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AMT – Johan's Problem Set 3

Solutions

Note that question 2 is optional and will not be discussed in class.

1 Broadband in a small village far away

A telecom operator contemplates providing a small village far a way with superfast fiber broadband. The cost would be a staggering 100 monetary units. But, thanks to the enormous capacity of fiber, everyone in the village would be able to do what they want online – all at the same time – without any annoying delays.

Every inhabitant in the village has a utility function, which can be written as $u_i = q_i + \theta_i \cdot f_i$ where q_i is the number of other goods that the individual consumes, $f_i = 1$ means that the individual has bought Internet access, $f_i = 0$ means that the individual does not have access, and θ_i is a personal preference parameter. Each person has a budget constraint given by $q_i + p \cdot f_i = w_i$, where $w_i \geq 100$ is the person's income. Notice that the price of other goods is one and that the price for Internet access is denoted *p*.

There are three inhabitants in the village, characterized by their personal tastes for Internet access, θ_1, θ_2 and θ_3. The telecom operator knows the *distribution* of personal preference parameters in the population. That is, the operator knows that a third of the inhabitants have parameter θ_1, a third have θ_2 and a third have θ_3. The operator does not, however, know who is who. Therefore the operator has to charge the same price *p* to all customers.

1. What type of good is fiber for this village?
2. Under what conditions *should* the facility be built?
3. Will the market be efficient if $\theta_1 = 90$, $\theta_2 = 45$ and $\theta_3 = 10$? Why/Why not?
4. Will the market be efficient if $\theta_1 = 95$, $\theta_2 = 55$ and $\theta_3 = 15$? Why/Why not?

(Hint: How much is person *i* at most willing to pay for Internet access?)

2 Kitchen duty

Philip and Madeleine stay in the same dormitory, sharing a kitchen. Both spend the evening studying diligently in their separate rooms. They both know that the oven really should be cleaned before the morning, when their new neighbor is expected to arrive. While both Philip and Madeleine would benefit from a cleaner oven, for practical reasons, only one of them could do the job. Whenever one of them hears that the kitchen door is opened, he or she understands that the neighbor is on the way to clean the oven.

Discuss whether the oven should be cleaned and if it will be done.

3 Bee keeping

Consider a small fruit-growing village with two people. Mr. Anderson and Mr. Peterson. Mr. Anderson's production function is given by

$$f^A = \alpha \cdot \log\left(t^A\right) + \beta \cdot \log\left(h^A + h^P\right)$$

where f^A is the number of fruits he produces, t^A is the number of fruit trees he keeps and $h^A + h^B$ is the total number of beehives in the village; α and $\beta > \alpha$ are two productivity parameters. Every fruit tree requires one time-unit of work per year and every beehive also requires one time-unit of work. Thus, Mr. Anderson's time constraint is given by $1 = t^A + h^A$. Mr. Anderson wants to maximize his own production of fruits. Similarly, Mr. Peterson's production function is given by

$$f^P = \alpha \cdot \log\left(t^P\right) + \beta \cdot \log\left(h^A + h^P\right)$$

and his time constraint is $1 = t^P + h^P$.

1. Describe how much time Mr. Anderson wishes to spend on bee keeping and how that depends on the time Mr. Peterson spends on bee keeping. Depict this relationship in a diagram with h^P on the horizontal axis and h^A on the vertical axis.
2. Describe how much time Mr. Peterson wishes to spend on bee keeping and how that depends on the time Mr. Anderson spends on bee keeping. Depict this relation in the same diagram.
3. How many beehives *will* the two fruit farmers keep?
4. How many beehives *should* the two farmers keep, in order to maximize the total amount of fruit produced in the village?
5. Explain the difference between what the two farmers do and what they should do.
6. Is bee keeping a public good or an activity with positive externalities?

4 Pollution

A profit-maximizing firm produces a commodity that causes emissions of a pollutant affecting the region where it is located. The firm sells its produce in the

world market at a given price p. The firm can both decide how much to produce, q, and how clean technology to use. Let e be the emissions per unit of production caused by the firm's production. A dirtier technology (a higher e) is associated with a lower marginal cost. The firm's cost function is given by $TC = (c - \alpha \cdot e) \cdot q + \frac{1}{2} \cdot q^2$, where α is a technological parameter. The available technologies can be represented by $e \in [L, H]$ where $L \geq 0$ is the cleanest technology and $\alpha \cdot H < c$ is the dirtiest. There are *M* people living in the region and they are all affected by the pollution. The cost of pollution to citizen *j* is given by $V_j = v_j \cdot \frac{1}{2} \cdot [e \cdot q]^2$. That is, citizen *j* would be willing to pay V_j to eliminate all pollution. Define $v = \sum_{j=1}^{M} v_j$.

1. What technology e will a firm choose and how much will it produce?
2. What would be the socially optimal choice of technology and production? (Hint: use the Compensation Principle.)
3. Is it possible for the Government to make the firm behave optimally using a tax on output? If so, what tax rate would do the job? What is such a tax called?
4. Is it possible for the Government to make the firm behave optimally by limiting the total amount of emissions? If so, what limit should it set?
5. Is it possible for the Government to make the firm behave optimally by taxing emissions? If so, what tax rate would do the job? (Hint: In case the firm is indifferent between two or more actions, you may assume that the firm takes the best such action from a social welfare point of view).