School of Business, Economics and Law
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General Equilibrium

II: Inequality, redistribution & tax distortions

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Agenda

- Inequality and taxes (first hour)
- Leisure (second hour)

A simple model of inequality

Inequality

- Before
  - All people identical
  - Produce $1/\phi_i$ units of good i per time-unit
  - Earn w per time-unit

Inequality

- Two types of people
  - High productive
    - Produce $(1+\delta) / \phi_i$ units of good i per hour worked
    - Half the population
  - Low productive
    - Produce $(1–\delta) / \phi_i$ units of good i per hour worked
    - Other half
- Q: Compute average productivity
  - Average $= ½ (1+\delta) / \phi_i + ½ (1-\delta) / \phi_i = 1/ \phi_i$
  - same as before

Inequality

- Wages
  - Hourly wages are proportional to productivity
    - If not, all firms would prefer to use only one type of labor
    - Allows us to analyze the model as if there is only one labor market

- Incomes (since all work 1

  - $I^H = w \bullet (1+\delta)$

  - $I^L = w \bullet (1-\delta)$

From now on, don't include profits, since they will be zero anyway

Inequality

- Demand (as before, but different incomes)

$$q_i^H = \alpha_i \cdot \frac{w \cdot (1+\delta)}{p_i}$$

$$q_i^L = \alpha_i \cdot \frac{w \cdot (1-\delta)}{p_i}$$

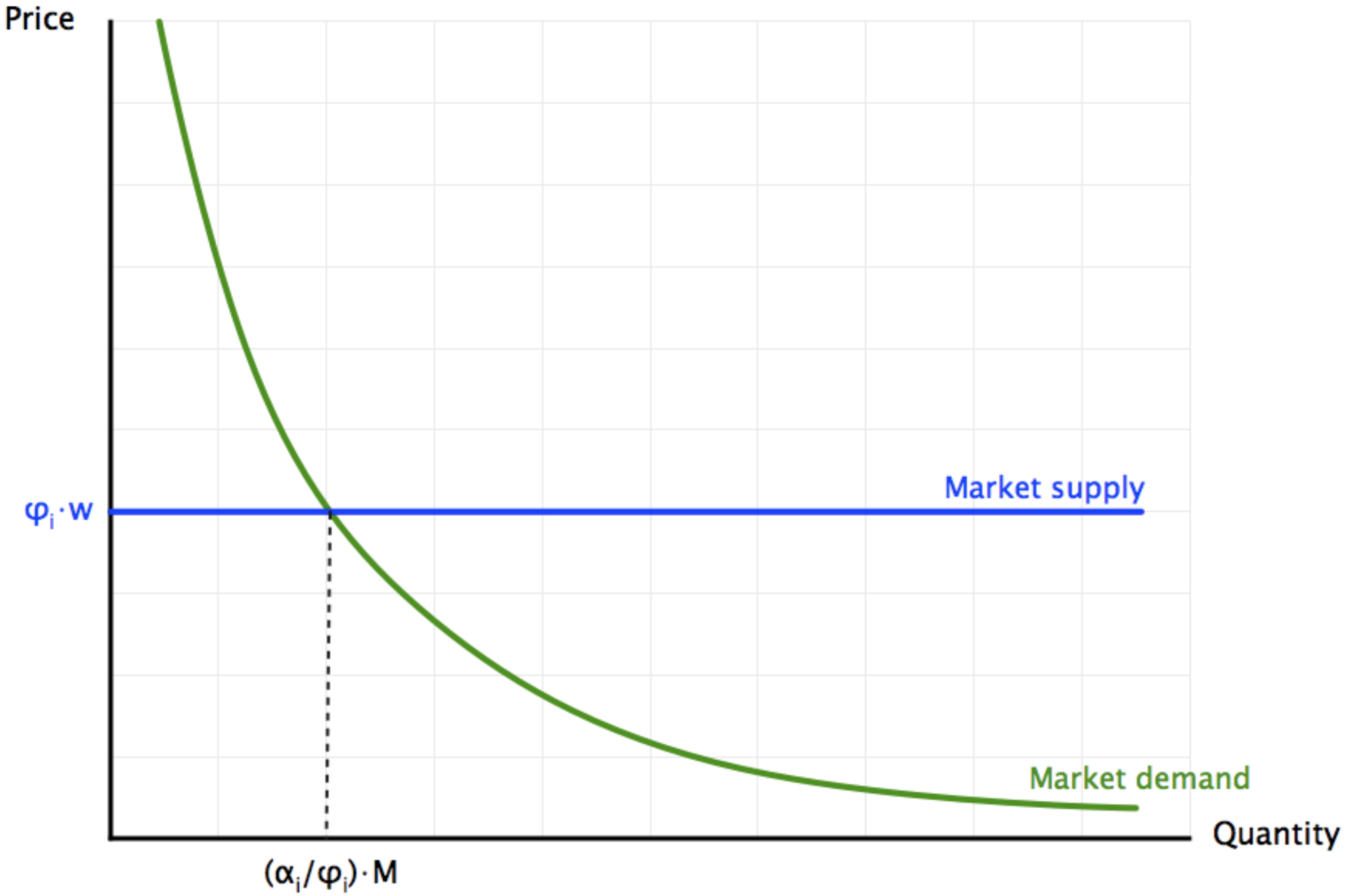
- Market demand (as before)

$$Q_i = \alpha_i \cdot \frac{w}{p_i} \cdot M$$

Inequality

- Market supply (as before)
  - Perfectly elastic at marginal cost: $\phi_i \bullet w$
  - Compute marginal cost:
    - It takes $l = \phi_i / (1 + \delta)$ high productive people to produce one unit of good i
    - Their wage is $w \bullet (1+\delta)$ per time-unit
    - The marginal cost is the product: $w \bullet \phi_i$

Inequality



Price
$\varphi_i \cdot w$
Market supply
Market demand
Quantity
$(\alpha_i/\varphi_i) \cdot M$

Inequality

- Equilibrium (as before)
  - $p_i = \phi_i \bullet w$
  - $Q_i = (\alpha_i/\phi_i) \bullet M$
- But with different distribution of consumption
  - $q_i^H = (\alpha_i/\phi_i) \bullet (1+\delta)$
  - $q_i^L = (\alpha_i/\phi_i) \bullet (1-\delta)$

Inequality

- Q: Efficiency
  - MRS = MRS since all consumers pay same prices
  - MRS = MRT since also firms are price-takers
- Distribution
  - People consume goods in proportion to their productivity

Redistribution

Redistribution

- Policy intervention
  - Same tax rate, t, on all goods
  - Total tax revenues, T, distributed as income support to the M/2 people with low income

Redistribution

- Marginal cost
  - Marginal production cost $w \cdot \varphi_i$
  - Tax per unit sold proportional to MC $w \cdot \varphi_i \cdot t$
  - Same as fixed proportion of price ("ad valorem")

Redistribution

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- Market supply
  - Perfectly elastic at total MC $w \cdot \varphi_i \cdot (1 + t)$

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  - Same as fixed proportion of price ("ad valorem")
- Market supply
  - Perfectly elastic at total MC $w \cdot \varphi_i \cdot (1+t)$
- Equilibrium price

$$p_i = w \cdot \varphi_i \cdot (1+t)$$

Redistribution

- Incomes

$$I^L = w \cdot (1-\delta) + \frac{T}{M/2}$$
$$I^H = w \cdot (1+\delta)$$

Redistribution

- Incomes

$$I^L = w \cdot (1-\delta) + \frac{T}{M/2}$$

$$I^H = w \cdot (1+\delta)$$

- Market demand

$$Q_i^D = \alpha_i \cdot \frac{I^H}{p_i} \cdot \frac{M}{2} + \alpha_i \cdot \frac{I^L}{p_i} \cdot \frac{M}{2} = \alpha_i \cdot \frac{w \cdot M + T}{p_i}$$

Redistribution

- Incomes

$$I^L = w \cdot (1 - \delta) + \frac{T}{M/2}$$

$$I^H = w \cdot (1 + \delta)$$

- Market demand

$$Q_i^D = \alpha_i \cdot \frac{I^H}{p_i} \cdot \frac{M}{2} + \alpha_i \cdot \frac{I^L}{p_i} \cdot \frac{M}{2} = \alpha_i \cdot \frac{w \cdot M + T}{p_i}$$

- Total tax revenue

$$T = \sum_{i=1}^{N} (w \cdot \varphi_i \cdot t) \cdot Q_i$$

Redistribution

- Equilibrium quantity and tax revenues

$$Q_i^D = \alpha_i \cdot \frac{w \cdot M + T}{p_i} \quad \text{where} \quad p_i = w \cdot \varphi_i \cdot (1 + t)$$

$$T = \sum_{i=1}^{N} w \cdot \varphi_i \cdot t \cdot Q_i$$

Quantities depend on total tax revenues

Total tax revenues depend on quantities

Redistribution

Substitute quantities into T

$$Q_i^D = \frac{\alpha_i}{\varphi_i} \cdot \frac{w \cdot M + T}{w \cdot (1 + t)}$$

$$T = \sum_{i=1}^{N} w \cdot \varphi_i \cdot t \cdot \left\{ \frac{\alpha_i}{\varphi_i} \cdot \frac{w \cdot M + T}{w \cdot (1 + t)} \right\}$$

Redistribution

Substitute quantities into T

$$Q_i^D = \frac{\alpha_i}{\varphi_i} \cdot \frac{w \cdot M + T}{w \cdot (1+t)}$$

$$T = \sum_{i=1}^{N} w \cdot \varphi_i \cdot t \cdot \left\{ \frac{\alpha_i}{\varphi_i} \cdot \frac{w \cdot M + T}{w \cdot (1+t)} \right\} = \frac{t}{1+t} \cdot (w \cdot M + T) \cdot \sum_{i=1}^{N} \alpha_i$$

Redistribution

Substitute quantities into T

$$Q_i^D = \frac{\alpha_i}{\varphi_i} \cdot \frac{w \cdot M + T}{w \cdot (1+t)}$$

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Single equation to be solved for equilibrium T

$$T = \frac{t}{1+t} \cdot (w \cdot M + T) \qquad \Rightarrow \qquad T = t \cdot w \cdot M$$

Redistribution

Substitute quantities into T

$$Q_i^D = \frac{\alpha_i}{\varphi_i} \cdot \frac{w \cdot M + T}{w \cdot (1+t)}$$

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Single equation to be solved for equilibrium T

$$T = \frac{t}{1+t} \cdot (w \cdot M + T) \qquad \Rightarrow \qquad T = t \cdot w \cdot M$$

Substitute back into quantities

$$Q_i^D = \frac{\alpha_i}{\varphi_i} \cdot \frac{w \cdot M + \{t \cdot w \cdot M\}}{w \cdot (1+t)} = \frac{\alpha_i}{\varphi_i} \cdot M$$

Redistribution

Equilibrium

$$p_i = w \cdot \varphi_i \cdot (1 + t)$$

$$Q_i = \frac{\alpha_i}{\varphi_i} \cdot M$$

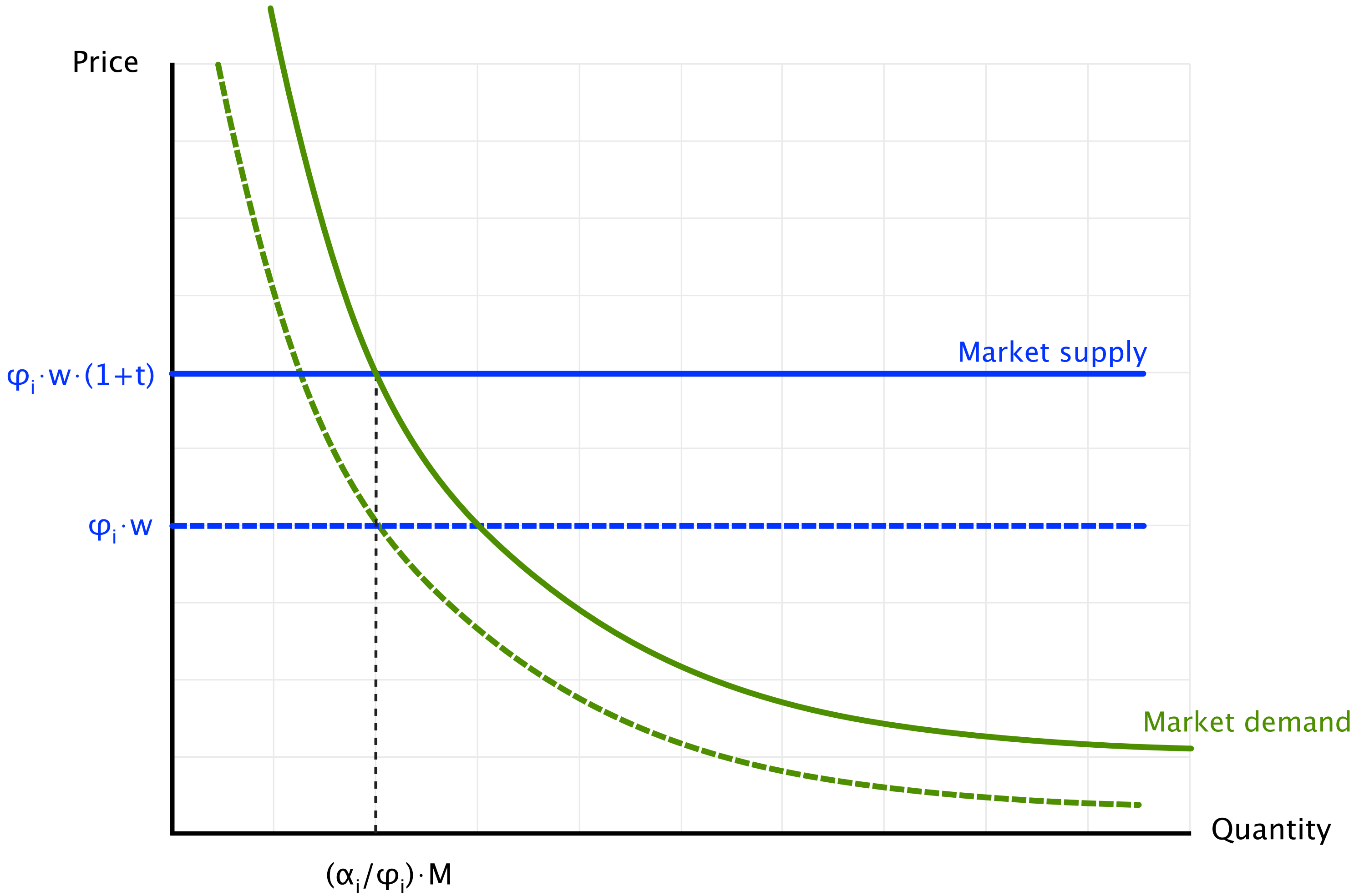
$$T = t \cdot w \cdot M$$

Same total quantity as without a tax!

- Price is higher

- But so is total income

Redistribution



Redistribution

- Q: Efficiency
  - Consumption

$$-MRS_{ij}^{h} = -MRS_{ij}^{g} = \frac{p_i}{p_j}$$

  - Activity levels

$$\frac{p_i}{p_j} = \frac{w \cdot \varphi_i \cdot (1+t)}{w \cdot \varphi_j \cdot (1+t)} = \frac{\varphi_i}{\varphi_j} = -MRT_{ij}$$

Redistribution

- Conclusion
  - Markets are efficient also with uniform commodity tax
  - Can also show that if t = δ, then all inequality removed
- Key assumption
  - Uniformity. If taxes are different $t_i \neq t_j$

$$\frac{p_i}{p_j} = \frac{w \cdot \varphi_i \cdot (1+t_i)}{w \cdot \varphi_j \cdot (1+t_j)} = \frac{\varphi_i \cdot (1+t_i)}{\varphi_j \cdot (1+t_j)} \neq \frac{\varphi_i}{\varphi_j} = -MRT_{ij}$$

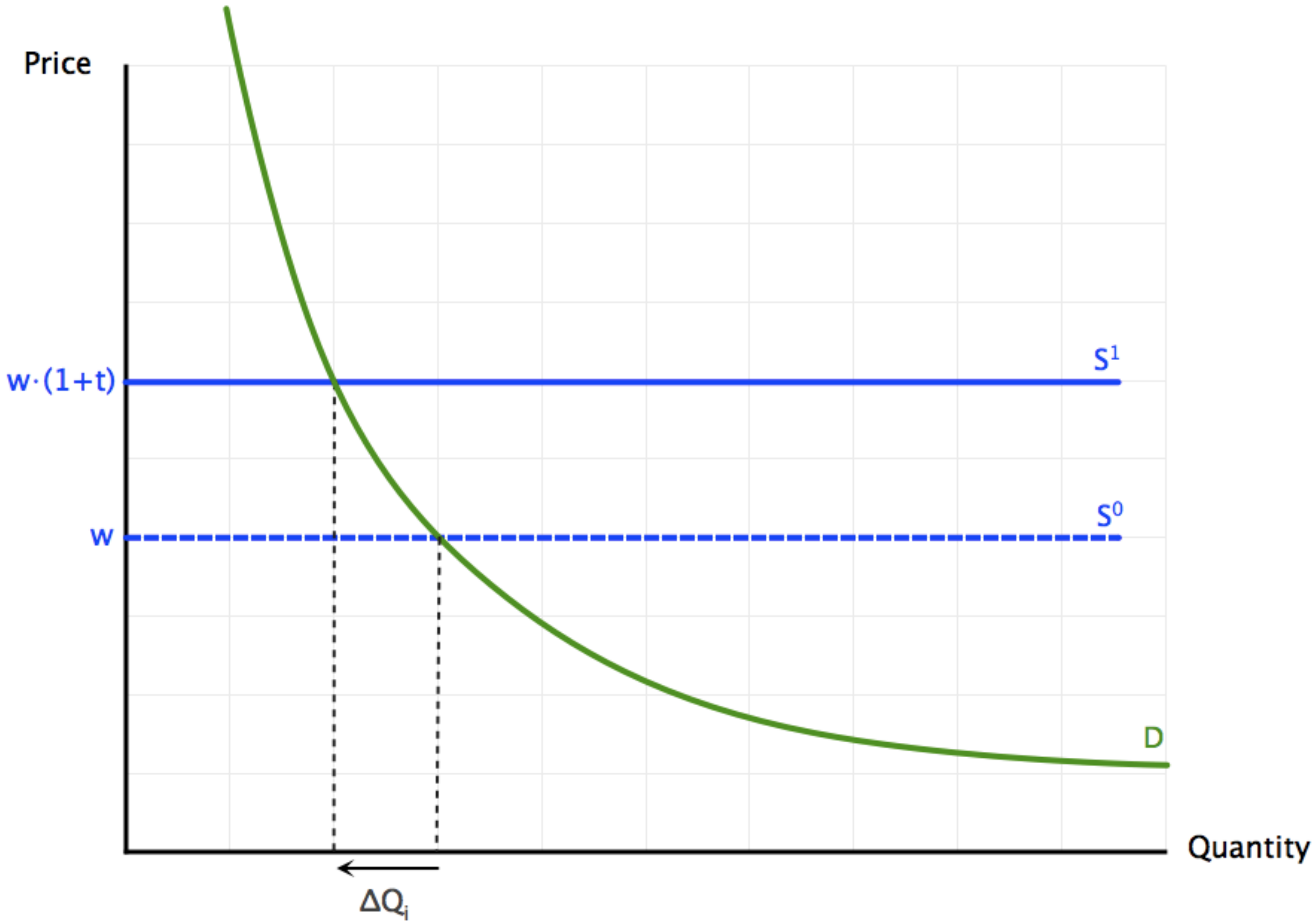
Redistribution

- General equilibrium analysis
  - Markets are efficient also with uniform commodity tax
- Partial equilibrium analysis
  - A tax on any market creates a dead weight loss
- Why is PE-analysis misleading?
  - Lets do it again
  - For simplicity: $\phi_i = 1$

Why is PE-analysis misleading?

- Experiment
  - Same tax t on all goods except one
  - Now introduce same tax t also on this good
- Effects on supply
  - Supply shifted up => price is increased
- Effects on demand
  - Total tax revenues increased
  - Negligible increase of demand in single market since N is very large

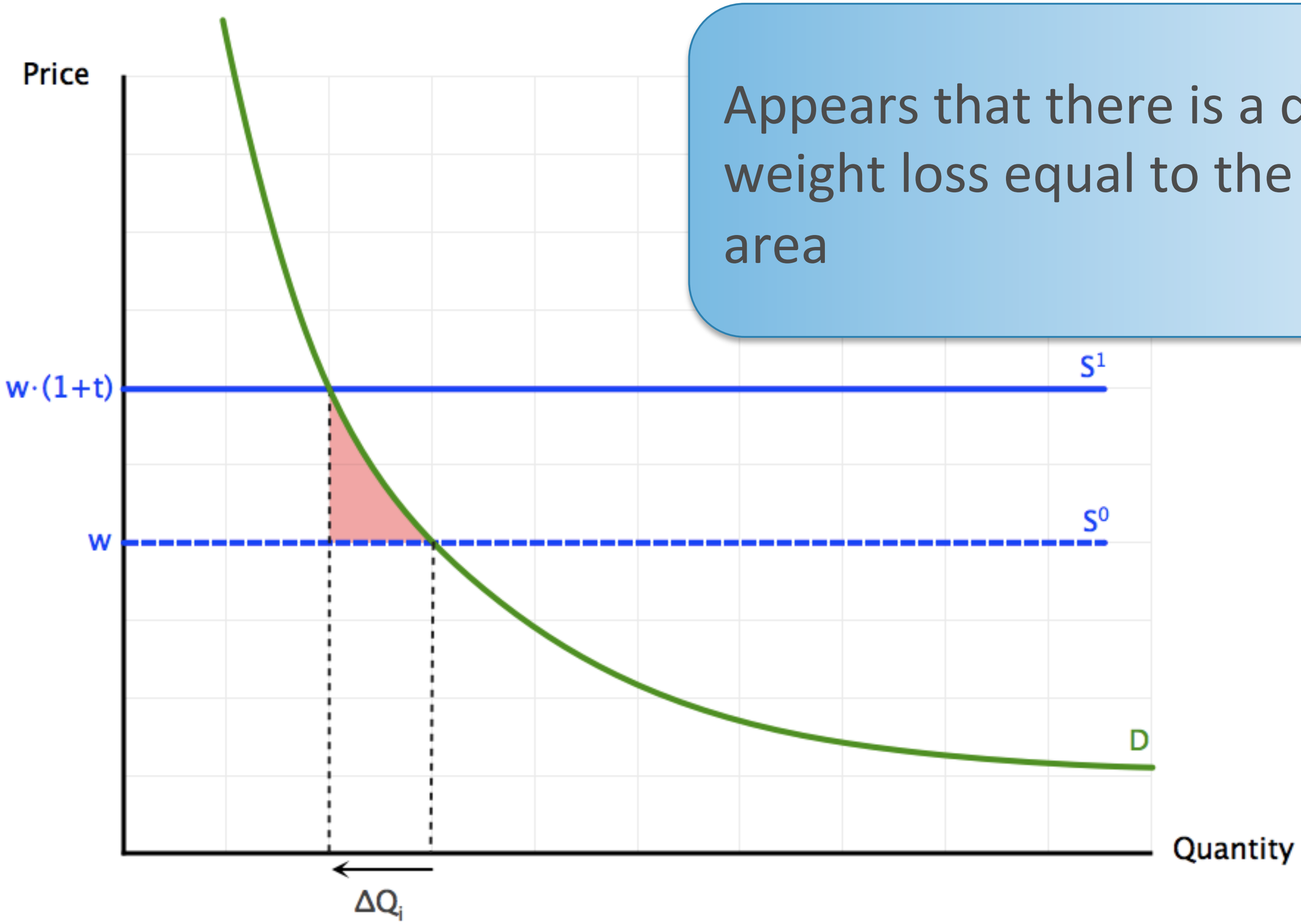
Why is PE-analysis misleading?



Why is PE-analysis misleading?

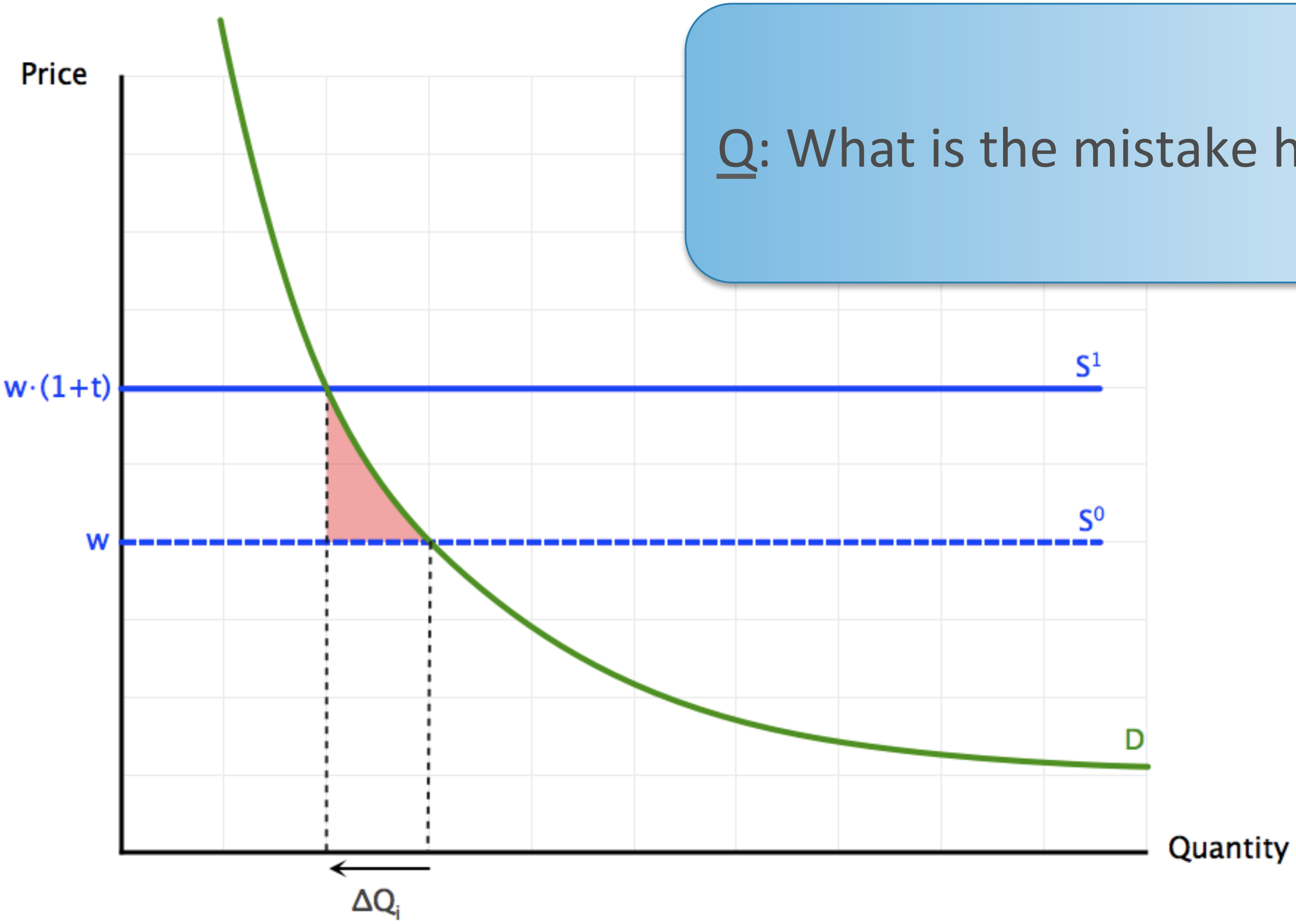
- Summary so far
  - Prices in all other markets unaffected: $p_i = w\,(1+t)$
  - Price in our market increased
  - Quantity in our market reduced

Why is PE-analysis misleading?



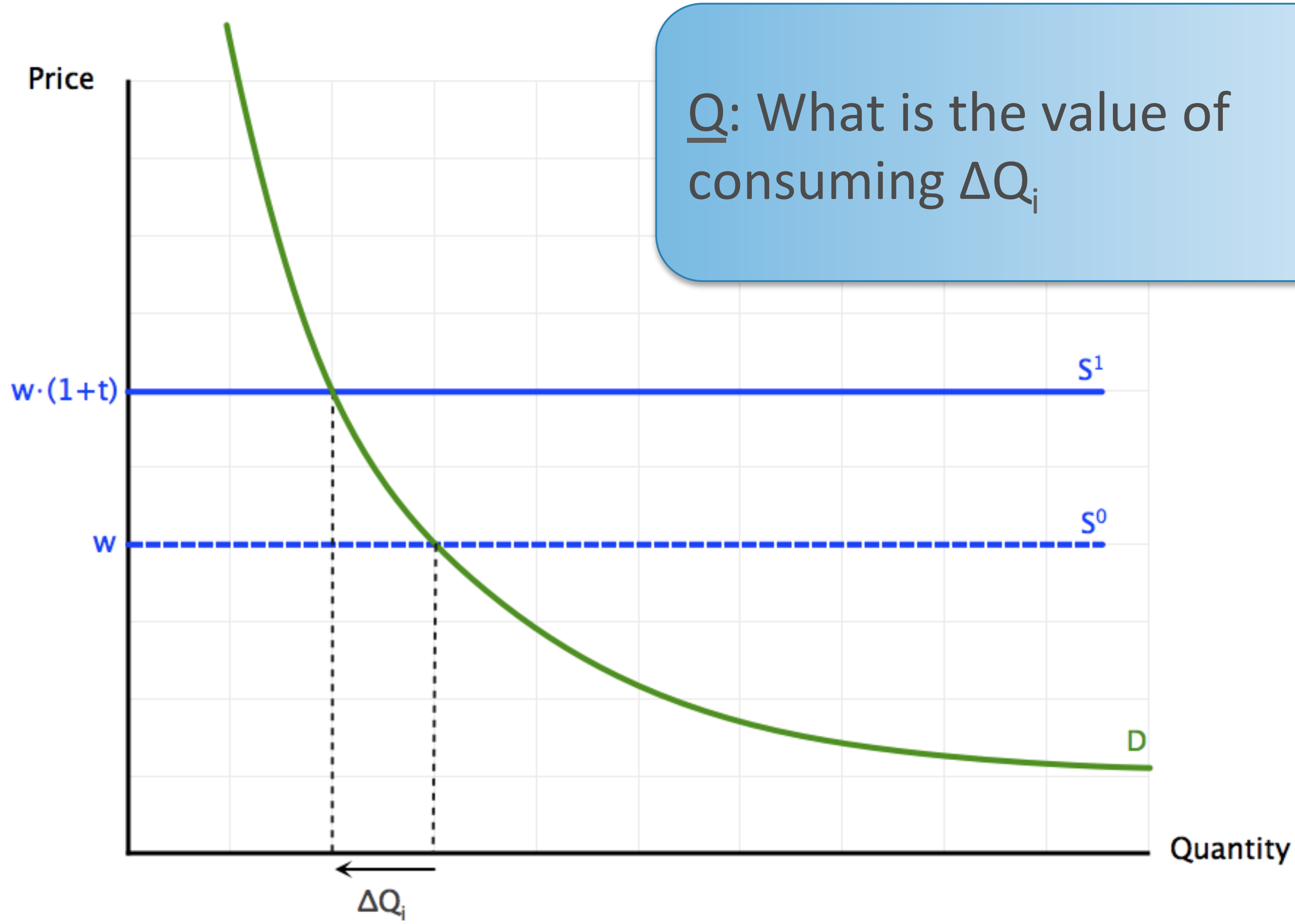
Appears that there is a dead weight loss equal to the red area

Why is PE-analysis misleading?



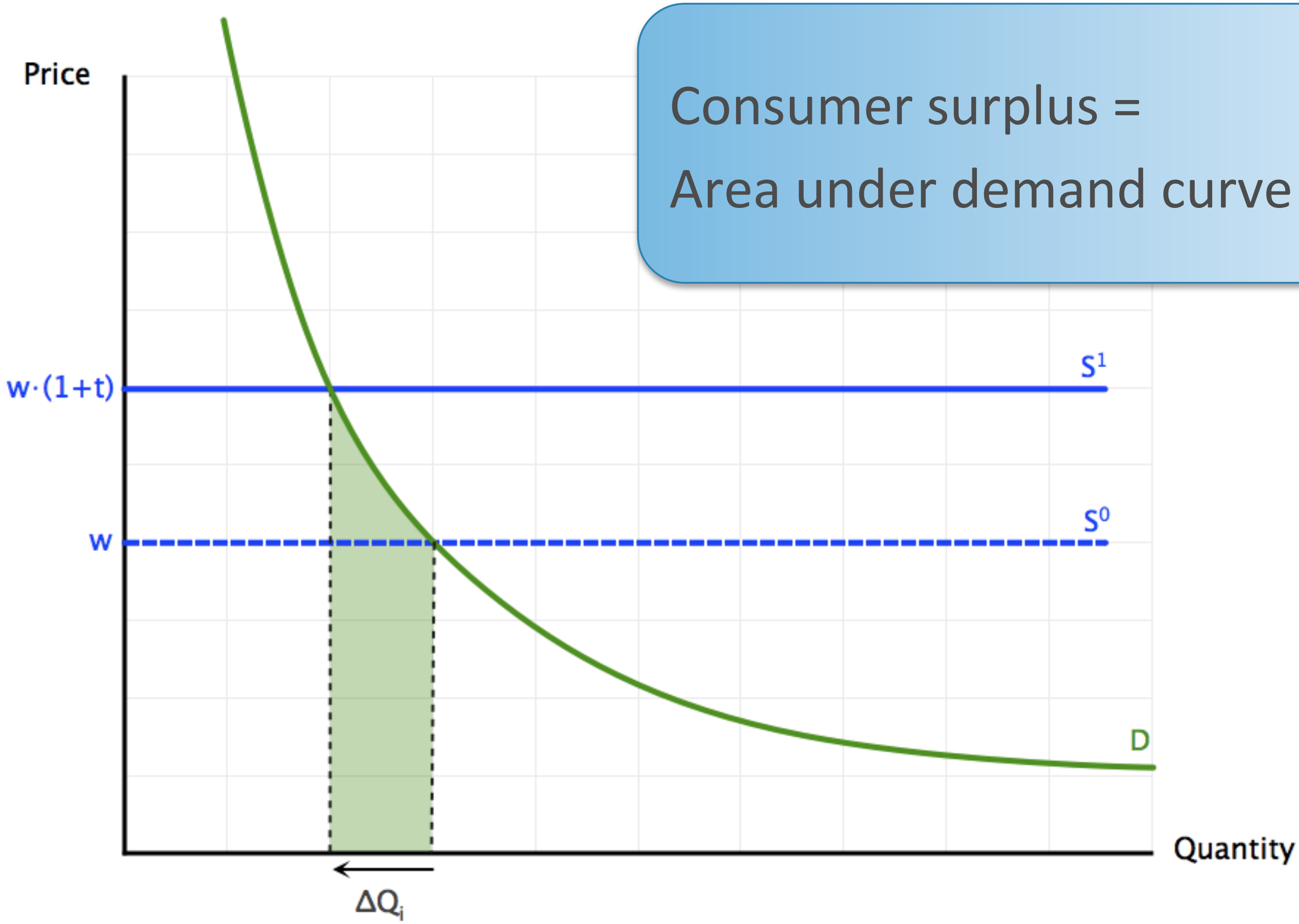
Q: What is the mistake here?

Why is PE-analysis misleading?



Q: What is the value of consuming ΔQ_i

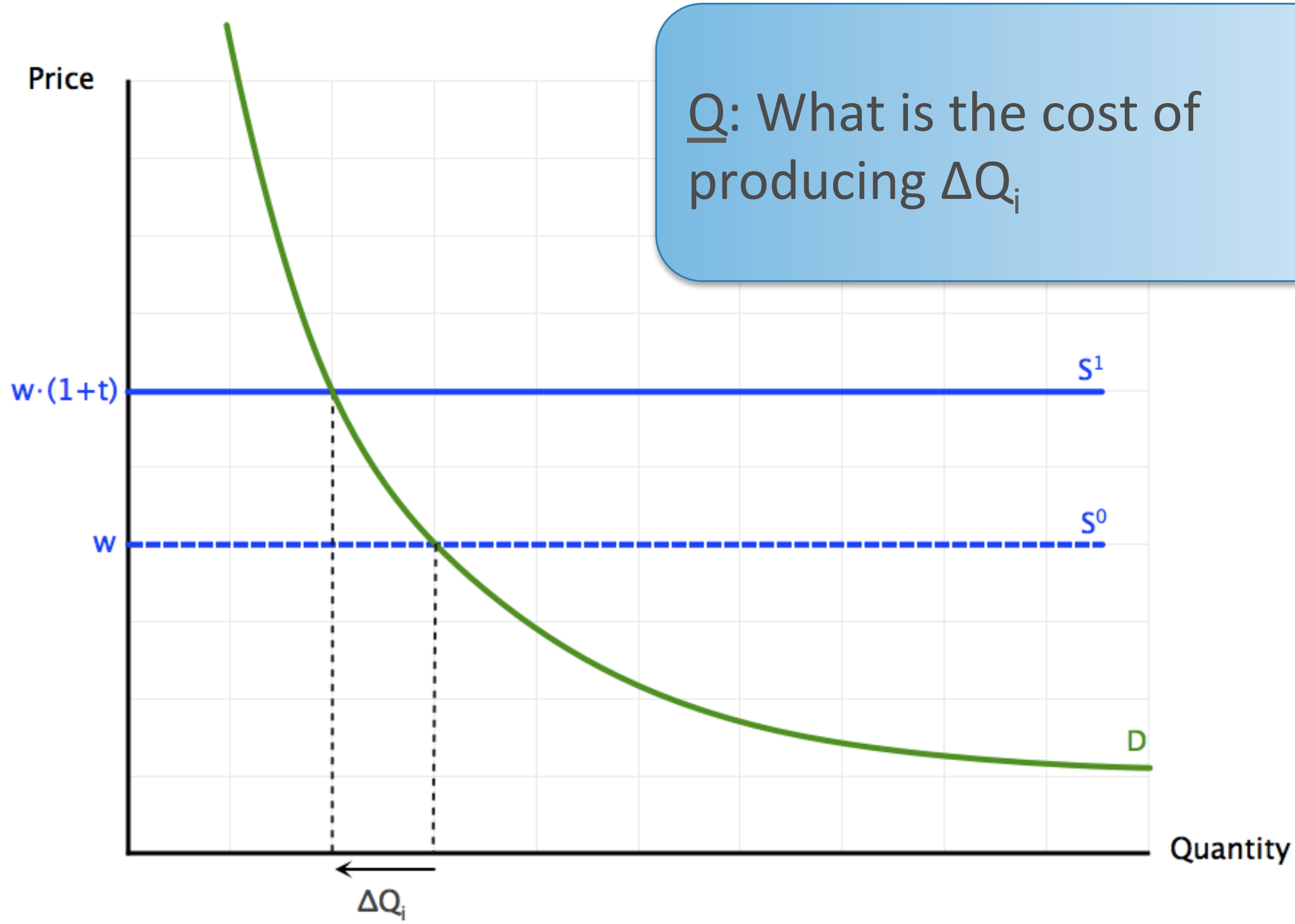
Why is PE-analysis misleading?



Consumer surplus =

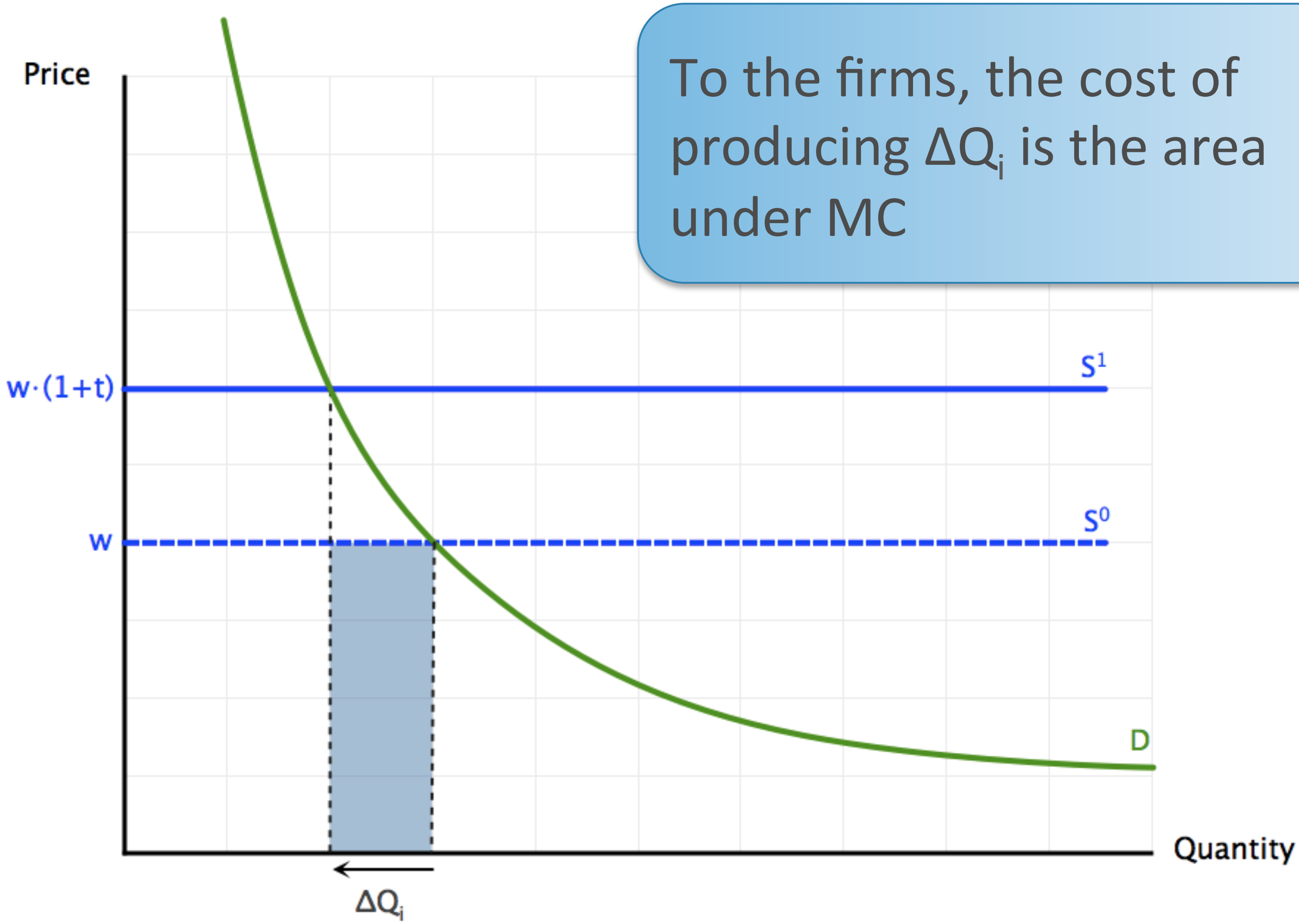
Area under demand curve

Why is PE-analysis misleading?



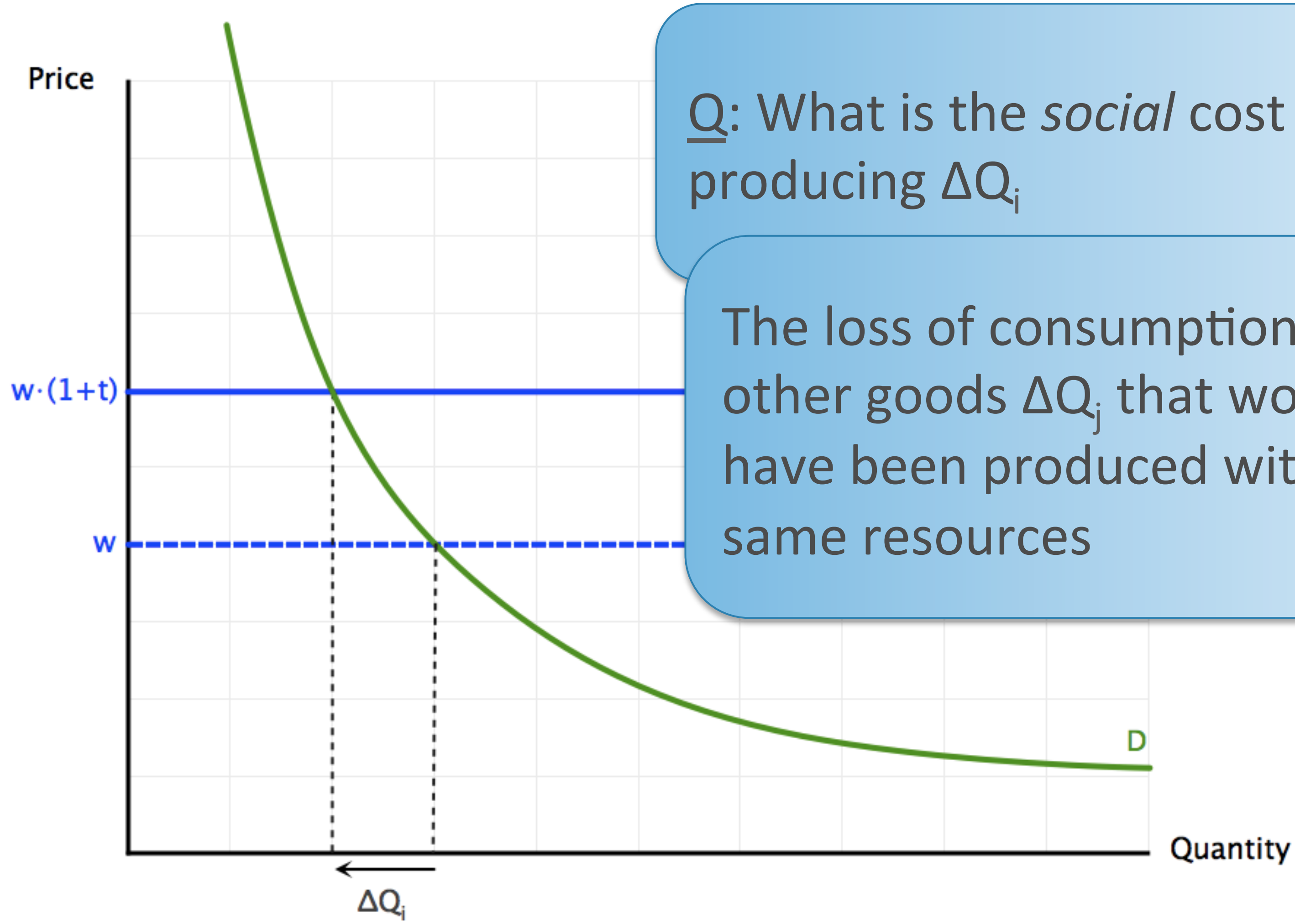
Q: What is the cost of producing ΔQ_i

Why is PE-analysis misleading?



To the firms, the cost of producing ΔQ_i is the area under MC

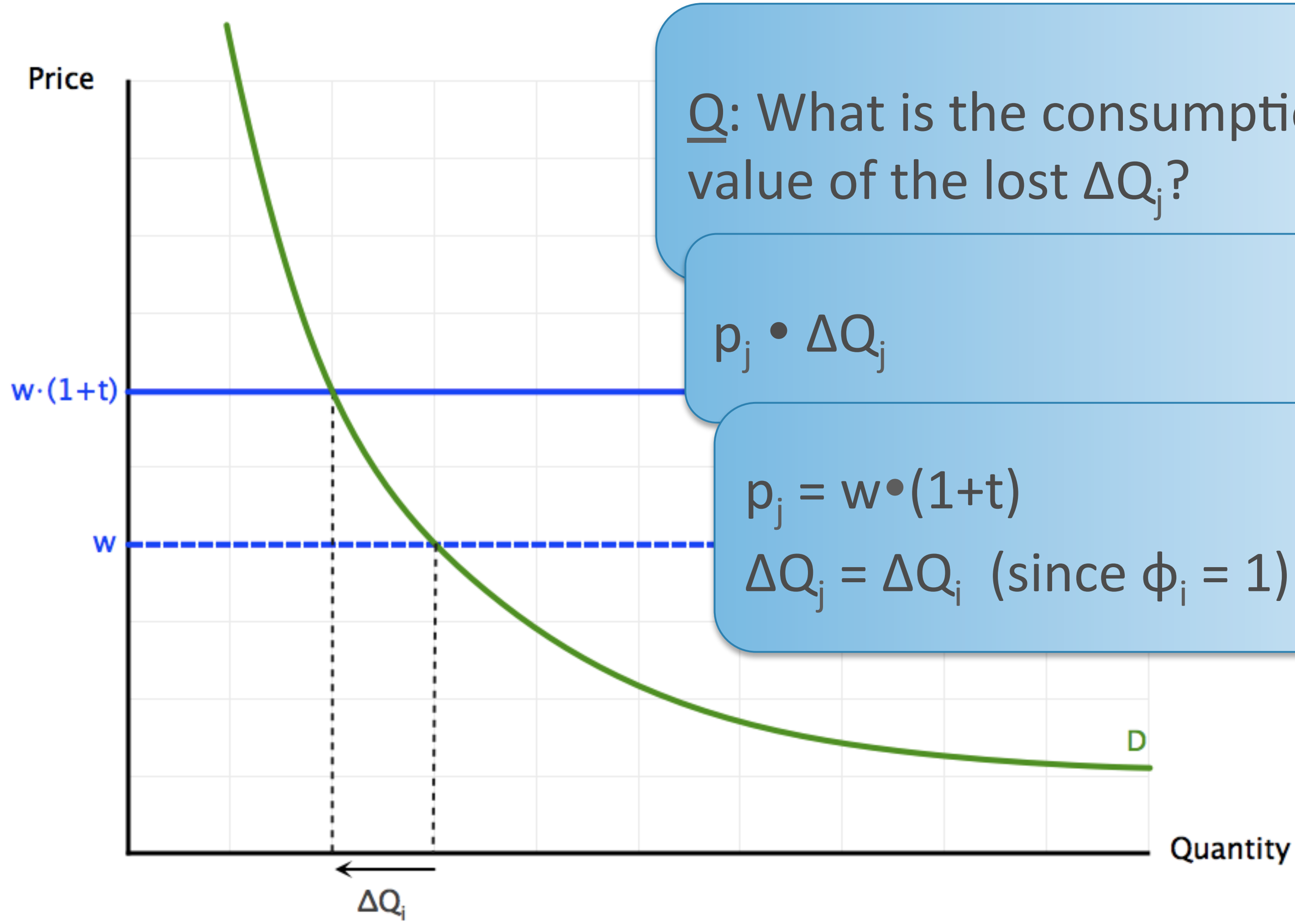
Why is PE-analysis misleading?



Q: What is the *social* cost of producing ΔQ_i

The loss of consumption of other goods ΔQ_j that would have been produced with the same resources

Why is PE-analysis misleading?



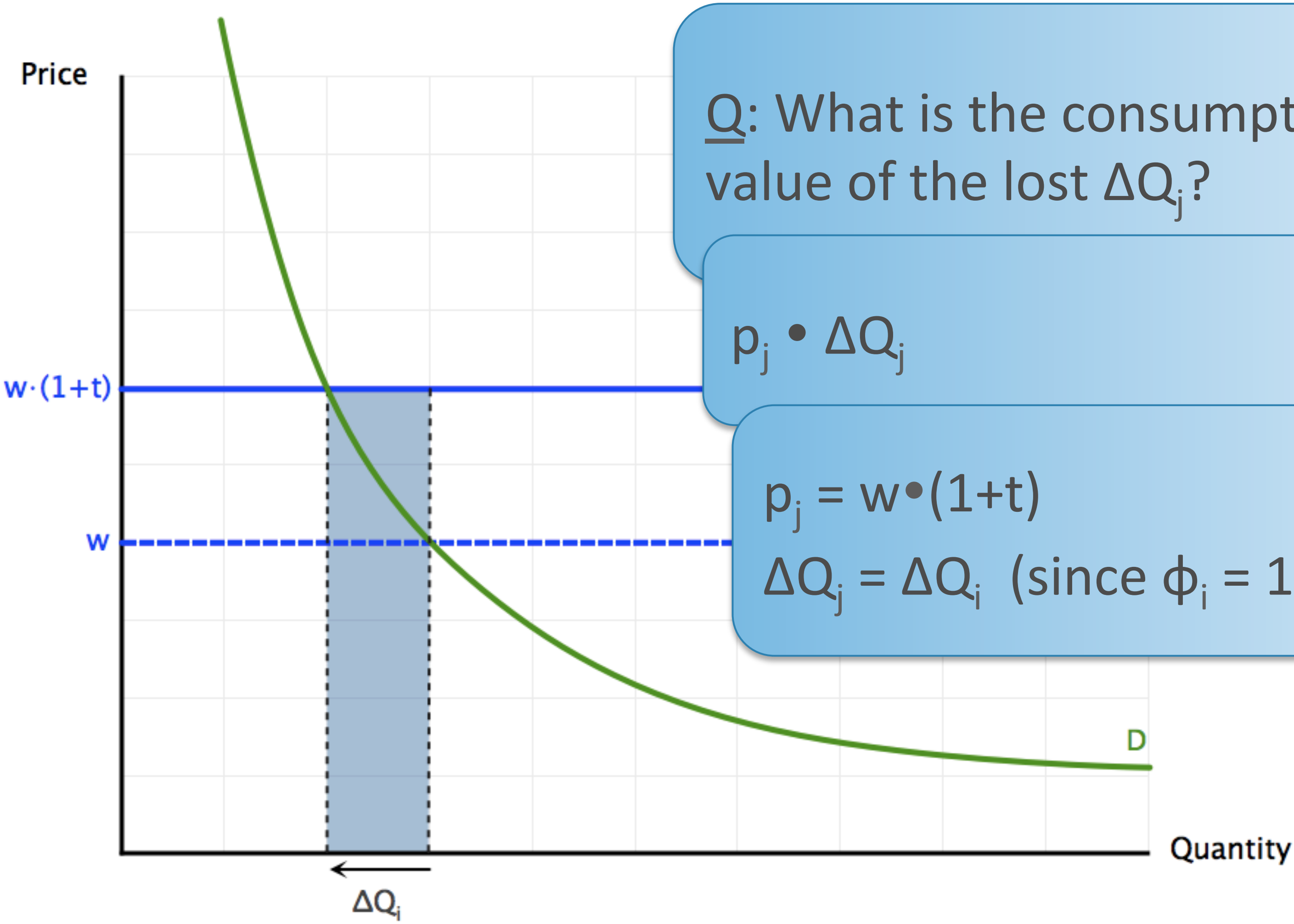
Q: What is the consumption value of the lost ΔQ_j?

$p_j \bullet \Delta Q_j$

$p_j = w \bullet (1+t)$

$\Delta Q_j = \Delta Q_i$ (since $\phi_i = 1$)

Why is PE-analysis misleading?



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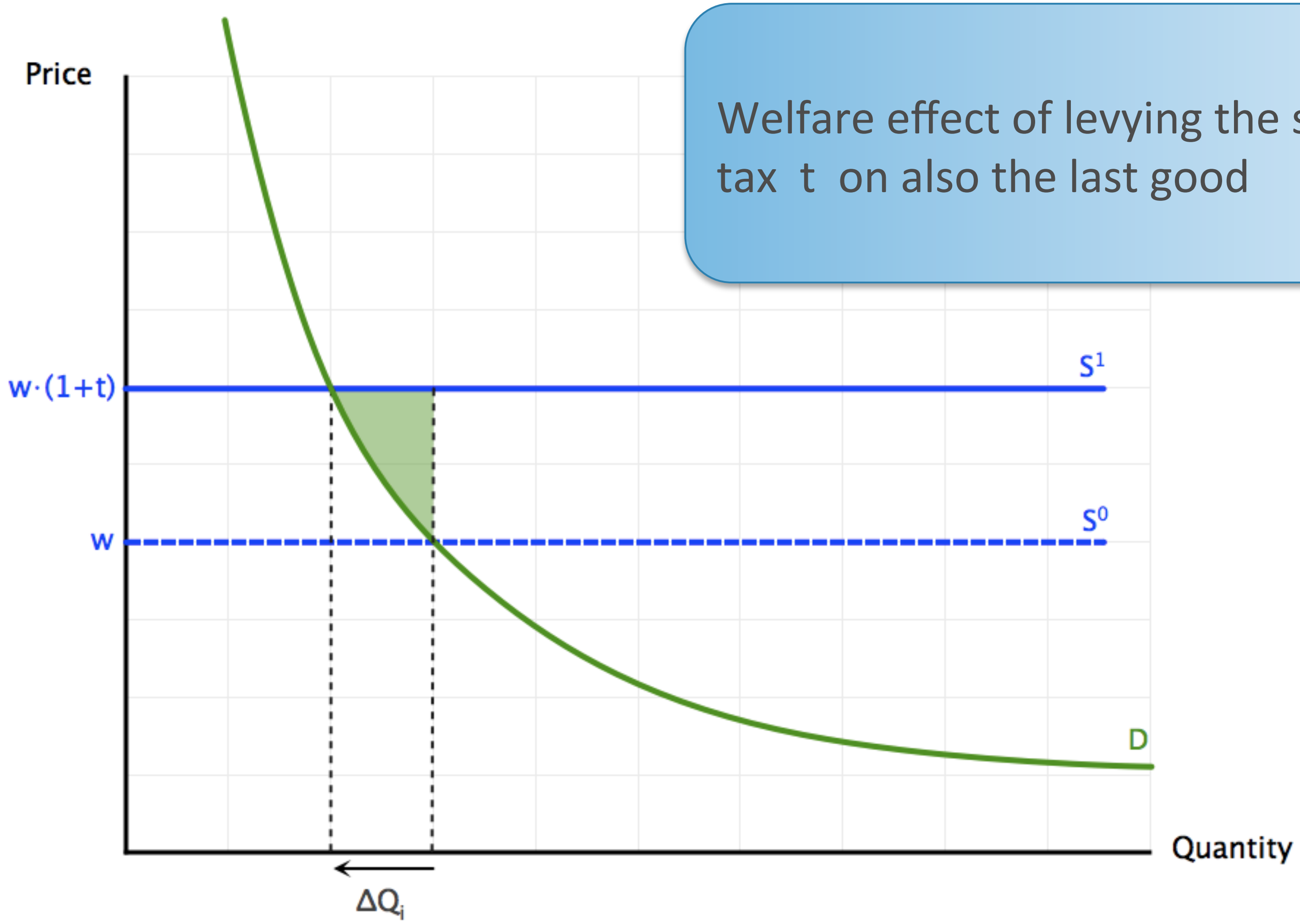
Why is PE-analysis misleading?



Welfare effect of levying the same tax t on also the last good

Why is PE-analysis misleading?

Welfare effect of levying a tax t on this good, when all other taxes are zero

Why is PE-analysis misleading?

- Conclusion 2
  - When assessing welfare using partial equilibrium analysis, we must ask if the firms' production costs really represent the social opportunity cost of the used resources

But…

But…

- Again: Conclusion 1
  - A uniform tax on all goods => No tax distortions
- But
  - We know that taxes create distortions
  - So, what to we miss?
- Q: Most important omission?
  - Governments cannot tax all goods
  - In particular: They cannot tax leisure

A simple model with leisure

Leisure

- Utility function, augmented with leisure

$$U = l^{\beta} \cdot \prod_{i=1}^{N} q_i^{\alpha}$$

  – For simplicity $\alpha_i = \alpha$

  – Normalization $\beta + N \cdot \alpha = 1$

Leisure

- Utility function, augmented with leisure

$$U = l^{\beta} \cdot \prod_{i=1}^{N} q_i^{\alpha}$$

- Budget

$$\sum_{i=1}^{N} p_i \cdot q_i = w \cdot (1-\delta) \cdot (1-l)$$

Leisure

- Utility function, augmented with leisure

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- Budget

$$\sum_{i=1}^{N} p_i \cdot q_i = w \cdot (1-\delta) \cdot (1-l)$$

  – Budget rewritten

$$\sum_{i=1}^{N} p_i \cdot q_i + w \cdot (1-\delta) \cdot l = w \cdot (1-\delta)$$

"Price of leisure"

Leisure

- Almost the same model as before

$$U = l^{\beta} \cdot \prod_{i=1}^{N} q_i^{\alpha}$$

$$\sum_{i=1}^{N} p_i \cdot q_i + w \cdot (1-\delta) \cdot l = w \cdot (1-\delta)$$

Leisure

- Conclusion
  - A uniform tax t on all "market goods" creates a dead weight loss, since leisure cannot be taxed
  - People will consume too little "market goods" and too much leisure
  - Expressed differently, they will work too little

Leisure

- Formally
  - High-productive consumers buy apples and leisure until:

$$MRS^{h}_{apple,leisure} \equiv -\frac{\partial U^{h} / \partial q_{apple}}{\partial U^{h} / \partial l} = -\frac{p_{apple}}{(1+\delta)\cdot w}$$

Leisure

- Formally
  - High-productive people produce 1+δ apples per time unit worked
  - Cost of apple in terms of leisure

$$MRT^{h}_{apple,leisure} = -\frac{1}{1+\delta}$$

Leisure

- Formally

$$-MRS^{h}_{apple,leisure} = \frac{p_{apple}}{(1+\delta)\cdot w} \qquad \frac{1}{1+\delta} = -MRT^{h}_{apple,leisure}$$

$$p_{apple} = w\cdot(1+t)$$

$$-MRS^{h}_{apple,leisure} = \frac{1+t}{1+\delta} > \frac{1}{1+\delta} = -MRT^{h}_{apple,leisure}$$

Leisure

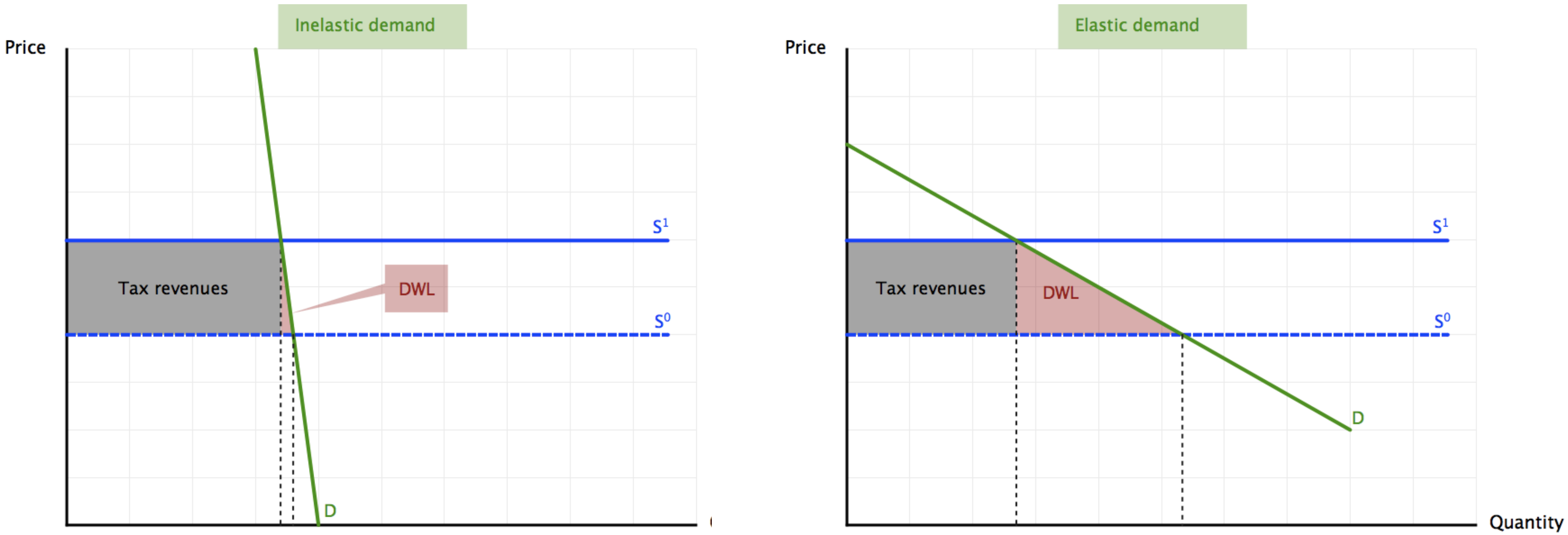
- Conclusion
  - In equilibrium, the value of an apple in terms of leisure is higher than the cost of an apple in terms of leisure
  - Still, people are not led to consume an additional apple, since they pay taxes on apples but not on leisure

Conclusions

- Conclusion
  - Taxes are not distortive *per se*
  - Distortions arise as a result of limitations on the tax system
  - Most important limitation is probably that we cannot tax leisure
    - Consumption of leisure (and home-produced goods) accounts for around 70 % of our “potential income”
      - Hours worked in a year: 1800
      - Total number of hours: 365*16 hours = 5840
      - Percentage: 1800/5840 = 0.3

Conclusions

- Partial solution I
  - – Higher tax on goods with inelastic demand



Conclusions

- Partial solution I
  - Higher tax on goods with inelastic demand

- Caveat
  - Inelastic demand ⇔ "Necessity good"
  - High tax on necessity goods => adverse effect on distribution

Conclusions

- Partial solution II
  - High tax on goods that are complements to leisure
    - Holiday travels, liquor and fine food
    - Not done in practice (?)
  - Low tax on goods that are substitutes to leisure (complements to work)
    - Cleaning services, working clothes
  - Even subsidies
    - Childcare, public transportation