Partial vs General

- Exercise: same analysis, but formally
  - Still: we don't do full general equilibrium analysis
  - Just "semi-partial" equilibrium analysis focusing on the two markets for chocolate and licorice.

Partial vs General

Equilibrium:

$$D_X(P_X, P_L) = S_X(P_X - T_X)$$
$$D_L(P_X, P_L) = S_L(P_L)$$

Two equations in two unknowns (P_X, P_L)

Define prices as functions of T_X

Partial vs General

Equilibrium:

$$D_X\left(P_X, P_L\right) = S_X\left(P_X - T_X\right)$$

$$D_L\left(P_X, P_L\right) = S_L\left(P_L\right)$$

Differentiate to find effect of an increase in tax (dT_X) on prices (dP_X and dP_L):

$$D_X^X \cdot dP_X + D_X^L \cdot dP_L = S'_X \cdot dP_X - S'_X \cdot dT$$

$$D_L^X \cdot dP_X + D_L^L \cdot dP_L = S'_L \cdot dP_L$$

Partial vs General

Equilibrium:

$$D_X\left(P_X, P_L\right) = S_X\left(P_X - T_X\right)$$

$$D_L\left(P_X, P_L\right) = S_L\left(P_L\right)$$

Differentiate to find effect of an increase in tax (dT_X) on prices (dP_X and dP_L):

$$D_X^X \cdot dP_X + D_X^L \cdot dP_L = S'_X \cdot dP_X - S'_X \cdot dT$$

$$D_L^X \cdot dP_X + D_L^L \cdot dP_L = S'_L \cdot dP_L$$

Collect terms:

$$\left[S'_X - D_X^X\right] \cdot dP_X - D_X^L \cdot dP_L = S'_X \cdot dT$$

$$\left[S'_L - D_L^L\right] \cdot dP_L - D_L^X \cdot dP_X = 0$$

Partial vs General

Divide by S = D, Multiply by P:

$$\left[\frac{S'_X \cdot P_X}{S_X} - \frac{D_X^X \cdot P_X}{D_X}\right] \cdot dP_X - \frac{D_X^L \cdot P_L}{D_X} \cdot \frac{P_X}{P_L} \cdot dP_L = \frac{S'_X \cdot P_X}{S_X} \cdot dT$$

$$\left[\frac{S'_L \cdot P_L}{S_L} - \frac{D_L^L \cdot P_L}{D_L}\right] \cdot dP_L - \frac{D_L^X \cdot P_X}{D_L} \cdot \frac{P_L}{P_X} \cdot dP_X = 0$$

Partial vs General

Divide by S = D, Multiply by P:

$$\left[\frac{S'_X \cdot P_X}{S_X} - \frac{D_X^X \cdot P_X}{D_X}\right] \cdot dP_X - \frac{D_X^L \cdot P_L}{D_X} \cdot \frac{P_X}{P_L} \cdot dP_L = \frac{S'_X \cdot P_X}{S_X} \cdot dT$$

$$\left[\frac{S'_L \cdot P_L}{S_L} - \frac{D_L^L \cdot P_L}{D_L}\right] \cdot dP_L - \frac{D_L^X \cdot P_X}{D_L} \cdot \frac{P_L}{P_X} \cdot dP_X = 0$$

Recall $P^* = P - T$

$$\left[\frac{S'_X \cdot (P_X - T_X)}{S_X} \cdot \frac{P_X}{P_X - T_X} - \frac{D_X^X \cdot P_X}{D_X}\right] \cdot dP_X - \frac{D_X^L \cdot P_L}{D_X} \cdot \frac{P_X}{P_L} \cdot dP_L = \frac{S'_X \cdot (P_X - T_X)}{S_X} \cdot \frac{P_X}{P_X - T_X} \cdot dT$$

$$\left[\frac{S'_L \cdot P_L}{S_L} - \frac{D_L^L \cdot P_L}{D_L}\right] \cdot dP_L - \frac{D_L^X \cdot P_X}{D_L} \cdot \frac{P_L}{P_X} \cdot dP_X = 0$$

Partial vs General

Recall definition of elasticities

$$\varepsilon_S^X = \frac{S'_X \cdot \left(P_X - T_X\right)}{S_X}$$ Price elasticity of supply

$$\varepsilon_D^{XX} = \frac{D_X^X \cdot P_X}{D_X}$$ Own-price elasticity of demand

$$\varepsilon_D^{XL} = \frac{D_X^L \cdot P_L}{D_X}$$ Cross-price elasticity of demand

Partial vs General

Substitute elasticities

$$\left[\varepsilon_S^X \cdot \frac{P_X}{P_X - T_X} - \varepsilon_D^{XX}\right] \cdot dP_X - \varepsilon_D^{XL} \cdot \frac{P_X}{P_L} \cdot dP_L = \varepsilon_S^X \cdot \frac{P_X}{P_X - T_X} \cdot dT$$

$$\left[\varepsilon_S^L - \varepsilon_D^{LL}\right] \cdot dP_L - \varepsilon_D^{LX} \cdot \frac{P_L}{P_X} \cdot dP_X = 0$$

Partial vs General

Substitute elasticities

$$\left[\varepsilon_S^X \cdot \frac{P_X}{P_X - T_X} - \varepsilon_D^{XX}\right] \cdot dP_X - \varepsilon_D^{XL} \cdot \frac{P_X}{P_L} \cdot dP_L = \varepsilon_S^X \cdot \frac{P_X}{P_X - T_X} \cdot dT$$

$$\left[\varepsilon_S^L - \varepsilon_D^{LL}\right] \cdot dP_L - \varepsilon_D^{LX} \cdot \frac{P_L}{P_X} \cdot dP_X = 0$$

Rearrange

$$\left[\varepsilon_S^X \cdot \frac{P_X}{P_X - T_X} - \varepsilon_D^{XX}\right] \cdot dP_X - \varepsilon_D^{XL} \cdot \frac{P_X}{P_L} \cdot dP_L = \varepsilon_S^X \cdot \frac{P_X}{P_X - T_X} \cdot dT$$

$$dP_L = \frac{\varepsilon_D^{LX}}{\varepsilon_S^L - \varepsilon_D^{LL}} \cdot \frac{P_L}{P_X} \cdot dP_X$$

Partial vs General

Substitute elasticities

$$\left[\varepsilon_S^X \cdot \frac{P_X}{P_X - T_X} - \varepsilon_D^{XX}\right] \cdot dP_X - \varepsilon_D^{XL} \cdot \frac{P_X}{P_L} \cdot dP_L = \varepsilon_S^X \cdot \frac{P_X}{P_X - T_X} \cdot dT$$

$$\left[\varepsilon_S^L - \varepsilon_D^{LL}\right] \cdot dP_L - \varepsilon_D^{LX} \cdot \frac{P_L}{P_X} \cdot dP_X = 0$$

Rearrange

$$\left[\varepsilon_S^X \cdot \frac{P_X}{P_X - T_X} - \varepsilon_D^{XX}\right] \cdot dP_X - \varepsilon_D^{XL} \cdot \frac{P_X}{P_L} \cdot dP_L = \varepsilon_S^X \cdot \frac{P_X}{P_X - T_X} \cdot dT$$

$$dP_L = \frac{\varepsilon_D^{LX}}{\varepsilon_S^L - \varepsilon_D^{LL}} \cdot \frac{P_L}{P_X} \cdot dP_X$$

Partial vs General

Substitute Licorice market into Chocolate market

$$\left[\varepsilon_S^X \cdot \frac{P_X}{P_X - T_X} - \varepsilon_D^{XX}\right] \cdot dP_X - \varepsilon_D^{XL} \cdot \frac{P_X}{P_L} \cdot \left\{\frac{\varepsilon_D^{LX}}{\varepsilon_S^L - \varepsilon_D^{LL}} \cdot \frac{P_L}{P_X} \cdot dP_X\right\} = \varepsilon_S^X \cdot \frac{P_X}{P_X - T_X} \cdot dT$$

Partial vs General

Substitute Licorice market into Chocolate market

$$\left[\varepsilon_S^X \cdot \frac{P_X}{P_X - T_X} - \varepsilon_D^{XX}\right] \cdot dP_X - \varepsilon_D^{XL} \cdot \frac{P_X}{P_L} \cdot \left\{ \frac{\varepsilon_D^{LX}}{\varepsilon_S^L - \varepsilon_D^{LL}} \cdot \frac{P_L}{P_X} \cdot dP_X \right\} = \varepsilon_S^X \cdot \frac{P_X}{P_X - T_X} \cdot dT$$

Rearrange

$$\left[\varepsilon_S^X \cdot \frac{P_X}{P_X - T_X} - \varepsilon_D^{XX} - \frac{\varepsilon_D^{XL} \cdot \varepsilon_D^{LX}}{\varepsilon_S^L - \varepsilon_D^{LL}}\right] \cdot dP_X = \varepsilon_S^X \cdot \frac{P_X}{P_X - T_X} \cdot dT$$

Partial vs General

Substitute Licorice market into Chocolate market

$$\left[\varepsilon_S^X \cdot \frac{P_X}{P_X - T_X} - \varepsilon_D^{XX}\right] \cdot dP_X - \varepsilon_D^{XL} \cdot \frac{P_X}{P_L} \cdot \left\{\frac{\varepsilon_D^{LX}}{\varepsilon_S^L - \varepsilon_D^{LL}} \cdot \frac{P_L}{P_X} \cdot dP_X\right\} = \varepsilon_S^X \cdot \frac{P_X}{P_X - T_X} \cdot dT$$

Rearrange

$$\left[\varepsilon_S^X \cdot \frac{P_X}{P_X - T_X} - \varepsilon_D^{XX} - \frac{\varepsilon_D^{XL} \cdot \varepsilon_D^{LX}}{\varepsilon_S^L - \varepsilon_D^{LL}}\right] \cdot dP_X = \varepsilon_S^X \cdot \frac{P_X}{P_X - T_X} \cdot dT$$

Rearrange

$$\frac{dP_X}{dT} = \frac{\varepsilon_S^X}{\varepsilon_S^X - \varepsilon_D^{XX} \cdot \frac{P_X - T_X}{P_X} - \frac{\varepsilon_D^{XL} \cdot \varepsilon_D^{LX}}{\varepsilon_S^L - \varepsilon_D^{LL}} \cdot \frac{P_X - T_X}{P_X}}$$

Partial vs General

Gereral

$$\frac{dP_X}{dT_X} = \frac{\varepsilon_S^X}{\varepsilon_S^X - \varepsilon_D^{XX} \cdot \frac{P_X - T_X}{P_X} - \frac{\varepsilon_D^{XL} \cdot \varepsilon_D^{LX}}{\varepsilon_S^L - \varepsilon_D^{LL}} \cdot \frac{P_X - T_X}{P_X}}$$

Partial

$$\frac{dP_X}{dT_X} = \frac{\varepsilon_S^X}{\varepsilon_S^X - \varepsilon_D^{XX} \cdot \frac{P_X - T_X}{P_X}}$$

$$\frac{\varepsilon_D^{XL} \cdot \varepsilon_D^{LX}}{\varepsilon_S^L - \varepsilon_D^{LL}} > 0$$

Partial equilibrium analysis underestimates the effect on price

Partial vs General

- Exercise: same analysis, but with explicit functional forms

Partial vs General

- Linear model
  - Consumers

$$U(l,x,z)=\alpha\cdot l+\alpha\cdot x-\tfrac{\beta}{2}\cdot l^2-\tfrac{\beta}{2}\cdot x^2-\sigma\cdot l\cdot x+z$$

$$p_l\cdot l+p_x\cdot x+z=I$$

  - Firms

$$C(l)=c\cdot l+\tfrac{\delta}{2}\cdot l^2$$

$$C(x)=c\cdot x+\tfrac{\delta}{2}\cdot x^2+t\cdot x$$

Simplify
-1 consumer
-1 chocolate firm
-1 licorice firm
-But, price takers

Partial vs General

- "Utility"

$$\tilde{U} = \alpha \cdot l + \alpha \cdot x - \tfrac{\beta}{2} \cdot l^2 - \tfrac{\beta}{2} \cdot x^2 - \sigma \cdot l \cdot x + \left\{ I - p_l \cdot l - p_x \cdot x \right\}$$

- FOC

$$\tfrac{\partial \tilde{U}}{\partial l} = \alpha - \beta \cdot l - \sigma \cdot x - p_l = 0$$
$$\tfrac{\partial \tilde{U}}{\partial x} = \alpha - \beta \cdot x - \sigma \cdot l - p_x = 0$$

- Inverse demand

$$p_l = \alpha - \beta \cdot l - \sigma \cdot x$$
$$p_x = \alpha - \beta \cdot x - \sigma \cdot l$$

To find demand functions, solve system of inverse demand functions for x and l.

Partial vs General

- Demand

$$l = A - B \cdot p_l + S \cdot p_x$$
$$x = A - B \cdot p_x + S \cdot p_l$$

$$A \equiv \frac{\beta - \sigma}{\beta^2 - \sigma^2} \cdot \alpha > 0 \qquad B \equiv \frac{\beta}{\beta^2 - \sigma^2} > 0 \qquad S \equiv \frac{\sigma}{\beta^2 - \sigma^2} > 0$$

Partial vs General

$$p_l = \alpha - \beta \cdot l - \sigma \cdot x$$
$$p_x = \alpha - \beta \cdot x - \sigma \cdot l$$

➔

$$\beta \cdot l + \sigma \cdot x = \alpha - p_l$$
$$\beta \cdot x + \sigma \cdot l = \alpha - p_x$$

➔

➔ $$x = \frac{\alpha - p_x - \sigma \cdot l}{\beta}$$

➔ $$\beta \cdot l + \sigma \cdot \left\{ \frac{\alpha - p_x - \sigma \cdot l}{\beta} \right\} = \alpha - p_l$$

➔ $$l = \frac{\beta - \sigma}{\beta^2 - \sigma^2} \cdot \alpha - \frac{\beta}{\beta^2 - \sigma^2} \cdot p_l + \frac{\sigma}{\beta^2 - \sigma^2} \cdot p_x$$

Partial vs General

Cost

$$C(l) = c \cdot l + \tfrac{\delta}{2} \cdot l^2$$

$$C(x) = (c + t) \cdot x + \tfrac{\delta}{2} \cdot x^2$$

Marginal cost

$$MC(l) = c + \delta \cdot l$$

$$MC(x) = (c + t) + \delta \cdot x$$

Inverse supply

$$p_l = c + \delta \cdot l$$

$$p_x = (c + t) + \delta \cdot x$$

Supply

$$l = -D \cdot c + D \cdot p_l$$

$$x = -D \cdot c - D \cdot t + D \cdot p_x$$

$$D \equiv 1 / \delta$$

Partial vs General

Supply

$$l = -D \cdot c + D \cdot p_l$$

$$x = -D \cdot c - D \cdot t + D \cdot p_x$$

Demand

$$l = A - B \cdot p_l + S \cdot p_x$$

$$x = A - B \cdot p_x + S \cdot p_l$$

Equilibrium

$$A - B \cdot p_l + S \cdot p_x = -D \cdot c + D \cdot p_l$$

$$A - B \cdot p_x + S \cdot p_l = -D \cdot c - D \cdot t + D \cdot p_x$$

Partial vs General

Equilibrium, rewritten

Licorice market: $\left(D+B\right)\cdot p_l - S\cdot p_x = \left(A + D\cdot c\right)$

Chocolate market: $\left(D+B\right)\cdot p_x - S\cdot p_l = \left(A + D\cdot c\right) + D\cdot t$

Effect of change in tax on chocolate

Licorice market: $\left(D+B\right)\cdot dp_l - S\cdot dp_x = 0$

Chocolate market: $\left(D+B\right)\cdot dp_x - S\cdot dp_l = D\cdot dt$

Partial vs General

Effect of change in tax on chocolate

Licorice market: $$\frac{dp_l}{dt} = \frac{S}{D+B} \cdot \frac{dp_x}{dt}$$

Chocolate market: $$\frac{dp_x}{dt} = \frac{D}{D+B} + \frac{S}{D+B} \cdot \frac{dp_l}{dt}$$

Partial equilibrium analysis falsely assumes dp_l=0

Chocolate market: $$\frac{dp_x}{dt} = \frac{D}{D+B}$$

Partial vs General

Effect of change in tax on chocolate

Licorice market: $$\frac{dp_l}{dt} = \frac{S}{D+B} \cdot \frac{dp_x}{dt}$$

Chocolate market: $$\frac{dp_x}{dt} = \frac{D}{D+B} + \frac{S}{D+B} \cdot \frac{dp_l}{dt}$$

General equilibrium analysis

Licorice market: $$\frac{dp_l}{dt} = \frac{S}{D+B} \cdot \frac{dp_x}{dt}$$

Chocolate market: $$\frac{dp_x}{dt} = \frac{D}{D+B} + \frac{S}{D+B} \cdot \left\{ \frac{S}{D+B} \cdot \frac{dp_x}{dt} \right\}$$

Partial vs General

General equilibrium analysis

Chocolate market: $$\frac{dp_x}{dt} = \frac{D}{(D+B) - \left[\frac{S^2}{D+B}\right]}$$

Partial vs General

General equilibrium analysis

$$\frac{dp_x}{dt} = \frac{D}{(D+B) - \left[\frac{S^2}{D+B}\right]}$$

Partial equilibrium analysis

$$\frac{dp_x}{dt} = \frac{D}{D+B}$$

Partial equilibrium analysis underestimates the effect on price

Partial vs General