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## 1 Cournot Duopoly with differentiated goods

Consider a differentiated goods market with 2 firms competing a la Cournot. Firm  $i$ 's inverse demand is given by:

$$p_i = \alpha - q_i - \sigma \cdot q_j.$$

where  $p_i$  is firm  $i$ 's price and  $q_i$  denotes the quantity of firm  $i$ 's product that the consumer consumes. Both firms have the same constant marginal cost  $c$ .

1. What is the interpretation of  $\sigma$ ? What does  $\sigma=0$ ,  $\sigma=1$  and  $0 < \sigma < 1$  mean?
2. What is the interpretation of  $\alpha$ ? Should  $\alpha$  be larger or smaller than  $c$ ?
3. Write down an expression for the firms' profits!
4. Find the best-reply functions!
5. Compute the equilibrium prices.
6. What determines the strength of competition? Hint: How does price depend on  $\sigma$ ? Explain!

## 2 Cournot Oligopoly

Consider a homogenous goods market with  $n$  firms competing a la Cournot. Let  $q_i$  be firm  $i$ 's quantity and  $Q = \sum_{i=1}^n q_i$  the aggregate quantity. The price is given by the inverse demand function, which is linear and given by  $p = \alpha - Q$ . The firms' marginal cost is constant and denoted  $c$ .

1. Compute the equilibrium price.
2. What determines the strength of competition?
3. Compute the firms' equilibrium profits.

### 3 Cournot duopoly with different costs

Consider a homogenous goods market with 2 firms competing a la Cournot. Let  $q_i$  be firm  $i$ 's quantity. The price is given by the inverse demand function, which is linear and given by  $p = \alpha - \beta \cdot (q_1 + q_2)$ . Firm  $i$ 's marginal cost is constant and denoted  $c_i$ . Firm 1's profit is thus given by:

$$\pi_1(q_1, q_2) = [\alpha - \beta \cdot (q_1 + q_2) - c_1] \cdot q_1.$$

1. Compute the firms' equilibrium quantities as functions of the marginal costs, i.e.  $q_1^*(c_1, c_2)$  and  $q_2^*(c_1, c_2)$ .
2. How do the firms' (relative) equilibrium quantities depend on their (relative) marginal costs?
3. How does the equilibrium price depend on firm 1's marginal costs? Explain why firm 1's marginal cost affects the equilibrium price.

### 4 Effects of a change in marginal cost (difficult)

Consider the same market as in the previous question. Notice that firm 1's equilibrium profit can be written as:

$$\pi_1^*(c_1, c_2) = \pi_1(q_1^*(c_1, c_2), q_2^*(c_1, c_2)) = [\alpha - \beta \cdot (q_1^*(c_1, c_2) + q_2^*(c_1, c_2)) - c_1] \cdot q_1^*(c_1, c_2).$$

Assume that firm 1's marginal cost is reduced by a very small amount. Explain how firm 1's profit is affected, making explicit use of the Envelope Theorem.

## 5 Bertrand duopoly with different costs

Consider a market with 1000 consumers. Everybody wants to buy one unit of the good and is willing to pay at most €  $v$  for that unit. Nobody wants a second unit.

There are two firms, called A and B, producing exactly the same good. They compete a la Bertrand. The consumers only buy from the firm with the lowest price and if they charge the same price half of the consumers buy from each firm.

The unit cost of production is constant for both firms. But firm A is more efficient. Firm A's unit cost is and given by  $c_A = 1 < 2 = c_B$

1. What prices will the two firms set?
2. How many units will they sell?
3. Compare the effect of a cost difference between Cournot and Bertrand!

## 6 Monopoly and Bertrand Oligopoly (Extra)

[Not to be discussed in class]

Consider a market with 1000 consumers. Everybody wants to buy one unit of the good and is willing to pay at most € 5 for that unit. Nobody wants a second unit.

The unit cost of production is constant and given by € 2. At the outset there is only one firm serving the market. But then a second firm enters and starts to produce an identical good. The two firms compete in prices a la Bertrand. The consumers only buy from the firm with the lowest price and if they charge the same price half of the consumers buy from each firm. The smallest monetary unit is 10 cents which means that the firms can only set their prices in multiples of € 0.1, e.g. 0, 0.1, 0.2, 0.3 and so on.

4. What is the monopoly price?
5. How many units does the monopolist sell?
6. Is the outcome efficient or inefficient? Explain!
7. What is the duopoly price? When answering this question, be careful to draw a diagram of the firm's best-reply functions.
8. Is competition good for welfare?