Advanced Microeconomic Theory II | Johan Stennek

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PS C - Dynamic Games with Applications

1 Divide & Choose

Two children are to split a cake. The first kid gets to split the cake in two pieces and the second kid gets to choose which piece he wants. Both children want as much cake as possible for themselves. This is understood by both.

For simplicity, you may assume that the first child cuts the cake into a left and a right piece and that the left piece must be 25%, 50% or 75% of the cake.

1. Write down the extensive form game tree!
2. Does this game have perfect information?
3. Does this game have complete information?
4. What is the appropriate solution concept?
5. Find all such equilibria!

If you have the time, you may also do the following exercises (not to be discussed in class):

6. Write down the normal form!
7. Find the Nash equilibria!

2 Centipede

Arne and Bertil own a highly successful startup in the it-sector. The company's value doubles every year. A potential problem is that every year one of the two owners gets the chance to steal 3/4 of the company's assets leaving 1/4 to the other. Each owner gets this chance every second year. If the assets are stolen, the company will have to discontinue its operations.

The first year, Arne gets the chance. He either takes out €3mn leaving €1mn to Bertil or lets the company survive. The second year, Bertil gets the chance. He either takes out €6mn and leaves €2mn to Arne or lets the company survive. This continues for (say) four years. If the company survives to year five, it is bought by Microsoft, in which case Arne gets €48mn and Bertil gets €16mn.

1. Write this up as an extensive form game.
2. Solve it
3. What would you have done if you where Arne?

3 Take it or leave it

A buyer and a seller meet to negotiate the price of a unique good. The buyer's valuation of the good is v_B and the seller's valuation of the good if he keeps it is $v_S < v_B$. These valuations are common knowledge. The two parties are in a hurry so they will have no time for haggling.

1. Assume that the seller gets to offer a price and that the buyer only can accept or reject it. Will there be trade? If so, what will the price be?
2. Assume that the buyer gets to offer a price and that the seller only can accept or reject it. Will there be trade? If so, what will the price be?

4 Two-Period Bargaining

Consider the same situation as above and assume that they have time for two periods of negotiations. The buyer's discount factor is δ_B and the seller's is δ_S.

1. Assume that the buyer gets to offer a price in period 1 and that the seller can make a counter offer in period 2. Will there be trade? If so, what will the price be? When will they come to an agreement? Compare with the outcome of "take-it-or-leave-it-bargaining".
2. Assume that the seller gets to offer a price in period 1 and that the buyer can make a counter offer in period 2. What will the price be? Compare with the outcome of "take-it-or-leave-it-bargaining".

If you have the time, try also to solve the game for three periods (not to be discussed in class).

5 Strategic investment in Bertrand duopoly

Consider a market with N consumers. Everybody wants to buy one unit of the good and is willing to pay at most € v for that unit. Nobody wants a second unit.

There are two firms producing exactly the same good. They compete a la Bertrand. The consumers only buy from the firm with the lowest price and if they charge the same price half of the consumers buy from each firm.

The marginal cost of production is constant for both firms. Both firms have had the same marginal cost c in the past.

Suddenly firm 1 discovers an opportunity to lower its marginal cost to $c_L < c$ by making an investment that costs €I. If the firm makes the investment, the other firm will be able to observe it. It also understands the effect on the marginal cost. The other firm cannot make any similar investment.

1. Will the firm make this investment?
2. What solution concept do you use?

6 Strategic investment in Cournot (not to be discussed in class)

Consider a monopoly firm with linear demand $p = \alpha - \beta \cdot q$ and constant marginal cost c. Recall that the monopolist will produce quantity $q^m(c) = \frac{1}{2 \cdot \beta} \cdot (a - c)$. It will thus make profit

$$\pi^m(c) = \left(\alpha - \beta \cdot q^m(c) - c\right) \cdot q^m(c) - I(c),$$

where $I(c)$ is the firm's cost of investments. In particular, before production starts, the firm can invest in modern machinery and reduce marginal cost. A lower marginal cost requires a higher investment, i.e. $I'(c) < 0$. (Notice that we think of the investment decision as a choice of marginal cost.)

1. Derive the monopolist's first-order condition for the optimal investment. Make explicit use of the Envelope Theorem.

2. Explain what the monopolist's gain from investment is.

Consider now the same market, but assume that there are 2 firms competing a la Cournot. Let q_i be firm *i*'s quantity. The price is given by the inverse demand function, which is linear and given by $p = \alpha - \beta \cdot (q_1 + q_2)$. Firm *i*'s marginal cost is constant and denoted c_i. Notice that firm 1's equilibrium profit can be written as:

$$\pi_1^*(c_1, c_2) = \left[\alpha - \beta \cdot \left(q_1^*(c_1, c_2) + q_2^*(c_1, c_2)\right) - c_1\right] \cdot q_1^*(c_1, c_2) - I(c_1)$$

where $I(c_1)$ is the firm's cost of investments. In particular, before production starts, firm 1 can invest in modern machinery and reduce marginal cost. A lower marginal cost requires a higher investment, i.e. $I'(c) < 0$.

3. Derive the duopolist's first-order condition for the optimal investment. Make explicit use of the Envelope Theorem.
4. Explain what the duopolist's gain from investment is.

7 Voluntary contributions to a public good

Consider a group of *n* people. It may be a family, a firm or a village. Everyone has an income of *I* and can split this income between buying private goods and contributing to a public good. Individual *i*'s contribution is denoted c_i and the total amount of public good produced is $g = \sum_{i=1}^{n} c_i$. Individual *i*'s utility is given by $u_i = \theta \cdot (I - c_i) + g$ where $I - c_i$ is *i*'s consumption of private goods (which sell at price equal to unity) and $\theta \in (1, n)$ is a preference parameter.

1. What would happen if this situation only occurs once? Be careful to explain your reasoning. (Choice of game-theoretic solution concept and so on.)
2. What would happen if this situation is repeated many times? Be careful to explain if you need to make additional assumptions to analyze this situation. Be careful to explain your choice of solution concept and so on.