School of Business, Economics and Law
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General Equilibrium

I: Role of prices

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Agenda

- General equilibrium (first hour)
- Welfare (second hour)

A simple model of general equilibrium

General equilibrium

- Definition
  - A set of prices (one for each market) such that all households and firms can realize their consumption and production plans in all markets at the same time.

Agents

- Households
  - Demand goods & supply labor
  - Identical Cobb-Douglas preferences over N goods
  - Identical incomes
- Firms
  - Demand labor & supply goods
  - Specialize in only one good
  - CRTS
- Note
  - Only one production factor

Households

Households

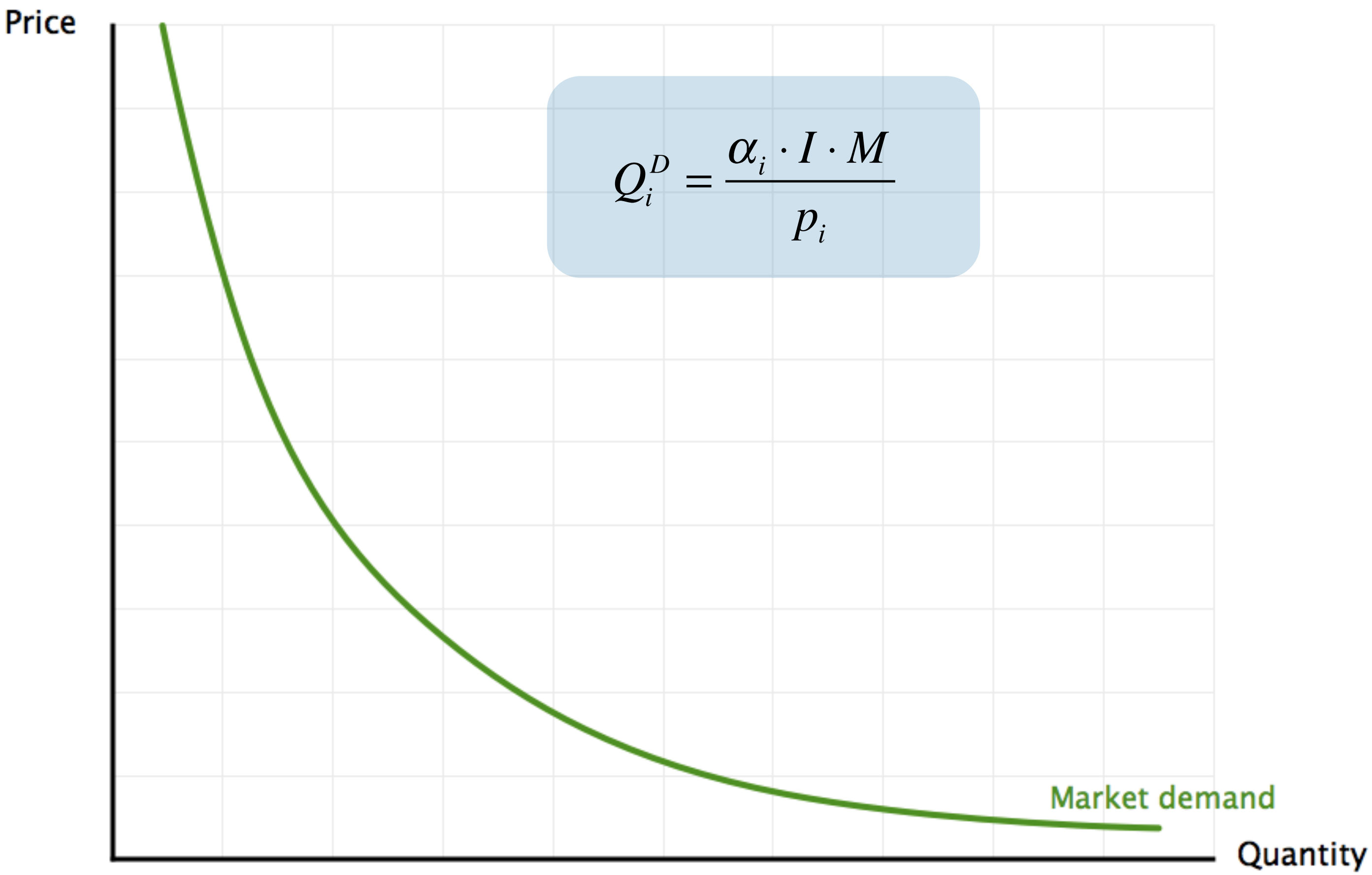
- Demand for goods
  - Cobb-Douglas preferences $U = \prod_{i=1}^{N} q_i^{\alpha_i}$ $\quad \sum \alpha_i = 1$
  - Budget constraint $\sum_{i=1}^{N} p_i \cdot q_i = I$

- Q: Derive market demand for good *i = 1…N*
  - *M* households
  - (4 min)

Households

- Market demand for goods
  - Lagrange: $$L = \prod_{i=1}^{n} q_i^{\alpha_i} + \lambda \cdot \left[I - \sum_{i=1}^{n} p_i \cdot q_i \right]$$
  - FOC: $$\frac{\partial L}{\partial q_i} = \alpha_i \cdot q_i^{-1} \cdot U - \lambda \cdot p_i = 0 \quad \Leftrightarrow \quad p_i \cdot q_i = \alpha_i \cdot \lambda^{-1} \cdot U$$
  - Substitute into budget: $$\lambda^{-1} \cdot U = I$$
  - Substitute back into FOC: $$q_i = \alpha_i \cdot \frac{I}{p_i}$$
  - Horizontal summation: $$Q_i^D = \alpha_i \cdot \frac{I \cdot M}{p_i}$$

Households



$$Q_i^D = \frac{\alpha_i \cdot I \cdot M}{p_i}$$

Households

- Supply of labor
  - No utility or disutility of working
  - Total time available per household = 1
  - Wage per time unit $w > 0$

- Q: Derive market supply of labor
  - M households
  - (3 min)

Households

- Market supply of labor
  - Individual household supplies 1 for any $w > 0$
  - Horizontal summation: $L^S = M$ for any $w > 0$

Households

- Assume
  - Everybody finds a job
    (we confirm that labor market clears later)
  - All households own 1/M of all firms
- Household income
  - $I = w + \pi/M$
- Demand
  - $$Q_i^D = \alpha_i \cdot \frac{w \cdot M + \Pi}{p_i}$$

Firms

Firms

- Supply of goods in market *i*
  - Production function: $q_i = \frac{1}{\varphi_i} \cdot l_i$

Example: If $\phi_i = 2$, then

- One unit of labor produces ½ a unit of output

Firms

- Supply of goods in market *i*
  - Factor requirement: $l_i = \varphi_i \cdot q_i$

Example: If $\phi_i = 2$, then

- it takes 2 units of labor to produce one unit of output

Firms

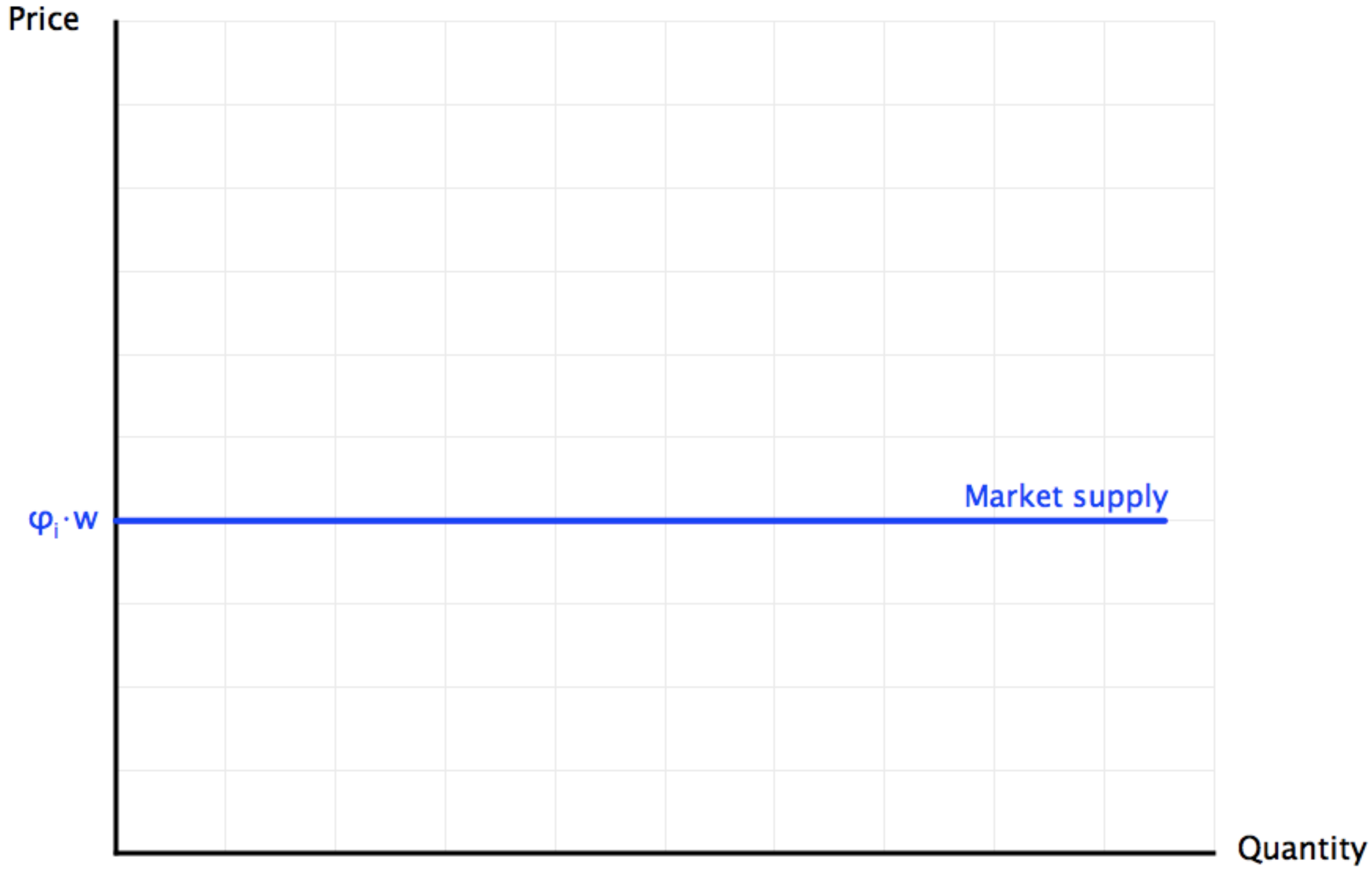
- Supply of goods in market *i*
  - Production function: $q_i = \frac{1}{\varphi_i} \cdot l_i$
  - *F* firms

- Q: Derive market supply of good *i*
  - Start with single firm
  - Find marginal cost
  - Display marginal cost curve in a figure
  - Then horizontal summation
  - (3 min)

Firms

- Market supply of good *i*
  - Constant marginal cost: $w \cdot \varphi_i$
  - Individual firm willing to supply any amount as long as $p_i \geq w \cdot \varphi_i$
  - Firm supply perfectly elastic at MC
  - Market supply perfectly elastic at MC

Firms



Firms

- Demand for labor
  - Discussed soon.

Product market equilibrium

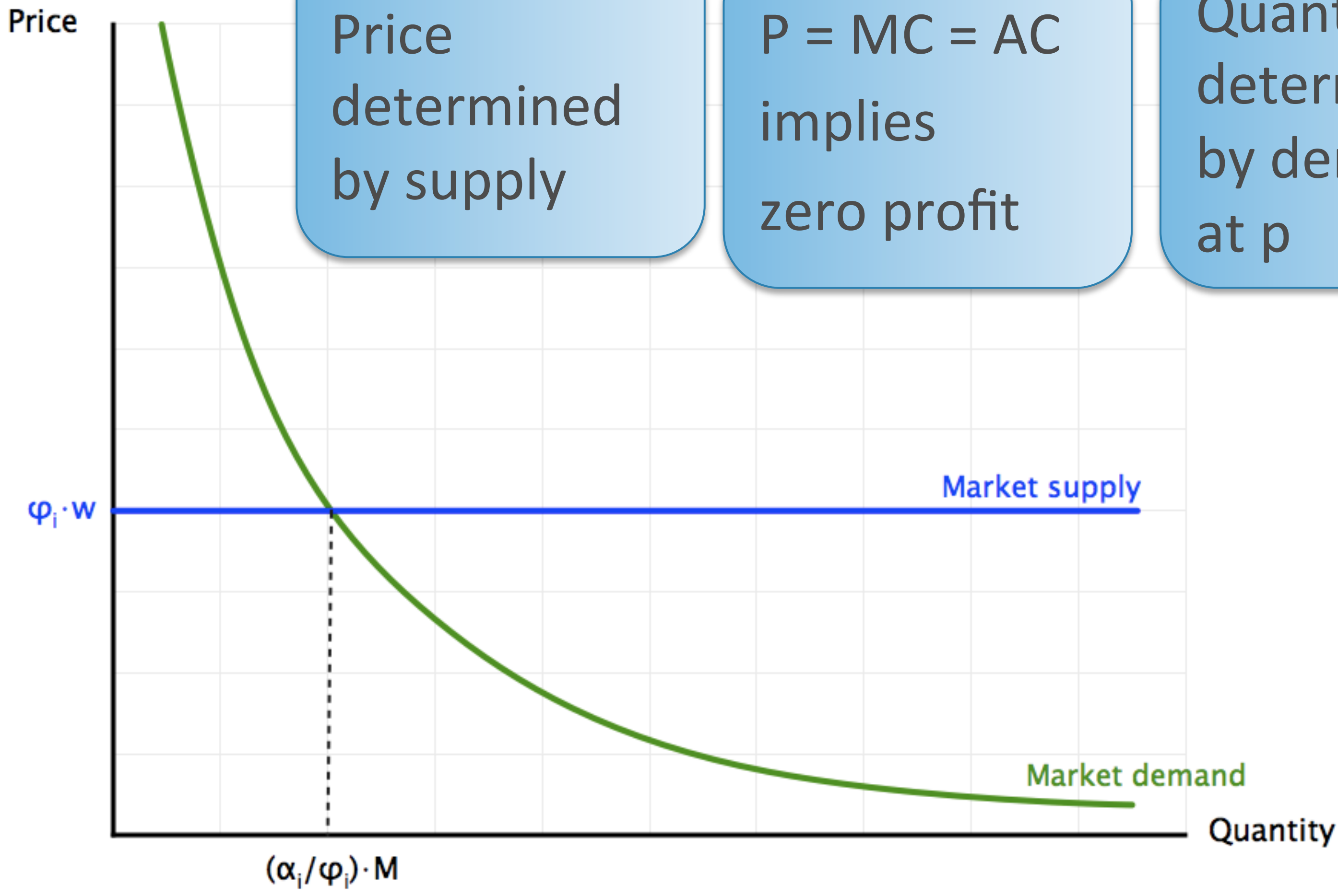
- Q: Find equilibrium
  - Hint: Start with figure
  - (4 min)

Product market equilibrium

Price determined by supply

P = MC = AC implies zero profit

Quantity determined by demand at p



Product market equilibrium

- Price: $p_i = w \cdot \varphi_i$
- Profit: $\pi = 0$
- Quantity
  - $Q_i^D = \alpha_i \cdot \dfrac{w \cdot M + \Pi}{p_i}$

Product market equilibrium

- Price: $p_i = w \cdot \varphi_i$
- Profit: π = 0
- Quantity
  - $$Q_i^D = \alpha_i \cdot \frac{w \cdot M + \Pi}{p_i} = \alpha_i \cdot \frac{w \cdot M}{p_i} =$$

Product market equilibrium

- Price: $p_i = w \cdot \varphi_i$
- Profit: π = 0
- Quantity
  - $Q_i^D = \alpha_i \cdot \frac{w \cdot M + \Pi}{p_i} = \alpha_i \cdot \frac{w \cdot M}{p_i} = \alpha_i \cdot \frac{w \cdot M}{w \cdot \varphi_i} =$

Product market equilibrium

- Price: $p_i = w \cdot \varphi_i$
- Profit: π = 0
- Quantity
  - $Q_i^D = \alpha_i \cdot \frac{w \cdot M + \Pi}{p_i} = \alpha_i \cdot \frac{w \cdot M}{p_i} = \alpha_i \cdot \frac{w \cdot M}{w \cdot \varphi_i} = \frac{\alpha_i}{\varphi_i} \cdot M$

Product market equilibrium

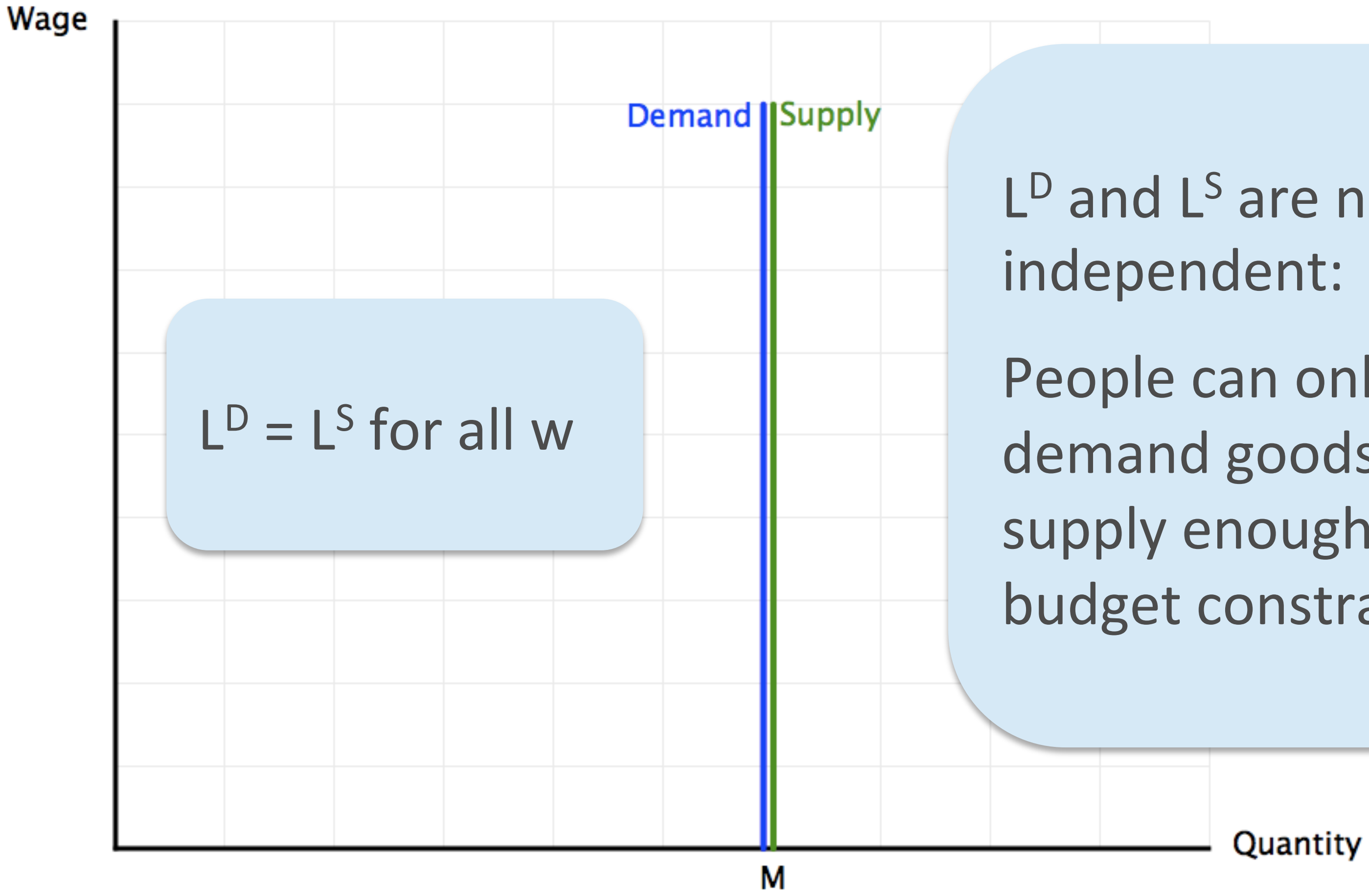
- Price: $p_i = w \cdot \varphi_i$
- Profit: π = 0
- Quantity
  - $Q_i^D = \alpha_i \cdot \frac{w \cdot M + \Pi}{p_i} = \alpha_i \cdot \frac{w \cdot M}{p_i} = \alpha_i \cdot \frac{w \cdot M}{w \cdot \varphi_i} = \frac{\alpha_i}{\varphi_i} \cdot M$
  - $q_i = \frac{\alpha_i}{\varphi_i}$

Labor market equilibrium

Labor market equilibrium

- Demand for labor
  - To produce Q_i^S firms in market *i* need $\varphi_i \cdot Q_i^S$ units of labor
  - Market demand $L^D = \sum \varphi_i \cdot Q_i^S$
  - Product market equilibrium $Q_i^S = Q_i^D = \frac{\alpha_i}{\varphi_i} \cdot M$
  - $L^D = \sum \varphi_i \cdot \left\{ \frac{\alpha_i}{\varphi_i} \cdot M \right\} = M \cdot \sum \alpha_i = M$
  - Independent of wage

Labor market equilibrium

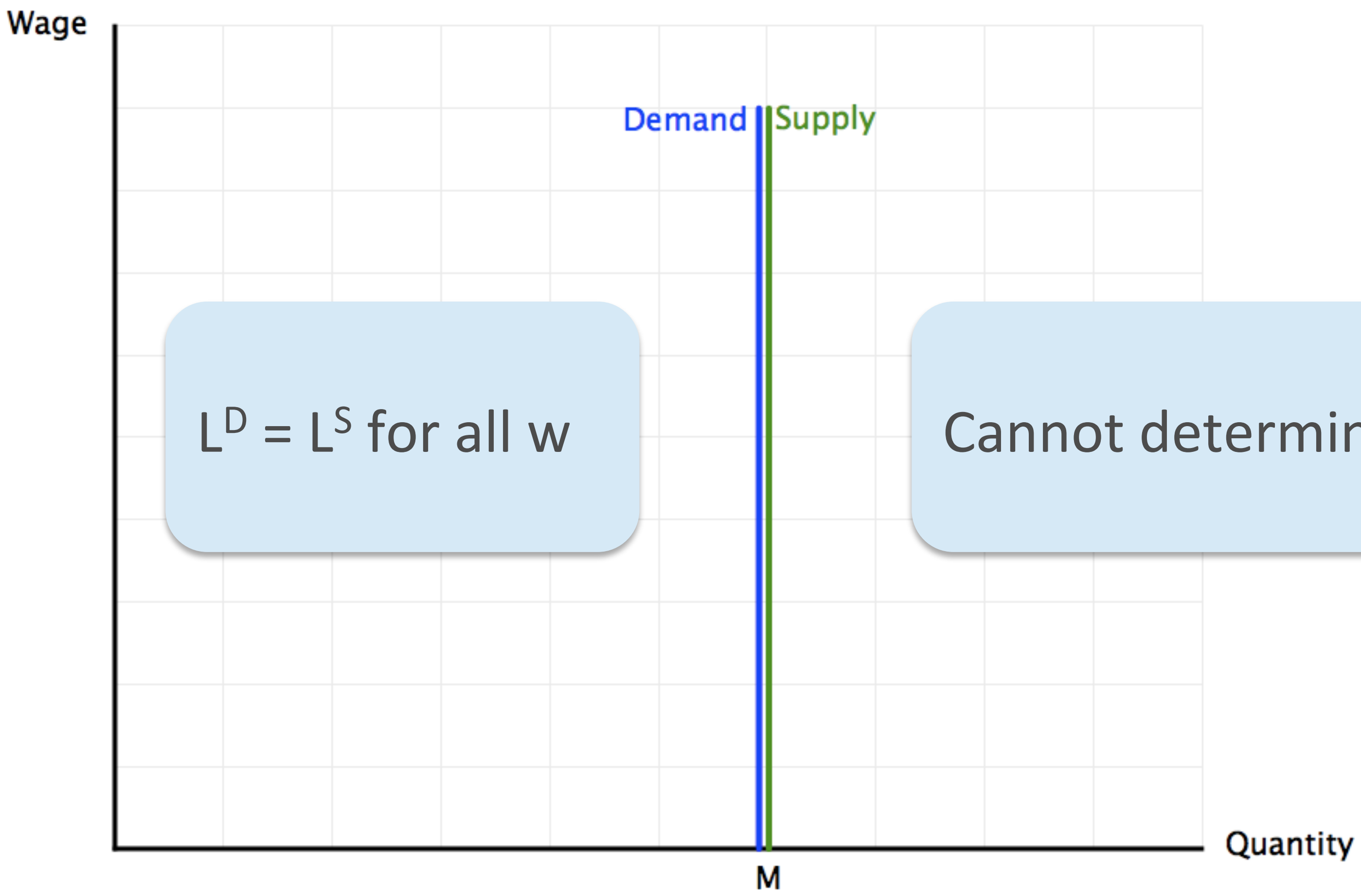


$L^D = L^S$ for all w

L^D and L^S are not independent:

People can only demand goods if they supply enough labor – budget constraint

Labor market equilibrium



$L^D = L^S$ for all w

Cannot determine w

Labor market equilibrium

- Conclusion
  - Whenever all product markets clear, labor demand equals labor supply, independent of the wage
  - This is instance of “Walras law”
  - We can leave the labor market from now on

General equilibrium

- We have determined all quantities
  - All people work full time and consume $q_i = \dfrac{\alpha_i}{\varphi_i}$
- But cannot determine w, only *relative* prices
  - $\dfrac{p_i}{w} = \varphi_i \qquad \dfrac{p_i}{p_j} = \dfrac{\varphi_i}{\varphi_j}$
  - where the wage is arbitrary

Role of simplifications

Role of simplifications

- Constant returns to scale
  - Need not (can not) determine number of firms in markets
  - Price determined by supply only

Role of simplifications

- Little interaction between markets in this model
  - Utility
    - Cobb-Douglas preferences
    - Goods are neither substitutes nor complements
  - Production function
    - No economies or diseconomies of scope
    - Factor requirements are independent
  - Only interaction
    - Limited amount of labor
  - For this reason we can find equilibrium in product markets separately

Efficiency

Pareto efficiency

- Q: Definition
  - An allocation is *Pareto efficient*, if it is impossible to improve welfare one person, without reducing welfare for somebody else

- Is the market allocation PE?
  - Efficient consumption (need to define MRS)
  - Efficient activity level (need to define MRT)

Marginal rate of substitution

- Q: Definition
  - Mr. Anderson's *marginal rate of substitution* between apples and pears is the number of pears that he is willing to give up for an additional apple.
  - Notation: $MRS_{apple,pear}^{Anderson}$
  - "The value of an apple in terms of pears"

Marginal rate of substitution

- Formula

$$MRS^{Anderson}_{apple,pear} = -\frac{\left(\dfrac{\partial U^{Anderson}}{\partial q_{apple}}\right)}{\left(\dfrac{\partial U^{Anderson}}{\partial q_{pear}}\right)}$$

  - If
    - Marginal utility of apples = 10
    - Marginal utility of pears = 2
  - Then
    - It takes 10/2 = 5 pears to replace an apple
  - "The value of an apple in terms of pears"

Marginal rate of substitution

- Recall
  - Mr. Anderson's utility

$$U^{\mathrm{A}:son} = U^{\mathrm{A}:son}\left(q_{apples}, q_{pears}\right)$$

  - What happens to Mr. Anderson's utility if he consumes more apples?

Marginal rate of substitution

- Recall
  - What happens to Mr. Anderson's utility if he consumes more apples?

$$dU^{A:son} = \frac{\partial U^{A:son}}{\partial q_{apples}} \cdot dq_{apples}$$

Change in utility

Marginal utility of apples

Change in consumption of apples

Marginal rate of substitution

- Recall
  - What happens to Mr. Anderson's utility if he consumes more pears?

$$dU^{A:son} = \frac{\partial U^{A:son}}{\partial q_{pears}} \cdot dq_{pears}$$

Change in utility

Marginal utility of pears

Change in consumption of pears

Marginal rate of substitution

- Compute MRS, step 1
  - What happens to Mr. Anderson's utility if he both consumes more apples and pears?

$$dU^{A:son} = \frac{\partial U^{A:son}}{\partial q_{apples}} \cdot dq_{apples} + \frac{\partial U^{A:son}}{\partial q_{pears}} \cdot dq_{pears}$$

This equation is called *the total differential*

Marginal rate of substitution

- Compute MRS, step 2
  - To find out how many pears Anderson is willing to give up in exchange for an apple, we need to find the $dq_{pears} < 0$ that keeps Anderson's utility constant $dU^{A:son} = 0$ when $dq_{apple} = 1$

$$dU^{A:son} = \frac{\partial U^{A:son}}{\partial q_{apples}} \cdot 1 + \frac{\partial U^{A:son}}{\partial q_{pears}} \cdot dq_{pears} = 0$$

Marginal rate of substitution

- Compute MRS, step 2
  - Equation: $$\frac{\partial U^{A:son}}{\partial q_{apple}} + \frac{\partial U^{A:son}}{\partial q_{pear}} \cdot dq_{pear} = 0$$
  - Solve: $$dq_{pear} = -\frac{\partial U^{A:son} / \partial q_{apple}}{\partial U^{A:son} / \partial q_{pear}}$$
  - Thus: $$MRS^{A:son}_{apple,pear} = -\frac{\partial U^{A:son} / \partial q_{apple}}{\partial U^{A:son} / \partial q_{pear}}$$

Marginal rate of substitution

- Conclusion
  - The value of an apple in terms of pears

$$MRS_{apple,pear}^{A:son} = -\frac{\partial U^{A:son} / \partial q_{apple}}{\partial U^{A:son} / \partial q_{pear}}$$

- Note
  - MRS differs between people
  - MRS depends on consumption

$$q_{apple} \uparrow \quad \Rightarrow \quad \partial U^{A:son} / \partial q_{apple} \downarrow \quad \Rightarrow \quad MRS_{apple,pear}^{A:son} \downarrow$$

Efficient consumption

- Q: Is the following situation Pareto efficient?
  - $MRS^{Anderson}_{apple,pear} = -2$
  - $MRS^{Peterson}_{apple,pear} = -1$

- No
  - For Anderson, the value of an apple in terms of pears is 2
  - For Peterson, the value of an apple in terms of pears is 1
  - If Peterson gives Anderson an apple, Anderson can give (say) 1.5 pears back. Both are better off

Efficient consumption

- Condition for efficient consumption
  - $MRS_{ij}^{h} = MRS_{ij}^{g}$
  - for all pairs of people *h* and *g*
  - and all pairs of goods *i* and *j*

Is the market efficient?

- Q: Does market induce efficient consumption?
  - FOC for utility maximization:
    - $$\frac{\partial U^h}{\partial q_i} - \lambda^h \cdot p_i = 0 \qquad \text{and} \qquad \frac{\partial U^h}{\partial q_j} - \lambda^h \cdot p_j = 0$$
  - Divide
    - $$MRS_{ij}^h \equiv -\frac{\partial U^h / \partial q_i}{\partial U^h / \partial q_j} = -\frac{p_i}{p_j}$$
  - Same for all consumers, if same prices

$$MRS_{ij}^h = MRS_{ij}^g$$

Is the market efficient?

- Conclusion
  - If consumers pay the same prices, they will be induced to distribute goods between themselves in an efficient manner.

Marginal rate of transformation

- Q: Definition
  - The *marginal rate of transformation* between apples and pears is the number of pears that we must stop producing to produce an additional apple.
  - Notation: $MRT_{apple,pear}$
  - "The cost of an apple in terms of pears"

Marginal rate of transformation

- Q: Compute $MRT_{car, moped}$ (1 min)
  - It takes 10 units of labor to build a car
  - It takes 2 units of labor to build a moped
- Answer
  - We could produce one more car by producing five mopeds less.
  - The cost of a car is five mopeds

Marginal rate of transformation

- Q: Compute $MRT_{1,2}$ (2 min)
  - Production function: $q_i = \frac{1}{\varphi_i} \cdot l_i$

Marginal rate of transformation

- Q: Compute $MRT_{1, 2}$
  - Production function: $q_i = \frac{1}{\varphi_i} \cdot l_i$
- Answer $L = \sum \varphi_i \cdot q_i$

Marginal rate of transformation

- Q: Compute $MRT_{1,2}$
  - Production function: $q_i = \frac{1}{\varphi_i} \cdot l_i$
- Answer

$$L = \sum \varphi_i \cdot q_i$$

$$dL = \varphi_1 \cdot dq_1 + \varphi_2 \cdot dq_2$$

Marginal rate of transformation

- Q: Compute $MRT_{1,2}$
  - Production function: $q_i = \frac{1}{\varphi_i} \cdot l_i$
- Answer

$$L = \sum \varphi_i \cdot q_i$$

$$dL = \varphi_1 \cdot dq_1 + \varphi_2 \cdot dq_2$$

$$dL = \varphi_1 \cdot 1 + \varphi_2 \cdot dq_2 = 0$$

Marginal rate of transformation

- Q: Compute $MRT_{1, 2}$
  - Production function: $q_i = \frac{1}{\varphi_i} \cdot l_i$
- Answer

$$L = \sum \varphi_i \cdot q_i$$
$$dL = \varphi_1 \cdot dq_1 + \varphi_2 \cdot dq_2$$
$$dL = \varphi_1 \cdot 1 + \varphi_2 \cdot dq_2 = 0$$
$$\varphi_2 \cdot dq_2 = -\varphi_1$$

Marginal rate of transformation

- Q: Compute $MRT_{1,2}$
  - Production function: $q_i = \frac{1}{\varphi_i} \cdot l_i$
- Answer

$$L = \sum \varphi_i \cdot q_i$$

$$dL = \varphi_1 \cdot dq_1 + \varphi_2 \cdot dq_2$$

$$dL = \varphi_1 \cdot 1 + \varphi_2 \cdot dq_2 = 0$$

$$\varphi_2 \cdot dq_2 = -\varphi_1$$

$$dq_2 = -\frac{\varphi_1}{\varphi_2}$$

Marginal rate of transformation

- Q: Compute $MRT_{1,2}$
  - Production function: $q_i = \frac{1}{\varphi_i} \cdot l_i$
- Answer

$$L = \sum \varphi_i \cdot q_i$$

$$dL = \varphi_1 \cdot dq_1 + \varphi_2 \cdot dq_2$$

$$dL = \varphi_1 \cdot 1 + \varphi_2 \cdot dq_2 = 0$$

$$\varphi_2 \cdot dq_2 = -\varphi_1$$

$$dq_2 = -\frac{\varphi_1}{\varphi_2}$$

$$MRT_{1,2} = -\frac{\varphi_1}{\varphi_2}$$

Marginal rate of transformation

- More generally
  - $MRT_{1,2} = -\frac{w \cdot \varphi_1}{w \cdot \varphi_2} = -\frac{MC_1}{MC_2}$
  - The cost of good 1 in terms of good 2 is given by the ratio of marginal costs

Efficient production

- Q: Is the following situation Pareto efficient? (2 min)
  - $MRS^{Anderson}_{apple,pear} = -2$
  - $MRT_{apple,pear} = -1$

- No
  - For Anderson, the value of an apple in terms of pears is 2
  - But it only costs one pear to produce an extra apple
  - Thus increase production of apples by 1 and reduce production of pears by 1

Efficient production

- Condition for efficient activity level
  - $MRT_{ij} = MRS_{ij}^{g}$
  - for all people *g*
  - and all pairs of goods *i* and *j*

Is the market efficient?

- Q: Does market induce efficient activity level? (3 min)
  - FOC for profit maximization:

$$MC_i = p_i \quad \Leftrightarrow \quad w \cdot \varphi_i = p_i$$

  - Divide

$$-\frac{w \cdot \varphi_i}{w \cdot \varphi_j} = -\frac{p_i}{p_j}$$

$$MRT_{ij} = -\frac{p_i}{p_j}$$

  - Note

$$MRT_{ij} = -\frac{p_i}{p_j} = MRS_{ij}^h$$

Is the market efficient?

- Conclusion
  - If all firms and consumer are price-takers, the market induces efficient activity level

Is the market efficient?

- 1st welfare theorem
  - The market is efficient

Is the market efficient?

- Equilibrium prices guarantee (by definition)
  - All agents' plans can be realized at the same time
  - If I just maximize my utility given prices:
    - I can rely on other people to buy the goods that I produce
    - I can rely on other people to produce the goods that I want
  - ➢ Coordination

- Equilibrium prices induce
  - Consumers to share a given amount of goods efficiently
  - Firms to produce an efficient amount of all goods
  - ➢ Efficiency