Advanced Microeconomic Theory II — Johan Stennek

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PS - Static Games

## 1 Game in Normal form

What do you need to specify to define a game in normal form?

Consider the game below:

a) What does the rows U and D signify?
b) What do the columns L and R signify?
c) What do the numbers 5 and 6 signify?

| | L | R |
|---|---|---|
| U | 0, 2 | 3, 1 |
| D | 7, 6 | 5, 4 |

## 2 Best reply

Consider the following game:

| | D | E | F |
|---|---|---|---|
| A | 0, 2 | 3, 1 | 4, 3 |
| B | 2, 4 | 0, 3 | 3, 2 |
| C | 1, 1 | 2, 0 | 2, 1 |

1. What is meant by a best reply?
2. What is meant by a best-reply function?
3. Identify the best-reply functions for both players!

## 3 Nash equilibrium

Consider the same game as in the previous question.

1. What is meant by a Nash equilibrium?
2. What are the Nash equilibria in pure strategies?

## 4 Nash equilibrium

For which values of a, b and c is the strategy profile (T, L) a pure strategy Nash equilibrium?

| | L | R |
|---|---|---|
| T | a, b | c, 2 |
| M | 1, 1 | 1, 0 |
| B | 3, 0 | 0, 1 |

## 5 A three-player game

Consider the following game:

| | L | R |
|---|---|---|
| U | 3, 2, 3 | 1, 2, 0 |
| D | 1, 2, 0 | 1, 2, 0 |
| A | | |

| | L | R |
|---|---|---|
| U | 1, 4, 2 | 1, 2, 0 |
| D | 1, 2, 0 | 3, 4, 1 |
| B | | |

Player 1's strategy space is {U, D}, player 2's strategy space is {L, R}, player 3's chooses one of the two payoff matrices: his strategy space is thus {A, B}. Find all pure-strategy Nash equilibria!

## 6 Voluntary contributions to public goods

Two individuals ( $i = A, B$ ) wish to finance a public project. Each individual is endowed with wealth $w_i > 1$ and is free to contribute any amount $c_i$ to the public project, with $0 \le c_i \le w_i$. The contributed money is used to produce a public good $y$ according to the production function:

$$y = \sqrt{c_A + c_B}\,.$$

Both individuals have the same utility function: $u_i = y + w_i - c_i$.

Assume that the two individuals have to decide on their contributions at the same time.

1. What is player A's strategy set?
2. Give an example of a strategy profile!
3. Find all pure-strategy Nash equilibria!

4. Are the contributions socially efficient?
5. Assume someone suggests that both players make socially efficient contributions. What will then happen?

## 7 Externality game

Anderson and Svenson are neighbors and they both love listening to music. Both own audio systems with a maximum loudness of 100 decibel.

Assume first that no sound can travel through the walls between the two apartments, no matter how loud they play. Each person's utility is then $U_i = 75 \cdot m_i - \frac{1}{2} \cdot m_i^2$ where $m_i$ is the loudness.

1. How loud will they play?

Assume now that sound does travel through the walls. Each person's utility is then $U_i = 75 \cdot m_i - \frac{1}{2} \cdot m_i^2 - 50 \cdot m_j + \frac{1}{4} \cdot m_i \cdot m_j$ where $m_j$ is the loudness of the neighbor's music.

2. Do Anderson and Svenson consider the neighbor's music a positive or a negative externality?
3. Find neighbor *i*'s best-reply function. Interpret!
4. How loud will they play? Compare with question 1.
5. If Anderson and Svenson could agree on a common decibel limit. What should that limit be? Compare with question 1 and 4.
6. Assume that Anderson and Svenson indeed make this agreement. How loud will they then play?