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Problem Set 1

Oligopoly, market shares and concentration indexes

* = Only these questions are discussed in class

1 Price Competition

This problem set gives you some experience in working with formal oligopoly models. We will use the linear Bertrand and the Cournot models and allow the firms to have different marginal costs.

We will also consider the case when the firms produce differentiated products. Instead of working with "localized competition" a la Hotelling, where goods are described by their characteristics and consumers have different preferences over characteristics, we will here assume that all consumers are identical but that they "love variety," i.e. that they like to consume a little of many different variants of the product rather than a lot of just one variant.

All the oligopoly models except Bertrand competition with homogenous goods can be solved using the same methodology: (i) define the firms' profit function, (ii) find the firms' first-order conditions (and, possibly, compute their best reply functions), (iii) solve the set of first-order conditions (or best reply functions) for the unknown prices (Bertrand) or quantities (Cournot).

To solve these exercises, you may need to revise the lecture on oligopoly in the Advanced Microeconomic Theory II course or to read the chapter on oligopoly in "How Markets Work."

1.1 Cournot Oligopoly with Homogenous Goods and Different Costs

This exercise investigates whether production costs are minimized in a Cournot oligopoly market.

Consider a homogenous goods market with n firms competing a la Cournot. Let q_i be firm i's quantity and $Q = \sum_{i=1}^{n} q_i$ the aggregate quantity. The price is given by the inverse demand function, which is linear and given by $p = \alpha - Q$. Firm i's marginal cost is constant and denoted c_i. Let $\overline{c} = \frac{1}{n}\sum_{i=1}^{n} c_i$ be the average cost in the market.

1. Compute the equilibrium price.
2. How does the equilibrium price depend on the *distribution* of costs (mean, variance et.c.)?

3. Compute firm i's quantity.
4. Under what conditions will firm *i* produce a positive output?
5. Keeping the number of firms in the market fixed, what determines firm i's market share $s_i = q_i/Q$?
6. Are production costs minimized in the market?

1.2 Bertrand Duopoly with Homogenous Goods and Different Costs

This exercise investigates whether production costs are minimized in a Bertrand oligopoly market.

Consider a homogenous goods market with 2 firms competing a la Bertrand. Let p_i be firm *i*'s price. Assume that firm 1's cost is c_L and that firm 2's cost is higher $c_H > c_L$. Market demand is given by $D(p)$. If the firms charge the same price, half of the consumers buy from each firm. Let p_1^m denote the price that firm 1 would choose if it were a monopolist and p_2^m the price that firm 2 would charge if it were a monopolist. Note that $p_2^m > p_1^m$ since firm 2's cost is higher.

1. Compute firm 2's best-reply function. Display it in a diagram with p_1 on the x-axis and p_2 on the y-axis. Hint: What price would firm 2 like to charge if firm 1 charges a price above p_2^m? What price would firm 2 like to charge if firm 1 charges a price at or below c_H? What price would firm 2 like to charge if firm 1 charges a price between firm 2's monopoly price and cost?[1]
2. Compute firm 1's best-reply function. Display it in the same diagram as above.
3. Find the Nash equilibrium. Hint: Use the diagram.
4. Is there any market power and, if so, what determines market power?

[1] To avoid some technical complications you should go back to see how we solved the Bertrand model with homogenous goods, but with identical costs. One problem is that sometimes several different prices may all give the highest possible profit. You may then select one of these prices for the best-reply function. Another problem is that sometimes firms want to undercut their rival's price with the smallest possible amount. You may then view $p_j - \varepsilon$ as the best reply (despite the fact that ε could be made arbitrarily small).

5. Are production costs minimized in the market?
6. Explain the difference to Cournot.

1.3 Cournot Oligopoly with Differentiated Goods

Consider a differentiated goods market with *n* firms competing a la Cournot. The consumer's utility is quadratic and given by

$$U = \alpha \sum_{i=1}^{n} q_i - \tfrac{1}{2} \sum_{i=1}^{n} q_i^2 - \sigma \sum_{i=1}^{n} q_i \sum_{j \neq i} q_j + y$$

where q_i denotes the quantity of firm i's product that the consumer consumes, y is the quantity consumed of an outside good (also serving as a numeraire) and $\sigma \in [0,1)$ is the degree of substitution between the q-goods. Let p_i be firm *i*'s price. All firms have the same constant marginal cost *c*, which is assumed to be lower than the demand intercept, i.e. $c < \alpha$. Firm *i*'s inverse demand is then given by:

$$p_i = \alpha - q_i - \sigma \sum_{j \neq i} q_j .$$

1. What does $\sigma = 0$ signify? What does $\sigma = 1$ signify?
2. Compute the equilibrium prices. If you wish, you may assume that the equilibrium is symmetric, i.e. that the firms produce the same quantity $q_i = q$. But, if you can, try to solve the problem without assuming symmetry.
3. What determines the strength of competition? Hint: How does price depend on the number of firms and the degree of substitutability?

1.4 Bertrand Oligopoly with Differentiated Goods *

Consider a differentiated goods market with *n* firms competing a la Bertrand. The consumer's utility is quadratic and given by

$$U = \alpha \sum_{i=1}^{n} q_i - \tfrac{1}{2} \sum_{i=1}^{n} q_i^2 - \sigma \sum_{i=1}^{n} q_i \sum_{j \neq i} q_j + y$$

where q_i denotes the quantity of firm i's product that the consumer consumes, y is the quantity consumed of an outside good (also serving as a numeraire) and $\sigma \in [0,1)$ is the degree of substitution between the q-goods.

Let p_i be firm *i*'s price. The indirect demand functions, used in the Cournot model above, are easy to derive. To derive the demand functions, one has to invert the system of linear indirect demand function. One may then show that firm *i's* demand is linear and given by

$$q_i = A - Bp_i + G\left(\tfrac{1}{n-1}\sum_{j\neq i} p_j\right)$$

where

$$\begin{aligned} A &= \frac{(1-\sigma)\alpha}{1+(n-2)\sigma-(n-1)\sigma^2} \geq 0 \\ B &= \frac{1+(n-2)\sigma}{1+(n-2)\sigma-(n-1)\sigma^2} \geq 0 \\ G &= \frac{(n-1)\sigma}{1+(n-2)\sigma-(n-1)\sigma^2} \geq 0 \end{aligned}.$$

(If you wish, you may try to show this yourself!) Note that if the goods would be perfect substitutes, ie if $\sigma = 1$, then the above expressions would entail division by zero, $1+(n-2)\sigma-(n-1)\sigma^2 = 0$, which is not allowed. The reason is that in this case we cannot solve for *separate* demand functions for the firms. That is, the demand for the individual goods cannot be determined.

Note also that if firm 1 would charge a very high price p_1 in comparison to the price charged by the competitors, firm 1 would not be able to sell anything, i.e. $q_1 = 0$. The demand function seems to suggest, however, that the quantity could be negative. But this is just because we have taken a short-cut here, simply assuming that the two prices will not be too different. Luckily, this simplification turns out to be okay in the analysis we will do here.

All firms have the same constant marginal cost *c*, which is assumed to be lower than the demand intercept, i.e. $c < \alpha$.

1. Compute the equilibrium prices (considering A, B and G as parameters). You may assume the equilibrium to be symmetric, i.e. that all firms charge the same price ($p_i = p$) in equilibrium.
2. Compute the equilibrium price as a function of the "deep parameters," i.e. α, σ, n.
3. What determines the strength of competition (market power) in the market?
    a. How does price depend on the number of firms? What happens to price if the number of firms is one, two or tends to infinity?
    b. How does price depend on the degree of substitution? What happens to price if goods are unrelated $\sigma = 0$ of almost perfect substitutes $\sigma \to 1$?

1.5 Comparing Cournot and Bertand

Compare the Cournot equilibrium price and the Bertrand equilibrium price, when firms produce differentiated products but have the same marginal cost. Which is higher?

2 Quality competition *

Many welfare sectors, such as childcare, education and health care, have been deregulated in recent decades. Private firms are allowed to enter these sectors and the users are allowed to choose their own supplier. Prices are often regulated and in some cases the government provides the users with vouchers. The main idea is that the firms will then compete in the quality of the services they offer.

We will study a local market with only two private firms and no public producer. The two firms are located at the two extreme ends of a street of length one and all the users live uniformly distributed over the street. The users' transportation cost is t per unit of distance. That is, it costs t to move from one end of the street to the other.

All users wish to consume one (and only one) unit of the welfare service. Let V_0 and V_1 denote the quality of the two firms, located at points 0 and 1. Then the utility of a user who lives at point x and who selects firm 0 is given by

$$U(0,x) = V_0 - t \cdot x .$$

If the user selects the other firm, the utility is

$$U(1,x) = V_1 - t \cdot (1-x).$$

A firm's profit is given by

$$\pi_i = (p - c \cdot V_i) \cdot q_i$$

where p is the voucher price that a firm receives from the government for every user it serves, $c \cdot V_i$ is the unit cost of production, which is higher the higher is the quality, and q_i is the number of users served by the firm.

The timing is as follows. First the two firms simultaneously choose what quality they wish to offer and then the users select a service provider.

1. Derive demand for both firms.
2. Find the Nash equilibrium of the quality competition game.

3. Explain how the equilibrium quality depends on the voucher price set by the government.
4. What determines the firms' equilibrium profits?

3 Market shares & Concentration indexes

It is common that regulatory authorities assume firms with a high market share to have more market power than firms with a lower market share. In fact many regulations are based on market shares. One example is European competition policy. The European Treaty includes a prohibition of “abuse of dominant position”, which means that dominant firms are not allowed to use certain strategies that are open to other firms. Dominant firms may for example not be allowed to price discriminate between different customers. To determine if a firm is dominant, the firm’s market share is important. It is e.g. unlikely that the European Commission would take any action against a firm with a market share below 25% (Motta, 2004).

Another example is the regulation of telecommunications in Europe. Telecom companies with significant market power (SMP) have certain obligations. They are required to interconnect their networks and to charge cost-based prices. Any company with more than 25% market share is presumed to have SMP.

Yet another example is the press subsidy in Sweden. Only the second largest newspaper in any geographical market is eligible.

It is also common that regulatory authorities assume that markets with high concentration, often measured by the so-called Herfindahl index (HHI), are less competitive than markets with low concentration. For example, US antitrust authorities will assess the legality of mergers based on the HHI. If the market is already highly concentrated, or if the market would become substantially more concentrated as a result of the merger, the authorities are much more likely to challenge the merger in court (US Federal Merger Guidelines).

This exercise aims at finding a foundation for these assumptions.

3.1 Basic Definitions *

Consider a homogenous goods market with inverse demand function $p = P(Q)$, relating the market price p to the quantity sold Q.

Market power

1. What do we mean by market power?
2. A common measure of a firm's market power is the Lerner index, which is defined as $L_i = (p - c_i)/p$, where c_i is the firm's marginal cost. Explain how this measure relates to the definition.
3. Why may public authorities be worried about market power?

Demand elasticity

4. What do we mean by price elasticity of demand?
5. Find an expression for the price elasticity of demand using the indirect demand function. Hint: Start by differentiating the demand function and recall that the percentage change in price is dp/p and the percentage change in quantity is dQ/Q.[2]
6. Show that the elasticity of demand is an important determinant of market power? Hint: Derive the inverse elasticity rule of monopoly pricing. Explain the intuition!

Concentration

The Herfindahl index is defined as $HHI = \sum_{i=1}^{n} s_i^2$, where s_i is firm *i*'s market share.

7. Compute the *HHI* for the case that all the *n* firms have equal market shares.
8. What happens to *HHI* if the smaller firm's market share increases in a duopoly market? Hint: In a duopoly market, $HHI = s_1^2 + (1 - s_1)^2$.

[2] Note that you may define the elasticity with or without a minus sign. You can choose whichever alternative you prefer.

3.2 Market Shares *

Consider a homogenous goods market with *n* firms competing a la Cournot. Let q_i be firm *i*'s quantity and $Q = \sum_{i=1}^{n} q_i$ the aggregate quantity in the market. The price is given by the inverse demand function, $p = P(Q)$. Firm *i*'s marginal cost is constant and denoted c_i.

1. Compute the Lerner index for a Cournot firm. Hint: Start with the firm's first order condition. Then use a similar procedure as when you derive the inverse elasticity rule for a monopoly firm.
2. How is a Cournot firm's market power (as measured by the Lerner index) related to its market share?
3. How does the model explain that different firms in the same market have different market shares?
4. Assuming that the regulatory authorities are worried about market power, why don't they base their policies directly on e.g. the Lerner index rather than on market shares?
5. Assume that you are monitoring a certain firm on behalf of some regulatory authority and that you observe that the firm's market share suddenly increases substantially. Should you be worried?

3.3 Concentration Indexes *

Consider the same market as in the previous question.

1. The (weighted) average Lerner index in a market can be defined as $L = \sum_{i=1}^{n} s_i L_i$ where $s_i = q_i / Q$ is firm *i*'s market share. Compute the average Lerner index for the Cournot market.
2. How is market power and concentration, as measured by the Herfindahl index, related?
3. Assume that you are monitoring a certain market on behalf of some regulatory authority and that you observe that the concentration suddenly increases substantially. Should you be worried?

3.4 Conduct Parameter (difficult)

Sometimes authorities try to measure the intensity of competition in a more direct way, using econometric techniques. Some empirical work is based on the idea of the so-called conduct parameter, which is meant to measure the intensity of competition in a (homogenous) goods market.

The equilibrium price is then assumed to satisfy the following equation:

$$P(Q) + \theta \cdot \frac{\partial P(Q)}{\partial Q} \cdot Q = c,$$

akin to a first order condition for profit maximization, where $P(Q)$ is inverse demand, Q is aggregate market output, c is marginal cost, and θ is the conduct parameter. Answer the following questions, to get some feeling for the conduct parameter:

1. For which value of θ would the above equation be a correct description of price under perfect competition or Bertrand competition?
2. For which value of θ would the above equation be a correct description of price under monopoly or a cartel?
3. For which value of θ would the above equation be a correct description of price under Cournot competition?

Assume now that you have econometric evidence showing that the elasticity of demand is around -2 and that you have access to an engineering report indicating that marginal cost is approximately 1.[3] The market price is 2.

4. What would you infer the conduct parameter to be?
5. Which market form seems to be the best description of the market?
6. How is this conclusion affected by knowing that there are 2 firms in the market?

[3] If your definition of demand elasticity includes a minus sign, then the demand elasticity should be +2.