to pay on all the goods they sell. The tax revenues T are distributed equally among all people with a low income.

3.4.1 Income

High-productive people continue to receive an income of $I^H = w\cdot(1+\delta)\cdot(1-l)$ but the low-productive now receive $I^L = w\cdot(1-\delta)\cdot(1-l)+\frac{T}{M/2}$.

3.4.2 Demand

Going through the individual optimization problems shows that high-productive people's demand function is given by:

$$q_i^H = \alpha\cdot(1+\delta)\cdot\frac{w}{p_i}.$$

and that demand for leisure is given by:

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[4] Note that it is not meaningful to compare two individuals' marginal rates of substitution between some (market) good *i* and leisure. The formula $MRS_{apple,leisure}^{high-productive} < MRS_{apple,leisure}^{low-productive}$ does not signify an inefficiency. In fact, it does not signify anything since leisure cannot be transferred like apples. If we would ask a high-productive person to give a low-productive person an apple and in exchange ask the low-productive person to do the high-productive person's work for *x* hours, the total production of goods (say apples) would be reduced at the same time.

$$l = \beta .$$

To find the demand for those with low productivity, we go through the whole optimization process again. The Lagrangian is given by

$$L = l^{\beta} \cdot \prod_{i=1}^{N} q_i^{\alpha} + \lambda \cdot \left[w \cdot (1-\delta) \cdot (1-l) + \frac{T}{M/2} - \sum_{i=1}^{N} p_i \cdot q_i \right]$$

where λ is the Lagrange multiplier. All the first-order conditions for the individual goods can be written:

$$\frac{\partial L}{\partial q_i} = \alpha \cdot q_i^{-1} \cdot U - \lambda \cdot p_i = 0 \qquad \Leftrightarrow \qquad p_i \cdot q_i = \alpha \cdot \lambda^{-1} \cdot U .$$

The first-order condition for leisure is given by

$$\frac{\partial L}{\partial l} = \beta \cdot l^{-1} \cdot U - \lambda \cdot w \cdot (1-\delta) = 0 \qquad \Leftrightarrow \qquad w \cdot (1-\delta) \cdot l = \beta \cdot \lambda^{-1} \cdot U$$

Substituting these conditions into the budget constraint gives

$$\sum_{i=1}^{n} p_i \cdot q_i + w \cdot (1-\delta) \cdot l = w \cdot (1-\delta) + \frac{T}{M/2} .$$

Solving for $\lambda^{-1} \cdot U$ yields

$$\lambda^{-1} \cdot U = w \cdot (1-\delta) + \frac{T}{M/2} .$$

Substituting the Lagrange multiplier back into the first-order conditions for the individual goods shows that low-productive people's demand function is given by:

$$q_i^L = \alpha \cdot \frac{w \cdot (1-\delta) + \frac{T}{M/2}}{p_i} .$$

The demand for leisure is given by

$$l = \beta \cdot \frac{w \cdot (1-\delta) + \frac{T}{M/2}}{w \cdot (1-\delta)} = \beta + \beta \cdot \frac{\frac{T}{M/2}}{w \cdot (1-\delta)} .$$

Notice that the income support from the government means that people with low productivity consume more goods than otherwise. They also consume more leisure than otherwise. Expressed differently they work less.

Total market demand for good *i* is given by:

$$Q_i^D = q_i^H \cdot \frac{M}{2} + q_i^L \cdot \frac{M}{2} = \alpha \cdot \frac{w \cdot M + T}{p_i} \cdot .$$

As before, market demand is determined by total consumer income in the country, i.e. $w \cdot M + T$.

3.4.3 Supply

The commodity tax implies that the firms' marginal cost is increased to $w \cdot (1+t)$. Expressed differently, market supply in all goods markets is shifted up by the commodity tax.

3.4.4 Equilibrium

Since market supply is perfectly elastic at the constant marginal cost and since the marginal cost is increased in proportion to the commodity tax, it follows that also the equilibrium price in every market must be increased by the same rate, i.e. $p_i = w \cdot (1+t)$.

Total tax revenues are given by $T = \sum_{i=1}^{N} w \cdot t \cdot Q_i$. To compute the equilibrium level of tax revenues, use the market demand function

$$Q_i^D = \alpha \cdot \frac{w \cdot M + T}{p_i}$$

and the equilibrium price $p_i = w \cdot (1+t)$ to rewrite the total tax revenues as

$$T = (N \cdot \alpha) \cdot \frac{t}{1+t} \cdot [w \cdot M + T].$$

Solving this expression for total tax revenues and using $\beta = 1 - N \cdot \alpha$ gives

$$T = \frac{(1-\beta) \cdot t}{1 + \beta \cdot t} \cdot w \cdot M .$$

Notice that if $\beta = 0$ this expression simplifies to the same expression as when people do not care about leisure.

Now let us determine how much is consumed of each good in equilibrium. To do so, substitute the equilibrium price and the equilibrium total tax revenues back into the market demand functions. The result is:

$$Q_i^D = \frac{\alpha}{1+\beta \cdot t} \cdot M .$$

It is straightforward to verify that the higher the tax rate t, the higher are the total tax revenues T in this economy. (This is not always the case since people work less and consume less goods when taxes are higher.) It is also straightforward to verify that the government can reduce any differences in consumption or utility between those with high productivity and those with low productivity by setting a sufficiently high tax rate.

3.4.5 Efficiency

When there is a tax on all market goods, but not on leisure, people are not led to strike the right balance between consuming goods and leisure.

To see this, recall that a high productive person works and eats apples to make sure that

$$MRS^h_{apple,leisure} \equiv -\frac{\partial U^h / \partial q_{apple}}{\partial U^h / \partial l} = -\frac{p_{apple}}{(1+\delta) \cdot w} .$$

Moreover, a high productive person produces $1+\delta$ apples per time-unit worked. Therefore, the cost of an apple in terms of leisure is

$$MRT^h_{apple,leisure} = -\frac{1}{1+\delta} .$$

As a result of taxation, the price of apples is given by $p_{apple} = w \cdot (1+t)$. It follows that

$$-MRS^h_{apple,leisure} = \frac{1+t}{1+\delta} > \frac{1}{1+\delta} = -MRT^h_{apple,leisure} .$$

That is, the value of apples in terms of leisure is $(1+t)/(1+\delta)$ while the cost of apples in terms of leisure is only $1/(1+\delta)$. A high-productive person should thus produce and eat at least one more apple. But people are not led to consume an additional apple, since they pay taxes on apples but not on leisure.

The same relation is true for those with low productivity. That is, all people are led to consume too few apples (and too little of all other market goods) and to consume too much leisure.

Another way to come to the same conclusion is to do the calculations in terms of money. Note that the value of an apple in terms of money is given by its price, $p_{apple} = w \cdot (1+t)$. The cost of producing the apple in terms of money is the wage per time-unit $w \cdot (1+\delta)$ times the time it takes to produce it $1/(1+\delta)$, i.e. w. Since the (monetary) value of consuming an extra apple is larger than the (monetary) cost of producing it, more apples should have been produced.

If it would be possible to tax leisure the same way as goods are taxed, there would be no distortion. Actually, we have already proved this above. Think of the model without leisure, but now reinterpret good $i=1$ as leisure. Then recall that when all goods, including $i=1$, are taxed the same way, the markets are fully efficient. Thus, the reason why commodity taxation is distortive is that governments cannot tax one of the "goods," namely leisure.

An income tax would have the same effect. The income tax reduces the price of labor, which is the same as reducing the price of leisure. Thus the relative price of consumption is increased.

3.5 Conclusion

3.5.1 Why taxes are distortive

It is widely understood that commodity taxes cause inefficiencies. But it is less widely understood that commodity taxes are not inefficient *per se*. It is the *differences* in tax rates between different goods that cause the inefficiencies. With different tax rates, the relative prices of goods deviate from the relative production costs. Raising a commodity tax that is lower than other taxes may actually improve efficiency.

The basic problem is that the government *cannot* tax all goods equally. In practice, the most important limitation is probably that the government cannot tax leisure to the same extent that goods sold in the market can be taxed.

Therefore commodity taxes distort peoples' trade-off between consuming the goods that they buy in the market and leisure (and the goods that they produce at home). Taxes make market goods "artificially" expensive in relation to leisure and home goods. Therefore people will tend to consume too little market goods and too much leisure. Expressed differently, they will tend to work too little.[5]

A tax on income causes the same distortion. A tax on income reduces the price on labor. Expressed differently, an income tax reduces the price on leisure relative the price of all other goods in the economy. Therefore people will tend to consume too much leisure and too little goods. Expressed differently, the will work too little.

Again, the most important reason why taxation is distortive is that governments in practice cannot tax leisure.

3.5.2 Why we need general equilibrium theory

Using partial equilibrium analysis, it appears that any tax on any individual good causes an inefficiency in that market. Imposing a tax on all goods should therefore cause inefficiencies in all markets. And the total inefficiency should simply be the sum of all inefficiencies. We have seen that this reasoning is incorrect.

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[5] In the model considered here, the high-productive people do not reduce their supply of labor (i.e. they do not increase their leisure) in response to the tax. The reason is that the taxes reduce the potential income (w) that high-productive people can use to consume goods and leisure. This so-called income effect counteracts the effect of the change in relative prices. But the point is that, from a welfare point of view, they should have increased it. These are two different comparisons.

A common mistake when assessing welfare using only partial-equilibrium analysis is to assume that all input prices reflect the cost of the resources used in producing the good under study. While the wage reflects the firms' costs of using labor, it does not necessarily reflect the *opportunity cost* of using labor to society. The social cost of producing a unit of some good may not be the labor used, but the loss of consumption of some other good.

For example, assume that the government levies the same tax on all goods (including leisure) except cars. If the firms' marginal costs of producing cars is € 20 000 and their marginal cost of producing mopeds is € 5 000, the country could produce an additional car by producing four mopeds less. The value of each of these mopeds to the consumers is the price they pay for the mopeds. Suppose that this price is € 6 000 as a result of the 20 percent tax on mopeds, then the social opportunity cost of an additional car is € 24 000. Since cars are not taxed, they sell at € 20 000. Thus the last cars produced and consumed impose a loss to society of € 4 000. It might then be better to also levy a 20-percent tax on cars. Then, fewer cars will be produced and consumed. Instead more mopeds will be produced and consumed. The new tax on cars results in a net gain to society.[6]

On the other hand, if the government cannot tax leisure, then even a uniform tax on all market goods will distort people's tradeoff between goods and leisure. So if the government would reduce the tax on all market goods simultaneously, more will be produced and consumed of all market goods and there will be less

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[6] More formally, if the marginal cost of producing *i* is MC_i and the marginal cost of producing *j* is MC_j a country could produce one more unit of *i* by producing MC_i / MC_j units fewer of good *j*. Since the consumers' valuation of *j* is p_j the opportunity cost of *i* is $p_j \cdot \left(MC_i / MC_j\right)$. When there is a tax on *j*, the price of *j* deviates from the marginal cost of producing the good, that is $p_j = MC_j \cdot \left(1 + t_j\right)$. Therefore the opportunity cost of *i* is $\left(1 + t_j\right) \cdot MC_i$. Expressed differently, the social opportunity cost of *i* is larger than the firms' cost of producing *i* as a result of the tax on *j*.

leisure time. Since the value of e.g. an extra apple is $p_{apple} = (1+t) \cdot MC_{apple}$ while the cost of producing it is only MC_{apple}, the change results in a net gain.

Note the difference between the two experiments. In the first experiment we investigated the effect of increasing the tax of a single good when all other goods are already taxed at the same rate. In this case, a tax increase creates a welfare gain. In the second experiment we discussed the removal of a uniform commodity tax when leisure could not be taxed. In this case, it is a reduction of taxes that creates a welfare gain.

The latter experiment does not entail that governments should remove all taxation, since they cannot tax leisure. Governments use tax revenues to redistribute income between people and to produce public goods. The benefits of these policies may be larger than the cost of the inefficiencies that result from taxation. What the latter experiment does show, however, is that the cost of public policies is higher than the government budget suggests.[7]

3.5.3 A partial solution to distortive taxes

The government can partially solve the problem arising from not being able to tax leisure directly by taxing leisure indirectly. The government then needs to tax different goods differently. A first strategy is to impose higher taxes on goods that are complements to leisure. Examples might include entertainment, Holiday travels, liquor and fine food. A second strategy is to impose lower taxes on goods that are substitutes to leisure (i.e. complements to work). Examples might include taxes on cleaning services or working clothes. In fact, there may even be a reason to subsidize some services such as childcare and public transportation.

An important drawback with the idea of taxing leisure indirectly is that is that it makes the tax system more complicated and more costly to administrate. Another drawback is that it may increase the possibility for people to evade

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[7] Say that the government increases taxes to finance a small project requiring the same small amount of resources that could produce an apple. Then the government budget increases by MC_{apple}. The opportunity cost of the project, however, is the apple foregone, i.e. $p_{apple} = (1+t) \cdot MC_{apple}$, which is larger.

taxes. Some people may for example be able to redefine (high-tax goods such as) Holiday travelling as (low-tax goods such as) commuting.

Note that to analyze the idea of taxing leisure indirectly in detail, we need to continue to work with general equilibrium models since the whole idea involves analyzing several market (goods that are substitutes and complements to leisure) at the same time.

3.5.4 Topics for discussion

3.5.4.1 Current differences in taxes

There are many examples of tax differences in Sweden today. What are the pros and cons of these differences? Think about the following examples:

1. The value added tax is lower on food and books than on most other goods.
2. There is a special tax on alcohol and tobacco.
3. Goods produced outside the European Union are subject to import taxes (tariffs).
4. The so-called jobskatteavdrag (approximately "earned income tax credit") implies that people pay a lower tax on the incomes they earn by working during the year than the tax senior citizens pay on their pension incomes.

3.5.4.2 Public transport

Sketch a formal proof that subsidizing public transport may improve efficiency if it is practically impossible to make cars bear their own cost. (Hint: Read section 2.2).

3.5.4.3 Dutch disease

A large natural gas field was discovered in the Netherlands in the late 1950-ies. Fifteen years later it was apparent that the country had experienced a decline in its manufacturing sector. What might be the reason? (Hint: This is a very difficult question, but try to analyze it, thinking about labor as a resource constraint.[8])

3.5.4.4 Monopoly distortions

In markets with limited competition, firms can raise prices above their marginal costs. What would be the social cost of such market power if mark-ups would be the same in all markets? (Hint: Notice the similarities between mark-ups and taxes.)

[8] When you have thought about it for a while, you may read the paragraph about direct and indirect deindustrialization at http://en.wikipedia.org/wiki/Dutch_disease.