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Problem Set 2

Calibrating an Entry Model

* = discussed in class

Introduction

Competition is intense if prices are close to marginal costs. But since prices are often difficult to observe (e.g. due to secret rebates) and since costs are almost never observable to an outsider, it is typically difficult to infer how strong competition is in a particular market. The purpose of this exercise is to infer the strength of competition without data on prices and costs. Instead only data on market size and concentration will be used.

Let us think about the widget market. The widget market is segmented into several separate geographical markets ("towns"). People in one town cannot shop for widgets in other towns as they are too far away. Concentration differs between towns: in some towns there are no firms, while in others there are one or two widget firms. In no town are there more than two firms. There are typically more firms in larger towns and it is often assumed that, as a result, prices are lower in larger towns. Nobody knows for sure, however.

An important reason for why concentration is rather high in the widget markets is that firms need to overcome some bureaucratic hurdles before entering a town. For instance, the firms must apply for municipal approval before starting operations at a certain location (zoning rights). The Government contemplates removing these hurdles to reduce entry costs in order to increase competition. First, however, the Government wants to know if competition is effective in the widget market. You are hired as a consultant to answer this question.

Profit Data *

The problem is that you have very limited data. You cannot observe prices and quantities. Those are business secrets. What you can observe though is the accounting profits in firms, both in monopoly and duopoly markets. Can you use these data to infer the intensity of competition? You should be able to say something since profits will be lower in duopoly markets than in monopoly markets only if firms really compete.

To give it a try, you are willing to make some assumptions that you find reasonable. You assume that the demand for widgets in a town is linear and given by

$$p_i = \alpha - \tfrac{1}{m} q_i - \tfrac{\sigma}{m} q_j ,$$

where *m* is the number of inhabitants in the town and σ is a measure of how close substitutes the consumers consider widgets from different firms to be. (If there is only one firm in a town $q_j = 0$.) Of course, you do not know neither α nor σ. You assume the marginal cost of producing widgets to be constant and equal to some unknown parameter *c*. There is also an unknown entry cost *F*. If there are two firms in a town, you assume them to compete a la Cournot.

According to your model of the widget market, the key determinant of competition is the degree of substitutability σ. Thus, your job will be to devise a strategy to infer σ from profits.

Question T1: Your first task is theoretical, to establish the determinants of profitability in the widget market according to the model. We focus on gross profit, i.e. the profit before deducting the entry costs, F.

- Compute the profit of monopoly firms, π^M. Hint: Start out by setting up the monopolist's profit function. Derive first-order condition. Solve for optimal quantity. Substitute optimal quantity back into profit function.
- Compute the profits of duopoly firms, π^D. Hint: As above, but you also need to solve the system of the two best-reply functions (or first-order conditions) for the two equilibrium quantities.
- How does firm profit depend on market size (i.e. the number of inhabitants), the number of firms and substitutability?
- Draw a diagram with market size m on the x-axis and money on the y-axis. Display how the monopoly and duopoly profits depend on market size in this diagram.

Now you know how the number of firms and the degree of substitutability determine profits. Next, you can *invert* this relationship to see what σ is required to explain the pattern of profits observed in the widget market.

Question E1: Your second task is empirical, to infer σ from profits, the number of firms and market size, by "calibrating" the model in Question T1. The accounting profits do not include entry costs, since those costs are historical.

- Describe how you can infer σ by comparing the firms' profits *per inhabitant* in monopoly and duopoly towns. Hint: What is the ratio $\left(\pi^M/m\right)/\left(\pi^D/m\right)$ according to theory?
- What σ would you infer if you observe that duopoly firms earn the same profit per inhabitant as monopoly firms?
- What observed profit levels would make you infer that widgets are perfect substitutes?

Alas, your preliminary report to the Government is heavily criticized on the ground that accounting profit is a poor measure economic profit. What should you do?

Data on concentration *

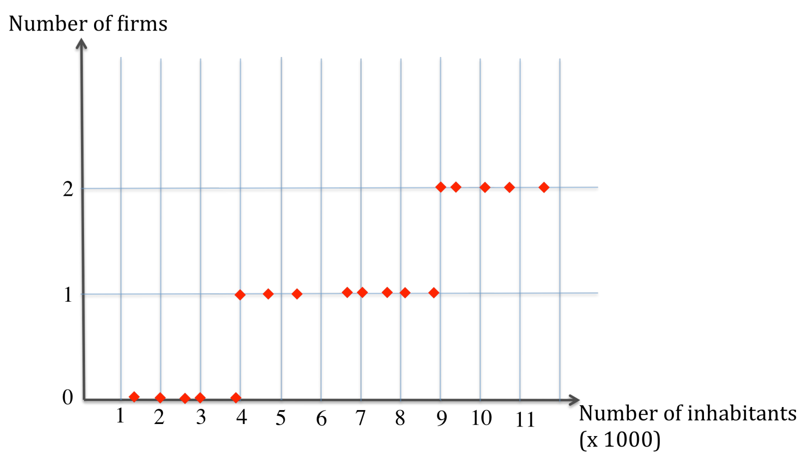
The only reliable data that you have access to is market size and the number of firms (concentration) in different towns. The question is: Will you be able to infer the degree of substitutability from this information only?

Let us try. As a start you need to reformulate your theory in terms of the observable variables. Instead of a theory explaining economic profit, which is unobservable, you need a theory explaining the number of firms. For this purpose, you may use an entry model. The simplest possible entry model builds on the assumption that new firms continue to enter a town until further entry is unprofitable.

Question T2: Your third task is theoretical, to establish the relationship between market size, the degree of substitutability and the number of firms in the widget market. Recall that the entry cost is denoted by F.

- What is the minimum number of inhabitants in a town for a first firm to enter? Denote this number by m_1. Hint: Use the zero-profit condition for a monopoly firm.
- What is the minimum number of inhabitants in a town for a second firm to enter? Denote this number by m_2. Hint: Use the zero-profit condition for a duopoly firm.
- Which is larger, m_1 or m_2? Explain!
- How do m_1 and m_2 depend on the degree of substitutability?
- Draw a diagram with market size m on the x-axis and money on the y-axis. Display the monopoly and duopoly profits in this diagram. Display the entry cost F. Indicate the locations of m_1 and m_2.
- Show that if $\sigma = 1$, then $m_2/2 > m_1$. Explain! Hint: Note that $m_2/2$ is the number of customer per firm in a market with just enough inhabitants for a second firm to enter.

Assume now that you have collected a dataset consisting of the number of firms and market size for a large number of towns. The data set is illustrated in the following figure.



The red diamonds represent towns in the dataset. Clearly there are more firms in larger towns, which is consistent with your theory.

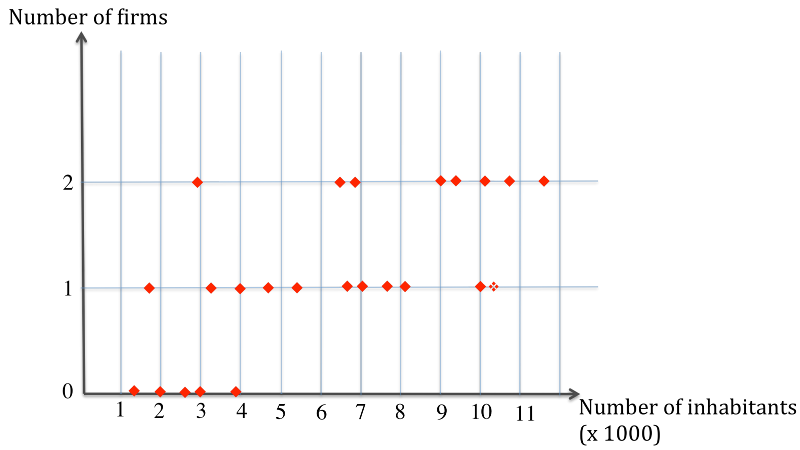
Question E2: Your fourth task is empirical, to infer σ from the dataset above, by "calibrating" the model in Question T2.

- According to the data, what is the minimum number of inhabitants in a town for a first firm to enter ($\hat{m}_1$)?
- According to the data, what is the minimum number of inhabitants in a town for a second firm to enter ($\hat{m}_2$)?
- Infer $F/\left(\frac{\alpha-c}{2}\right)^2$ from $\hat{m}_1$ using the relationship between market size and entry derived in Question T2.
- Similarly, infer $F/\left(\frac{\alpha-c}{2+\sigma}\right)^2$ from $\hat{m}_2$.
- Finally, infer σ from your estimates of $F/\left(\frac{\alpha-c}{2}\right)^2$ and $F/\left(\frac{\alpha-c}{2+\sigma}\right)^2$.
- How close substitutes are widgets?

Sorry, but the dataset you just worked on is not the correct one. It was cleaned for teaching purposes.

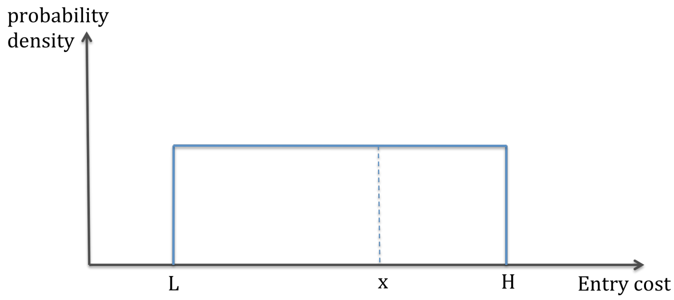
Fuzzy data on concentration

The correct dataset is, of course, much more complex. Even if it is evident that larger towns tend to have more firms on average, it is not unusual that towns of the same size have different number of firms.



How can we reconcile this complexity with our theory?

One possibility is that the entry cost (F) is different in different towns. So, let us assume that all firms in the same town face the same entry cost, but that the entry cost may differ between different towns. Let us also assume for simplicity that the entry cost is distributed uniformly over some interval $[L,H]$. This distribution is described by the following figure.



Recall that with a uniform distribution, the probability that the entry cost is below some value *x* is given by $\frac{x-L}{H-L}$.

Question T3: Your fifth task is theoretical, to establish the relationship between market size and the *distribution* of concentration (number of firms) in the widget market. Think of all the towns of a certain size *m*. Some of these towns will have zero firms due to very high entry costs. Some other towns will have one firm since entry costs are moderately high. Yet other towns will have two firms due to really low entry costs. There is thus a distribution of concentration levels even if we only study towns of the same size.

- Consider all the towns with the same size $m = m'$.[1] These towns may have different entry costs. Clearly all towns with very high entry costs will not see any entry. Other towns with a smaller entry cost will see one firm enter the market. Compute the threshold entry cost for one firm to enter the market. Expressed differently, what is the smallest entry cost that a town could have and still not get any entry? Denote this entry cost by F_0.

  Hint: Use the zero-profit condition.
- Towns with even smaller entry costs will see two firms enter the market. Compute the threshold entry cost for a second firm to enter the market. Denote this entry cost by F_1.

[1] In this question it is convenient to distinguish between the variable m and the specific number m'.

- Draw a diagram with market size on the x-axis and money on the y-axis. Display the monopoly and duopoly profits in this diagram. Draw a vertical line at m'. Indicate F_0 and F_1 in the figure.
- Also indicate F_0 and F_1 in the probability density figure above.
- Also indicate in the figure the probability that there are no, one or two firms in a market of size *m*.
- Compute the probability that there are no, one or two firms in a market of size *m*. Denote these probabilities by s_0, s_1 and s_2. Focus on an example where $L < F_1 < F_0 < H$.
- How do the probabilities depend on market size?
- How do the (observable) probabilities depend on the (unobservable) degree of substitution?

To simplify things, you will work with an extraction from the correct dataset, which is described by the following table. As you can see, this data concerns towns of two sizes only. It describes the number of towns with no, one or two firms for both categories.

	Towns with 1000 inhabitants	Towns with 2000 inhabitants
Number of towns with no firm	270	180
Number of towns with one firm	50	100
Number of towns with two firms	40	80
Total number of towns	360	360

Question E3: Your sixth task is empirical, to infer σ from the data on concentration, by "calibrating" the model in Question T3. Since we have so many observations we assume that we can observe the probability of no, one and two firms without error. Let s_0^{1000} denote the observed probability that there are no firms in a town of 1000 inhabitants. Clearly this probability is 75%. And so on.

- Use $s_0^{1000} - s_0^{2000}$ to infer $\frac{1000\left(\frac{\alpha-c}{2}\right)^2}{H-L}$. Hint: What is the relationship according to theory?
- Use $s_2^{2000} - s_2^{1000}$ to infer $\frac{1000\left(\frac{\alpha-c}{2+\sigma}\right)^2}{H-L}$.
- Infer σ from $\frac{1000\left(\frac{\alpha-c}{2}\right)^2}{H-L}$ and $\frac{1000\left(\frac{\alpha-c}{2+\sigma}\right)^2}{H-L}$.
- How close substitutes are widgets?

Policy Conclusions *

It is time to sum up your report to the Government, to draw the policy conclusions

Question P1: Given your estimate of the strength of competition in the widget market, do you think that it would make sense for the Government to make an effort to reduce entry costs?

You have succeeded to estimate the strength of competition from a dataset consisting only of the number of firms and market size. Congratulations—That is pure magic! Or, economics, as we prefer to call it.

Not all Governments officials are aficionados of economic methods, however.

Question P2: Please go through the exercise again and list all the assumptions that you had to make on the way. How important is e.g. the assumption on a particular oligopoly model? What is your take—should we rely on the report?

Epilogue

In this exercise you inferred σ by calibrating the model. The more normal procedure would be to estimate σ using the maximum likelihood estimator. That will be the topic of a later lecture. The estimation procedure is less transparent than the calibration done here. The basic intuition for why the estimation procedure succeeds in identifying the strength of competition is, however, the same.