Advanced Microeconomic Theory II | Johan Stennek

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PS D - Information

1 Versioning

Microware develops and sells software, among them the word-processing program Sentence. This program has terrific quality. The basic features are better designed than in any alternative software on the market, and there are several more advanced features not found in any other product. The costs of developing the program have already been taken and the cost of producing a DVD with a copy of the software is c which is rather small.

There are two types of consumers. Business people have a high valuation of quality; they are willing to pay H for the basic features and h for the advanced features. They receive utility $u_H = (H+h) - p$ if they buy a license of Sentence at price p. Students have a low valuation of quality; they are willing to pay $L < H$ for basic features and $l < h$ for the advanced features. They only receive utility $u_L = (L+l) - p$ if they buy. Any consumer not buying the software receives zero utility (this is the utility received if using the malfunctioning freeware that anyone can download from the Internet). There are equally many business people as students in the population.

1. Show that the monopolist will set $p = H + h$ if $c \geq (2L - H) + (2l - h)$. Assume this condition to be fulfilled.

Microware suddenly realizes that they can offer a student version of Sentence, with the advanced features removed. This does not affect the cost of producing the software.

2. Show that Microware can charge at most $p_s = L$ for the student version, if the students are to prefer buying the student version to not buying the program at all. What is this condition usually called?
3. Show that Microware can charge at most $p \leq L + h$ for the full version, in order to discourage the business people from buying the student version. What is this condition usually called?
4. What trade-off does Microware face, when deciding whether to market a student version in addition to the full-featured version?

5. Show that Microware indeed markets the student version if $c \le 2L - H$. Assume this condition to be fulfilled.

6. Assume that the cost of a DVD is so small that it can be approximated to zero, $c \approx 0$. Then, the conditions assumed so far can be written $2L \ge H$ and $h - 2l \ge 2L - H$. Try to give these conditions an economic interpretation.

7. Finally show that the business people actually prefer to buy the full-feature version to not buying at all and that the students prefer the student version to the full-feature version. What are these conditions usually called?

2 First-price auction

Consider a first-price, sealed-bid auction. There are two bidders, labeled $i = 1, 2$. Bidder i has valuation v_i for the good—that is, if bidder i gets the good and pays the price p, then i's payoff is $v_i - p$. The two bidders' valuations are private information. The valuations are independently and uniformly distributed on $[0, 1]$. Bids are constrained to be nonnegative. The bidders simultaneously submit their bids. The higher bidder wins the good and pays the price she bid; the other bidder gets and pays nothing. In case of a tie, the winner is determined by a flip of a coin. The bidders are risk-neutral. All of this is common knowledge.

1. What is a strategy in this game? Why?
2. When thinking about increasing her bid slightly, what trade-off does a player face?
3. Assume that player 2 uses the strategy to always bid half of his valuation. Describe the probability distribution over player 2's bids?
4. Write down an expression for player 1's expected payoff.
5. Show that player 1 then has a best reply, which is to also bid half of his valuation.
6. Derive a linear Bayesian Nash equilibrium of this game!
7. Will the bidders "tell the truth"?
8. Assess welfare!

3 First-price auction with N bidders (if time permits)

Consider now a first-price, sealed-bid auction with N bidders. All assumptions are as above. Assume that all players 2, …, N use the strategy to bid the same fraction of their valuations: $b_i = z \cdot v_i$

1. What is the probability that player 1 wins the auction if he bids b_1?
   (Hint: When flipping a coin, the probability of Head is ½. When flipping the coin twice the probability of only Heads is $(½)^2$ = ¼. When flipping the coin T times the probability of only Heads is $(½)^T$.)
2. What is player 1's expected utility?
3. Show that player 1 then has a best reply, which is to also bid a fraction of his valuation. What fraction? What if N=2? What if N is very large?
4. Derive a linear Bayesian Nash equilibrium of this game!