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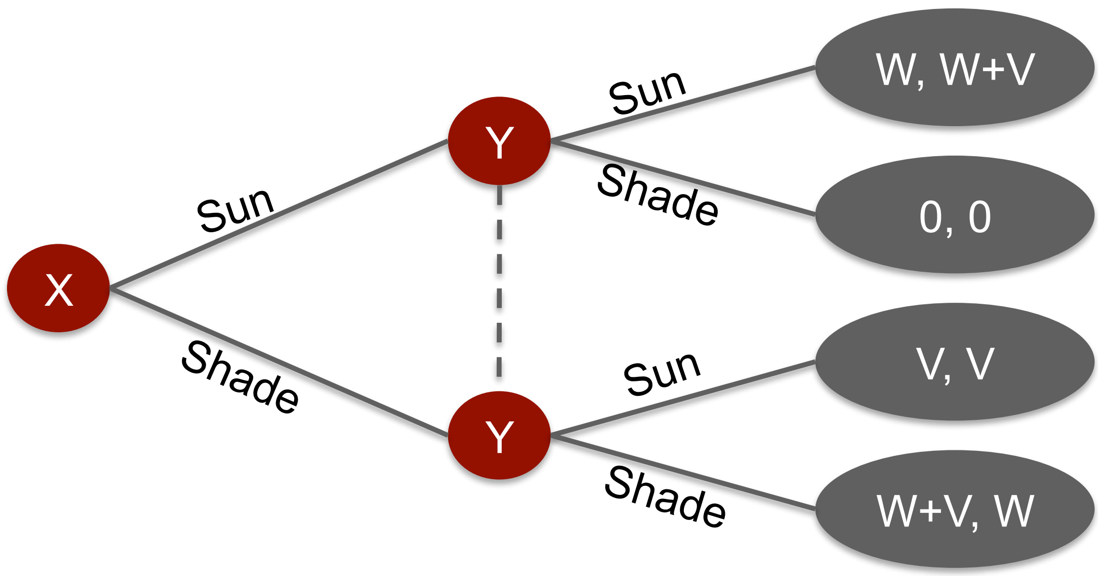
Sun or Shade 2: Simultaneous choices

X and Y need to talk. They can either meet at a restaurant called The Sun or at a restaurant called The Shade. Y prefers to meet at The Sun and X prefers to meet at The Shade. But both prefer to meet at the other person's favorite restaurant to sit alone in their own favorite restaurant. The problem is that the two players have to choose restaurant simultaneously, i.e. each player has make up his mind before knowing what restaurant the other person chose.

We can describe this situation as an extensive form game, using the following game tree, where the first payoff belongs to X and the second to Y:

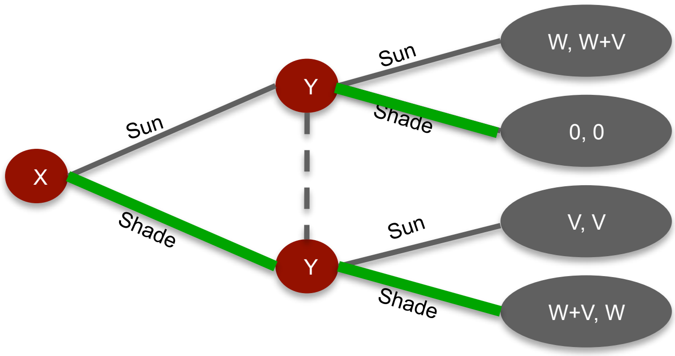


To draw a game tree, we need to put one of the players first, even though both players make their choices at the same time. But to indicate that Y does not know X's choice, before making his own, we have entered the dashed line between Y's two nodes. That is, the dashed line means that Y does not know which node he is at. The two nodes belong to the same *information set*.

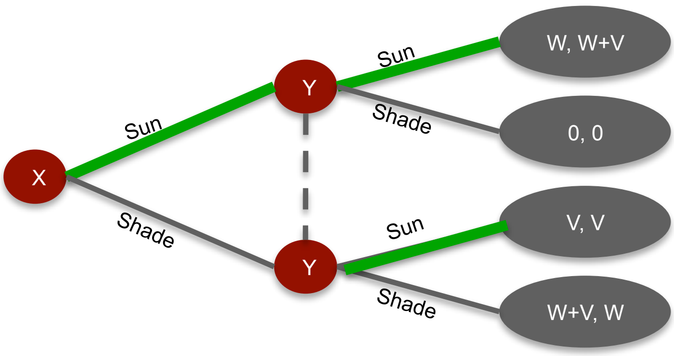
Since X and Y make their choices simultaneously, it is not possible to predict what Y will do before we know what X will do. Similarly, we cannot predict X's choice before we predict Y's choice. It is thus not possible to use backwards induction to analyze this game. We need to analyze the whole game at the same time, meaning that we predict both X's and Y's decisions at the same time. This means that we have to search for a Nash equilibrium for the game as a whole.

Recalling that $W > V > 0$, it is easy to see that there are two Nash equilibria (in pure strategies), namely that both players go to The Sun or that both players go to The Shade. Expressed differently, if it would be commonly know that both will go to The Sun, they would indeed do so. And if it would be commonly known that both will go to The Shade, they would indeed do so. As outside analysts, we cannot make a more precise prediction than to say that they will both go to the same restaurant. But, we can't say which restaurant they will go to.

We can illustrate the equilibria in the game tree, by marking the choices the players are required to take. First, we have the equilibrium when both go The Shade:



Next we have the equilibrim where both go to The Sun:

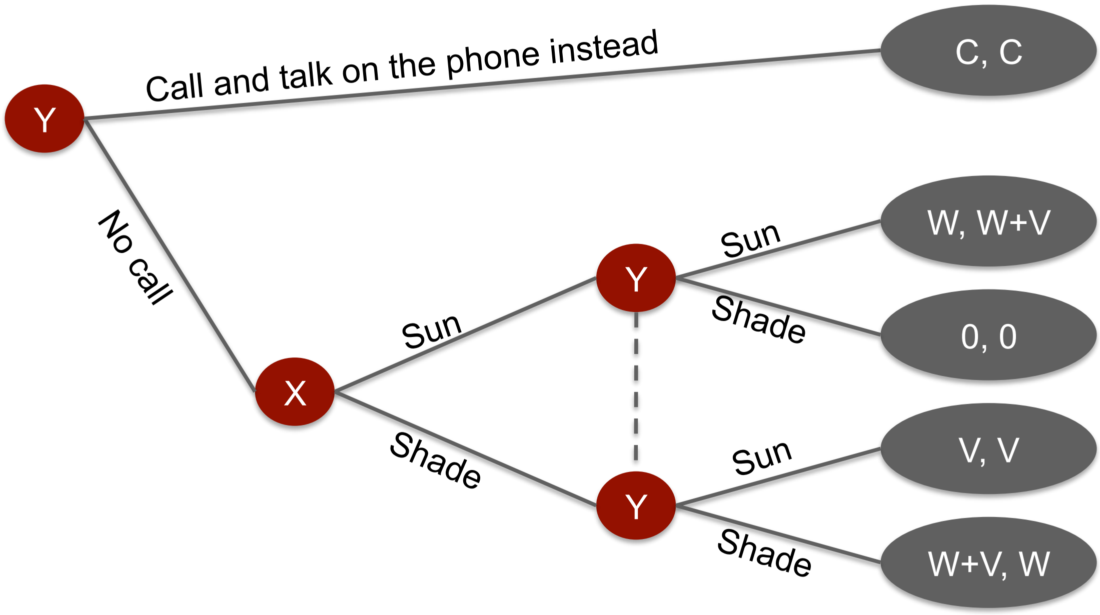


Notice that Y has to make the *same* choice at both his decision nodes, since he does not know X's choice.

Sun or Shade 3: Simultaneous and sequential choices in the same game

Assume now that Y can make a phone call to X already the day before they are supposed to meet. Then, they can discuss their matters without having to meet at some restaurant. Suppose they would then both get payoff C and that $W < C < W+V$.

We can represent this new situation with a new game tree, where the first payoff still belongs to X and the second to Y:

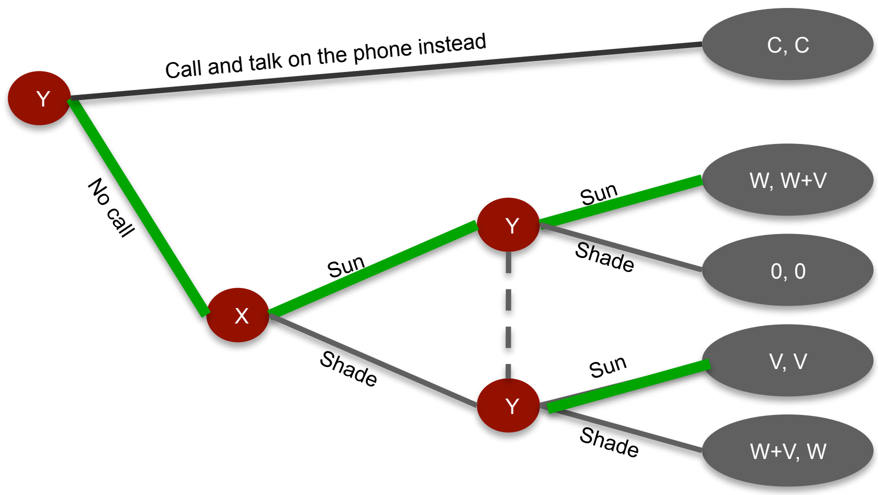


Remember that when we predict behavior in a situation where the players make all their decisions simultaneously, we use the idea of a Nash equilibrium. When we predict behavior in a situation where there are no simultaneous decisions, we use the idea of Backwards Induction. In the game above, however, we have both simultaneous decisions (which restaurant to go to) and sequential decisions (Y's choice to call comes before).

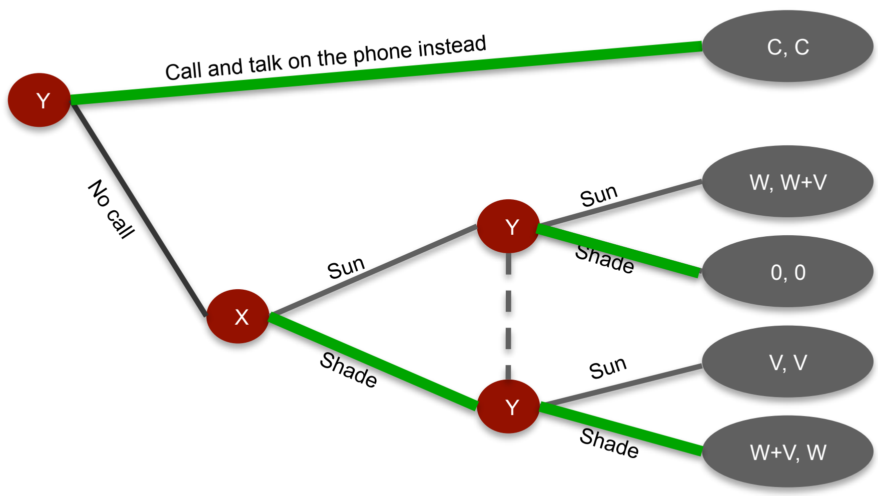
Subgame Perfect Equilibrium

To analyze the situation in the game tree above, we need to use the concept of Subgame Perfect Equilibrium (SPE). This means that we need to combine Nash equilibrium (to predict the two player's simultaneous choices at day 2) with backwards induction (starting with the choices of restaurant at day 2, before we predict if Y's choice at day 1).

As it turns out, there are two subgame perfect equilibria. If it is commonly known that the players will meet at The Sun at date 2 (if Y doesn't make the call), then Y would not make the call. The reason is that $W+V > C$. This equilibrium is described here:



If it is commonly know that the players will meet at The Shade at day 2 (if Y doesn't make the call), then Y would make the call, since $C > W$. This equilibrium is described here:



Again, the conclusion is that we (as analysts) cannot make precise predictions. The only outcome that we can rule out is that the two players would go to different restaurants at day 2.

Forward induction (Difficult - not compulsory)

However, some game theorists argue that only the first equilibrium above (when Y does not call) is a reasonable prediction.

So, let's study the second equilibrium a bit closer. The reason why Y makes the call is that he expects to end up at The Shade otherwise. But, what would really happen if Y doesn't make the call?

Some game theorists argue that if Y would not make the call, contrary to X's expectation, then X must ask himself what the reason for this deviation could be. One possible reason is that Y simply made a mistake and that Y during the rest of the game will follow the equilibrium, *i.e.* to go to The Shade. Then, X should also go to The Shade.

But a more plausible reason is that Y is perfectly rational, making no mistakes. Now, if Y actively chose not call, it must mean that Y expects to get a higher payoff by not calling than by calling. But that must mean that Y believes that they will meet at The Sun and not at The Shade. In this case, it is in X's best interest to also go to The Sun.

Thus, if X believes that Y cannot make mistakes, then both players should go to The Sun in case Y doesn't make the call. And, knowing this, Y will not make the call. Thus, there is only one plausible equilibrium: Y will not make the call and both players will go to The Sun.