Johan Stennek, 14 October 2014

AMT – Johan's Problem Set 1

Notice:

- Question 1 is only for preparation and will not be discussed in class.
- Question 7 will only be discussed in class if time permits.

1 How quantity demanded and supplied vary with price

The quantity that consumers wish to buy and consume of a good (Q) typically depends on the price of that good (P). We represent this relationship by the demand function: $Q = D(P)$, with first derivative $D_p(P)$.

- Assume that price is increased by ΔP. How many units less will the consumers then buy?
- Assume that price is increased by $\Delta P / P$ percent. By how many percent will the consumers then reduce their consumption? Describe this result in ordinary language.

The quantity that firms wish to produce and sell of a good (Q) typically depends on the price of that good (P). We represent this relationship by the supply function: $Q = S(P)$, with first derivative $S_p(P)$.

- Assume that price is increased by ΔP. How many units more will the firms then wish to sell?
- Assume that price is increased by $\Delta P / P$ percent. By how many percent will the firms then increase their production? Describe this result in ordinary language.

2 Effect of entry on price

Your consultancy firm O'Kinky is contacted by representatives of an industry that is likely to meet increased competition in the near future. Their intelligence

reports reveal that the number of firms is likely to be increased by 5 % as a result of new entry. They wish to hire you to predict by how many percent price is likely to drop.

To answer the question, you build a simple model of the industry. You denote the market demand function by $D(P)$. For simplicity you assume that there are N identical firms, and denote their individual supply functions by $s(P)$. You derive the market supply function by horizontal summation. It is given by $S(P) = N \cdot s(P)$. You find econometric evidence that reveals that the elasticity of supply in this industry is around 8 and that the elasticity of demand is around -2.

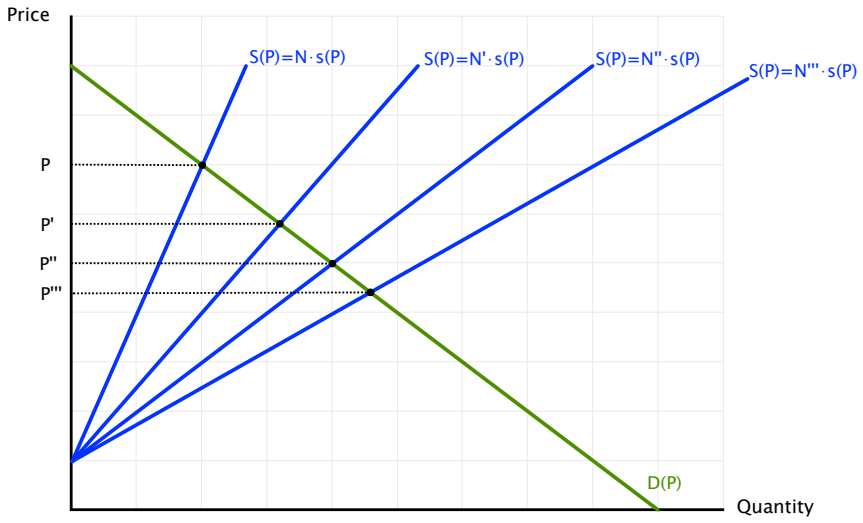
You recall that the equilibrium price is defined as the price that allows all firms and all consumers to realize their consumption and production plans at the same time. Expressed differently, it is the price that equates market demand and market supply.

The equilibrium price is therefore the price P that solves the following equation:

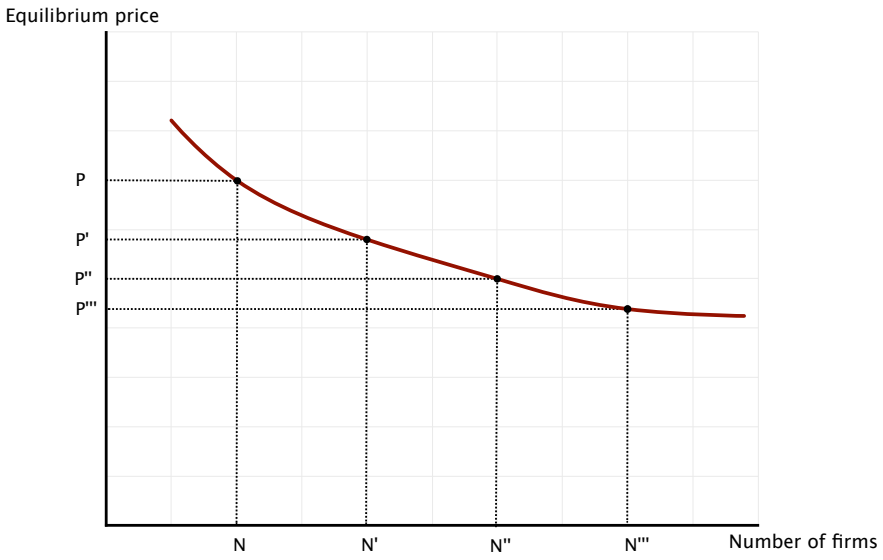
$$D(P) = N \cdot s(P).$$

Notice that this is a single equation in one unknown, namely the price, P. The equation also contains an exogenous variable, namely the number of firms, N.

You argue that if the number of firms changes, as a result of entry or exit, there will be a different price that clears the market. For instance if the number of firms increases to N' the equilibrium price is reduced to P' as given by $D(P') = N' \cdot s(P')$. This is described by the following figure:



Notice that there is an equilibrium price associated with any number of firms and that the price is decreasing as the number of firms is increased. This relation is described by the following figure:



We may therefore say that the "supply-equals-demand-equation" defines the equilibrium price as a function of the number of firms.

To investigate by how many percent the price is changed ($\Delta P / P$) when the number of firms is changed by $\Delta N / N$ percent, you start out by differentiating the "supply-equals-demand-equation". You denote the first derivative of the demand function with respect to price by $D_p(P)$ and the first derivative of firm supply with respect to price by $s_p(P)$. How do you proceed then?

3 Subsidizing milk I

The government aims to increase the consumption of milk, in an effort to improve public health. For this reason the government plans to introduce a consumption subsidy. The current price of milk, absent any subsidies is € 1.4 per liter. The effectiveness of the policy is determined by how much the consumer price falls. Since the government's target is to push it down to € 1.1, it appears that the subsidy should be € 0.3. You work as an economist at the ministry of finance and object that it isn't that simple. You argue that the price may be affected by the subsidy. As a result, you are given the job to compute how large the subsidy must be to meet the government's target.

To answer the question, you start out setting up a simple model. Despite the fact that there are rather few sellers of milk in the country, you believe that it is reasonable to think of the milk market as perfectly competitive. You denote the producer price by P and the consumers by P^C. The consumer price is simply the producer price minus the subsidy, $P^C = P - X$. You denote the demand for milk by $D\left(P^C\right)$ and supply by $S(P)$, with partial derivatives $D_p\left(P^C\right)$ and $S_P(P)$. The equilibrium price is given by $S(P) = D(P - X)$.

You gather some econometric evidence that suggests that the elasticities of demand and supply are given by

$$\varepsilon_D = \frac{D_p\left(P^C\right)\cdot\left(P^C\right)}{D\left(P^C\right)} = -0.5$$

$$\varepsilon_S = \frac{S_p(P)\cdot P}{S(P)} = 0$$

When you have your results, you need to communicate them to the minister who knows some economics, but has little time and taste for mathematical models. Therefore you decide to explain your analysis using a standard demand – supply diagram.

4 Subsidizing milk II

While working on your report to the minister, you realize that the econometric evidence on the elasticity of supply is based on the short-run supply curve. What it captures is simply the fact that it is difficult to increase the number of milk cows within a year. But in a time frame of, say, four years, you figure it would be possible to vary the number of cows freely. How would you represent this idea within the model you are working with? By how much would you need to subside milk to reduce the price by € 0.3 in the long run? Explain your results also in a figure.

5 Long run equilibrium price of revolutionary light bulbs

A new revolutionary light bulb is invented by researchers at a public university. The new light bulb requires extremely little energy and it is generally expected that it will take over the whole world market. The university, which is only motivated by the environmental gains, makes the technology public knowledge and does not file for patent protection. Other benefits is that the new light bulb can be produced locally in small-scale factories and that it is quite small, with a diameter of only 2 cm.

You are hired to compute the expected long run equilibrium price. You should base your analysis on the following description of the market. (Hint: what information do you really need?)

Technology

Every firm entering the market has access to the same production technology. The technology can be described by the production function $Q=(L-z)^{1/2}+(K-z)^{1/2}$ where L is labor input, K is capital input and $z>0$ is the minimum input to start production. Alternatively the technology can be described by the long-run cost function $C(Q)=z\cdot(w+r)+Q^2$ where w and r denote the price of labor and capital. The short-run cost function is given by

$$C(Q,\bar{K}) = w \cdot z + w \cdot \left[Q - \left(\bar{K} - z \right)^{1/2} \right]^2 + r \cdot \bar{K}$$

given a capital stock of $\bar{K}$.

Demand

Even if the new light bulb will take over the whole world market, the exact level of demand is uncertain, as with all new products.

Some experts have estimated that demand for the light bulb will be given by $D(p) = \sin\left\{\xi^{-1}\right\} \cdot c \cdot p^{-1}$ where c is the speed of light and ξ is the radius of the light bulb in centimeters.

Other experts believe that demand will be given by $D(p) = c + (\xi - 1) \cdot \sqrt{\Omega} + p^{-1}$ where Ω is the smallest number such that $\Omega = a + b + c$ where *a*, *b*, and *c* are positive integers satisfying $a^n + b^n = c^n$ for some integer $n > 2$.

Yet other experts argue that the two demand functions are equally likely according to their own subjective probability distribution.

6 Efficiency

Consider a perfectly competitive market.

- Argue that it would be impossible to reallocate consumption of the good between different consumers to improve overall welfare in society. What welfare criterion do you use?
- Argue that it would be impossible to reallocate production of the good between different producers to improve overall welfare in society. What welfare criterion do you use?
- Argue that it would be impossible to increase or reduce total production and consumption of the good to improve overall welfare in society. What welfare criterion do you use?

7 Cost of public funds

The minister of infrastructure investigates the costs and benefits of modernizing all the roads in the country. The ministry commissions a report from a marketing consultancy to estimate the value of the modernization program. The consultancy collects survey data indicating that all citizens have the same valuation and that the total value to society would be B. Another report from an engineering firm indicates that the production cost would be C. Since $C < B$, the minister of infrastructure proposes that the Government should go ahead with the program.

The powerful minister of finance is not convinced, however. To finance the cost of the road, the Government would have to levy a tax. The only possibility is to

levy the tax on good G, which is consumed by all the citizens in the country. The ministry of finance commissions a report on this market from an economic consultant. The consultant estimates that demand for this good is linear and given by $Q^D = 7 - P$ and that supply is also linear and given by $Q^S = 1 + P$.

The minister of finance asks you for advice: Should the program be initiated or not? You may answer the question graphically and assume that the cost of the program is $C = 4$. (Hint: Start out computing the tax rate *t*, needed to finance the program.)