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Public Goods

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Agenda

- First hour
  - Public goods vs private goods
  - (Club goods)
  - Voluntary contributions
- Second hour
  - Efficiency
  - Public provision
  - Why public policy

Different types of goods

- Apples
  - We buy apples in markets from farmers who grow them for profits
- Defense services
  - We receive defense services form the government and pay for them indirectly by trough taxes
- What is the reason for this difference?

Different types of goods

- Apples are *private goods*
  - **Rival consumption**: If I eat an apple, there is one apple less for others to eat
  - **Excludable consumption**: People who don't pay can be prevented from consuming apples
  - Other examples: food, clothes, housing, …

Different types of goods

- Theory of perfect competition shows
  - Markets provide the efficient amount of private goods, assuming competition and so on.

Different types of goods

- Defense is a *public good*
  - **Non-rival consumption**: Once produced, the cost of another person consuming it is zero (or very low)
  - **Non-excludable consumption**: It is impossible or very costly to prevent people from consuming the good

Different types of goods

- Q: Why problems to market?
  1. Since firms cannot exclude consumers, they cannot charge for consumption
  2. And if they would be able to charge a price $P > 0$, some people with valuation $P > V > 0$ would be excluded – an inefficiency.

Different types of goods

	Excludable	Non-excludable
Rival	**Private goods** (food)	**Common goods** (fish stock)
Non-rival	**Club goods** (TV)	**Public goods** (defense)

Different types of goods

	Excludable	Non-excludable
Rival	**Private goods** (food)	**Common goods** (fish stock)
Non-rival	**Club goods** (TV)	**Public goods** (defense)

Different types of goods

	Excludable	Non-excludable
Rival	Private goods (food)	Common goods (fish stock)
Non-rival	Club goods (TV)	Public goods (defense)

Non-rival

- I can watch even if you watch
- Once produced, marginal cost of consumption = 0
- Special case of IRTS

Different types of goods

	Excludable	Non-excludable
Rival	**Private goods** (food)	**Common goods** (fish stock)
Non-rival	**Club goods** (TV)	**Public goods** (defense)

Excludability

- Recent development (Cable, Satellite, Digital terrestrial, Internet)
- Today: people who do not pay can be excluded from watching
- Possible for private firms to provide TV

Different types of goods

	Excludable	Non-excludable
Rival	**Private goods** (food)	**Common goods** (fish stock)
Non-rival	**Club goods** (TV)	**Public goods** (defense)

Different types of goods

	Excludable	Non-excludable
Rival	Private goods (food)	**Common goods** (fish stock)
Non-rival	Club goods (TV)	Public goods (defense)

Non-excludable

- Fish move freely in the Baltic sea
- When on Swedish territorial water, it may be caught by Swedes
- But we cannot prevent Russians to catch it in Russia

Different types of goods

	Excludable	Non-excludable
Rival	**Private goods** (food)	**Common goods** (fish stock)
Non-rival	**Club goods** (TV)	**Public goods** (defense)

Tragedy of the commons

-Weak incentives for maintaining the stock

-Better for us to catch it today, otherwise the Russians will

Different types of goods

	Excludable	Non-excludable
Rival	Private goods (food)	**Common goods** (fish stock)
Non-rival	Club goods (TV)	Public goods (defense)

Very similar to negative externalities

Different types of goods

- Q: Classify recorded music
  - Yesterday
    - Music embodied in a record (rival, excludable)
  - Today
    - Similar to public good, since difficult to prevent copying
  - Note: depends on state of technology

Different types of goods

- Q: Classify light house
  - Public: All boats can see it
  - But if signal can be jammed, so that boats need to buy decoder, then club good

Different types of goods

- Q: Classify scenic view
  - Public: We can all enjoy it
  - If the view point can be fenced => club good
  - If the view point congested => common good

Different types of goods

- Q: Classify information
  (e.g. know-how behind medicine)
  - Public good:
    - Knowledge is non-rival
    - Non-excludable: firms can reverse-engineer
  - Caveat: Patent => punished if copied first 20 years

Different types of goods

- Q: Classify charitable giving
  - Public good: Most people feel good when other people don't suffer
  - Note: no clear distinction between market failure and redistribution

Different types of goods

- Q: Classify hospital care
  - Private good: Consumption is rival and excludable
  - Note: Government also provides some private goods

Different types of goods

- Q: Classify road maintenance
  - Public good: All drivers benefit
  - Note: Public goods may be produced by private sub-contractors

Different types of goods

- Q: Classify schooling
  - Public aspect: We all benefit from others being able to read and write

Club goods

Example: TV

Club goods

- Club goods
  - Non-rival
  - Excludable
- Assume
  - Indivisible club good, either produced or not
  - Cost of production, C
  - Three users
  - Willingness to pay: $V_1 \geq V_2 \geq V_3$

Club goods

- Q: Should the good be produced or not?
  - Yes, if: $V_1 + V_2 + V_3 \geq C$
  - Samuelson rule

Club goods

- Possible for monopolist to provide the good?
  - Assume $V_1 \geq C$.
    - If $P = V_1$ => $Q = 1$ => costs at least covered
  - Assume $V_2 \geq \frac{1}{2} \cdot C$.
    - If $P = V_2$ => $Q = 2$ => costs at least covered
  - Assume $V_3 \geq \frac{1}{3} \cdot C$.
    - If $P = V_3$ => $Q = 3$ => costs at least covered
  - Otherwise, private provision impossible

Club goods

- Possible for monopoly to provide the good to all consumers?
  - If $V_3 \geq ⅓ \cdot C$, then $P = V_3$ makes all buy
- Caveat
  - Costs are covered, but
  - Higher price may give higher profit

Club goods

- Numerical example– No production
  - Assume
    - $C = 12$
    - $V_1 = 11$, $V_2 = 5$, $V_3 = 3$
  - Then
    - Should be provided since $V_1 + V_2 + V_3 \geq C$, but
    - If $P = 11$, Revenues = 11
    - If $P = 5$, Revenues = 10
    - If $P = 3$, Revenues = 9

Club goods

- Numerical example– Suppressed consumption
  - Assume
    - $C = 12$
    - $V_1 = 11, V_2 = 6, V_3 = 3$
  - Then
    - Should be provided since $V_1 + V_2 + V_3 \geq C$
    - Monopolist covers costs if P = 6, Revenues = 12, but
    - Person 3 is excluded

Club goods

- Conclusion
  - Efficient provision (Samuelson rule)
    - Provide club good if sum of valuations exceed cost
  - Market solution entails risk of under-provision
    - Suppressed production - Even a monopolist may not be able to charge enough to cover cost
    - Suppressed consumption – Price may be higher than lowest valuation
- Public policy
  - One of the arguments for "tax-financed" public service, also today when consumption is excludable.

Public goods

Public goods

- Q: Can we rely on firms to provide public goods?
  - Profit-maximizing firms do not provide public goods, since they cannot charge a price
- Q: What about *voluntary* contributions by users?
  - Let's check!

Voluntary user contributions

- Model
  - Two people: $i = 1, 2$
  - Private consumption: q_i
  - Contribution to public good: c_i
  - Budget: $q_i + c_i = m$
  - Production function for public good $g = \frac{1}{k} \cdot (c_1 + c_2)$
  - Utility $U_i = q_i^{0.5} + g^{0.5}$

k = marginal cost of producing public good

Voluntary user contributions

- Model
  - The two people choose their contributions simultaneously
  - They cannot communicate or observe one another

Voluntary user contributions

- Person 1's decision problem
  - $$\max_{c_1} U_1 = (m - c_1)^{0.5} + \left(\frac{c_1 + c_2}{k}\right)^{0.5}$$

Voluntary user contributions

- Person 1's decision problem
  - $$\max_{c_1} U_1 = (m - c_1)^{0.5} + \left(\frac{c_1 + c_2}{k}\right)^{0.5}$$

Increasing the contribution increases the public good, but reduces private consumption

Voluntary user contributions

- Person 1's decision problem
  - $$\max_{c_1} U_1 = (m - c_1)^{0.5} + \left(\frac{c_1 + c_2}{k}\right)^{0.5}$$

Optimal contribution depends on the cost of producing the public good, k. *This cost is known*

Voluntary user contributions

- Person 1's decision problem
  - $$\max_{c_1} U_1 = (m - c_1)^{0.5} + \left(\frac{c_1 + c_2}{k}\right)^{0.5}$$

Person 1's preferred contribution also depends on person 2's contribution! *This contribution is not known.*

Voluntary user contributions

- Decision problem
  - $$\max_{c_1} U_1 = (m - c_1)^{0.5} + \left(\frac{c_1 + c_2}{k}\right)^{0.5}$$
- Interdependent decisions
  - Donor 1’s utility depends on donor 2’s contribution
  - Donor 1’s best choice depends on donor 2’s contribution
- Theory of interdependent decision-making
  - a.k.a. Game Theory
  - Solution concept: Nash equilibrium

Voluntary user contributions

- Q: Define Nash equilibrium
  - Both donors do as good as they can, given what the other donor is doing
    - Donor 1 selects c_1 to maximize U_1 given c_2
    - Donor 2 selects c_2 to maximize U_2 given c_1
  - Then, nobody has an incentive to change behavior

Voluntary user contributions

- Person 1's decision problem

$$\max_{c_1} U_1 = (m - c_1)^{0.5} + \left(\frac{c_1 + c_2}{k}\right)^{0.5}$$

- Person 1's first-order condition

$$\frac{dU_1}{dc_1} = -\frac{1}{2} \cdot (m - c_1)^{-0.5} + \frac{1}{2} \cdot \left(\frac{c_1 + c_2}{k}\right)^{-0.5} \cdot \frac{1}{k} = 0$$

- Now solve FOC for c_1

Voluntary user contributions

$$\frac{\partial U_1}{\partial c_1} = -\frac{1}{2} \cdot \left(m - c_1\right)^{-0.5} + \frac{1}{2} \cdot \left(\frac{c_1 + c_2}{k}\right)^{-0.5} \cdot \frac{1}{k} = 0$$

Voluntary user contributions

$$\frac{\partial U_1}{\partial c_1} = -\frac{1}{2} \cdot (m - c_1)^{-0.5} + \frac{1}{2} \cdot \left(\frac{c_1 + c_2}{k} \right)^{-0.5} \cdot \frac{1}{k} = 0$$

$$(m - c_1)^{-0.5} = \left(\frac{c_1 + c_2}{k} \right)^{-0.5} \cdot \frac{1}{k}$$

Voluntary user contributions

$$\frac{\partial U_1}{\partial c_1} = -\frac{1}{2} \cdot (m - c_1)^{-0.5} + \frac{1}{2} \cdot \left(\frac{c_1 + c_2}{k}\right)^{-0.5} \cdot \frac{1}{k} = 0$$

$$(m - c_1)^{-0.5} = \left(\frac{c_1 + c_2}{k}\right)^{-0.5} \cdot \frac{1}{k}$$

$$(m - c_1)^{0.5} = \left(\frac{c_1 + c_2}{k}\right)^{0.5} \cdot k$$

Voluntary user contributions

$$\frac{\partial U_1}{\partial c_1} = -\frac{1}{2}\cdot\left(m-c_1\right)^{-0.5} + \frac{1}{2}\cdot\left(\frac{c_1+c_2}{k}\right)^{-0.5}\cdot\frac{1}{k} = 0$$

$$\left(m-c_1\right)^{-0.5} = \left(\frac{c_1+c_2}{k}\right)^{-0.5}\cdot\frac{1}{k}$$

$$\left(m-c_1\right)^{0.5} = \left(\frac{c_1+c_2}{k}\right)^{0.5}\cdot k$$

$$\left(m-c_1\right) = \left(\frac{c_1+c_2}{k}\right)\cdot k^2$$

Voluntary user contributions

$$\frac{\partial U_1}{\partial c_1} = -\frac{1}{2} \cdot (m - c_1)^{-0.5} + \frac{1}{2} \cdot \left(\frac{c_1 + c_2}{k} \right)^{-0.5} \cdot \frac{1}{k} = 0$$

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$$(m - c_1)^{0.5} = \left(\frac{c_1 + c_2}{k} \right)^{0.5} \cdot k$$

$$(m - c_1) = \left(\frac{c_1 + c_2}{k} \right) \cdot k^2$$

$$m - c_2 \cdot k = c_1 \cdot (1 + k)$$

Voluntary user contributions

$$\frac{\partial U_1}{\partial c_1} = -\frac{1}{2} \cdot (m - c_1)^{-0.5} + \frac{1}{2} \cdot \left(\frac{c_1 + c_2}{k} \right)^{-0.5} \cdot \frac{1}{k} = 0$$

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$$(m - c_1) = \left(\frac{c_1 + c_2}{k} \right) \cdot k^2$$

$$m - c_2 \cdot k = c_1 \cdot (1 + k)$$

$$c_1 = \frac{m}{1 + k} - \frac{k}{1 + k} \cdot c_2$$

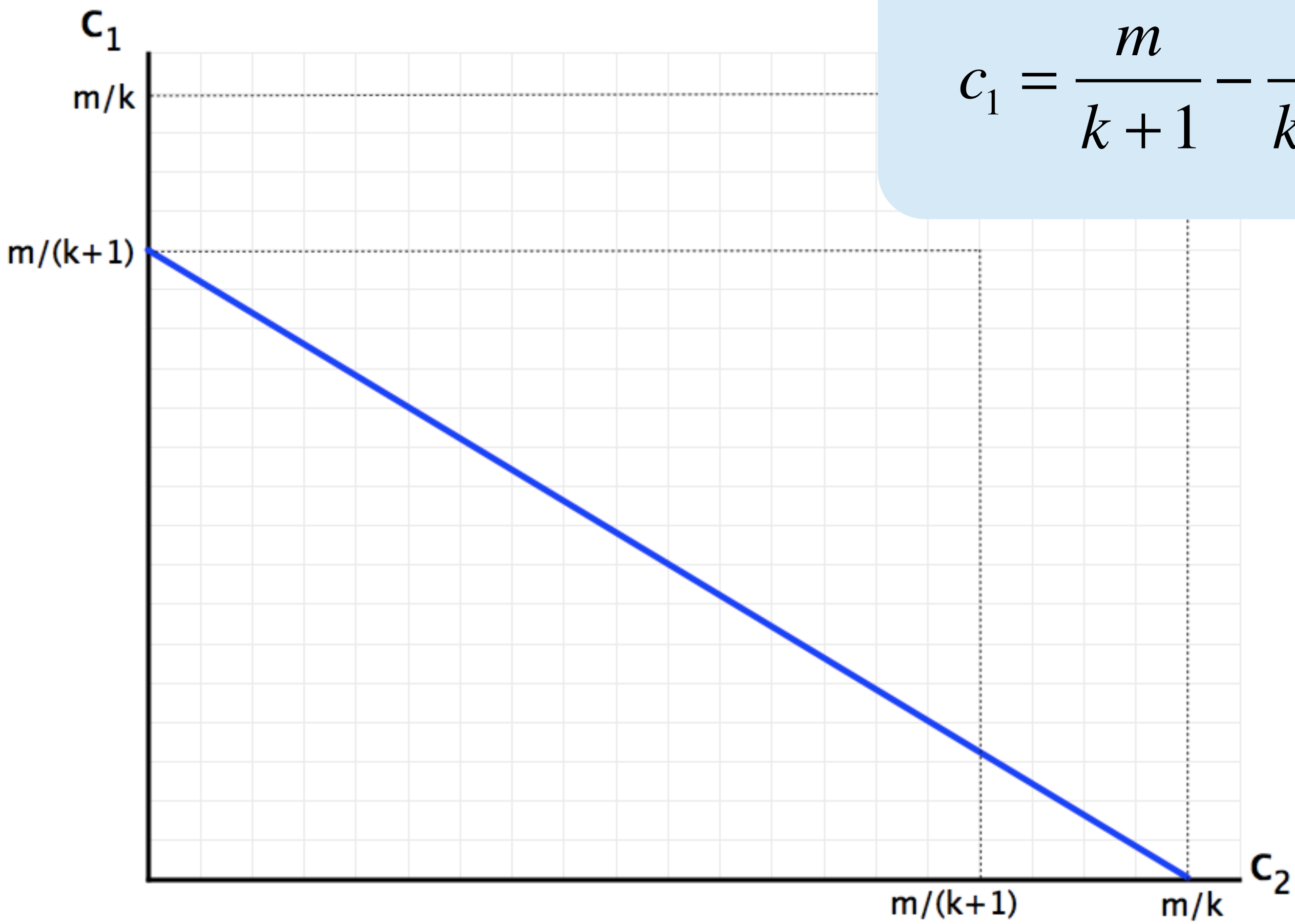
Voluntary user contributions

- User 1's best-reply function
  - Describes user 1's best choice for every possible choice that user 2 could make

$$c_1 = \frac{m}{k+1} - \frac{k}{k+1} \cdot c_2$$

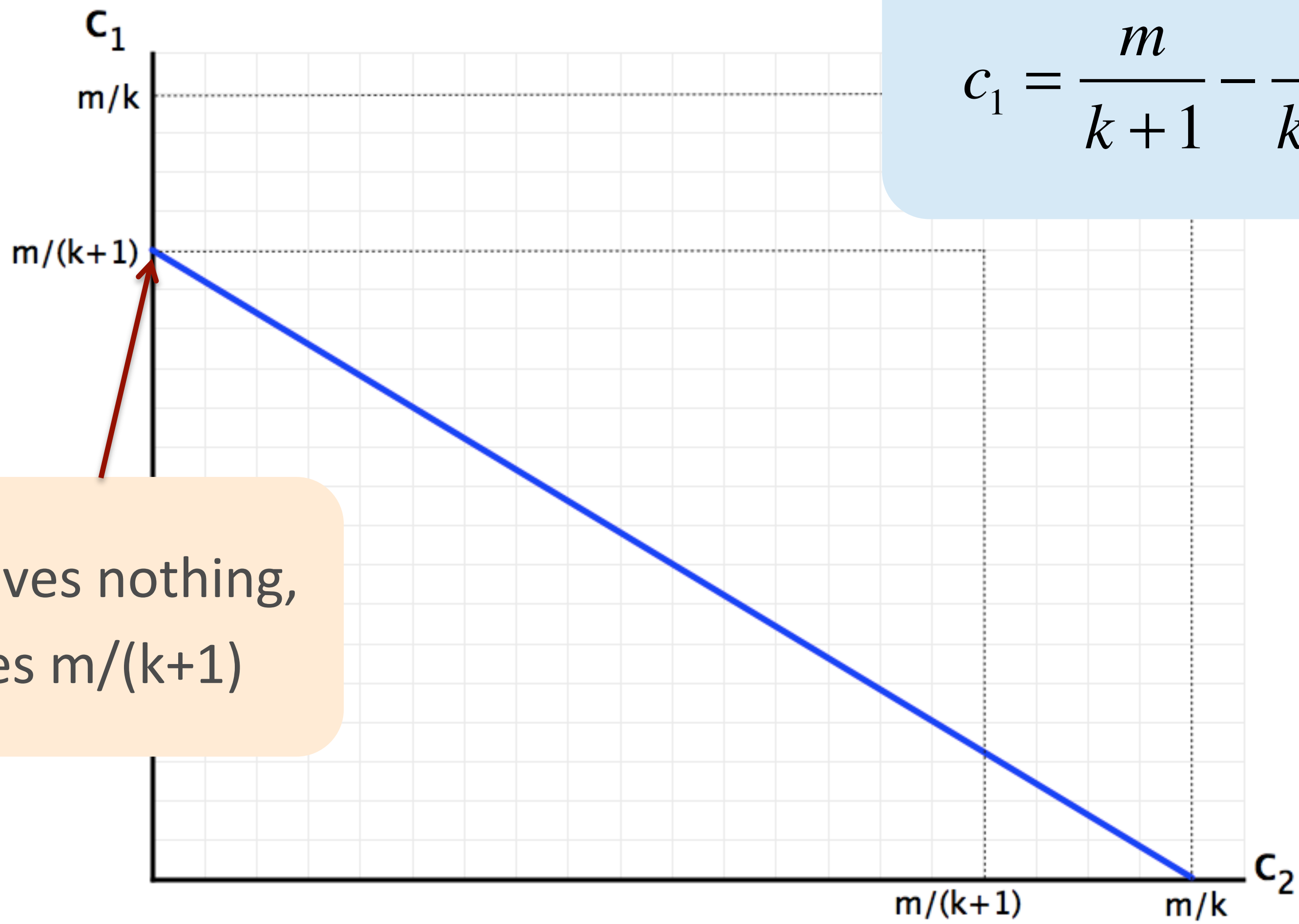
  - User 1 contributes less, the more the other user contributes

Voluntary user contributions



$$c_1 = \frac{m}{k+1} - \frac{k}{k+1} \cdot c_2$$

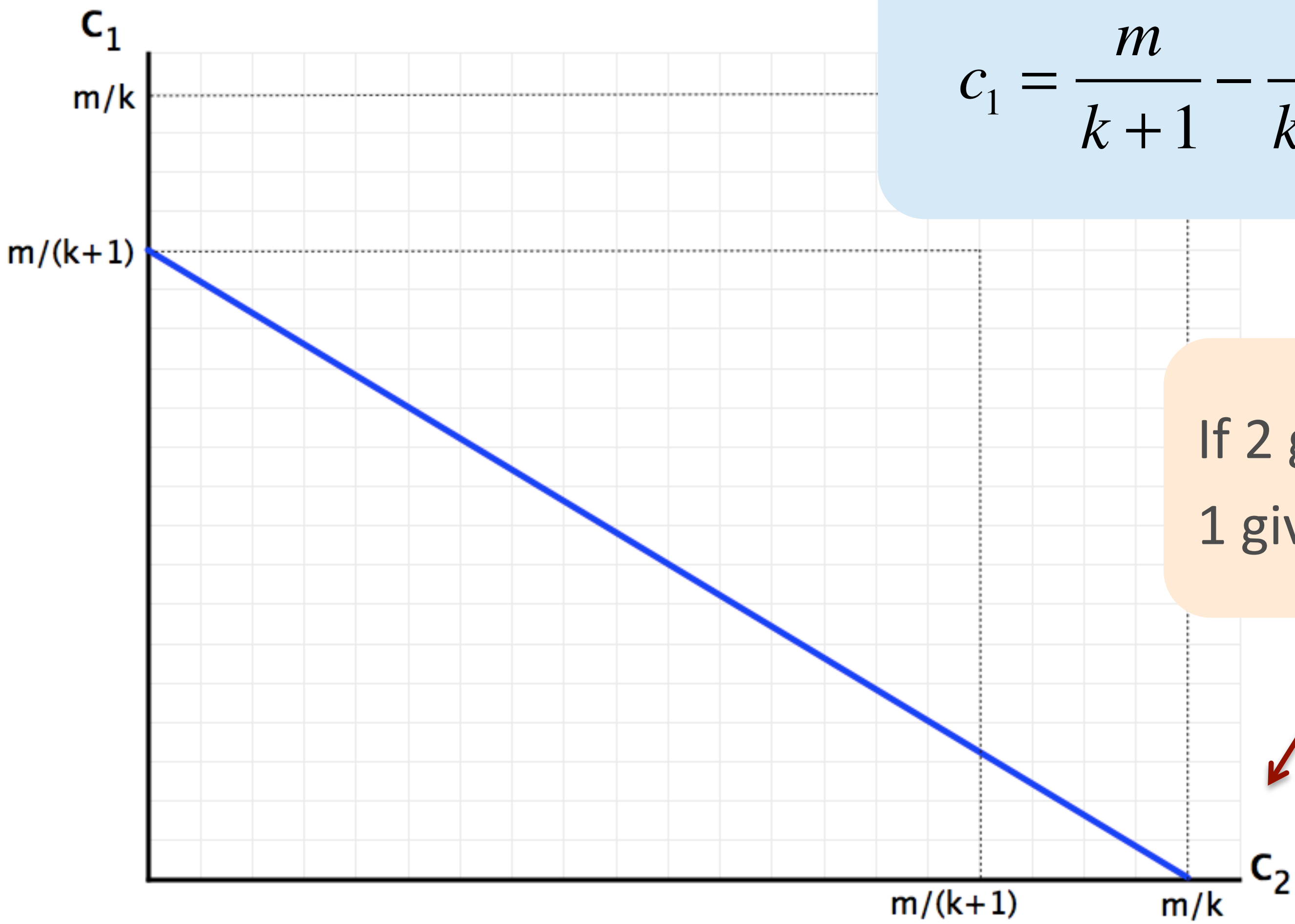
Voluntary user contributions



$$c_1 = \frac{m}{k+1} - \frac{k}{k+1} \cdot c_2$$

If 2 gives nothing,
1 gives m/(k+1)

Voluntary user contributions



$$c_1 = \frac{m}{k+1} - \frac{k}{k+1} \cdot c_2$$

If 2 gives m/k,
1 gives nothing

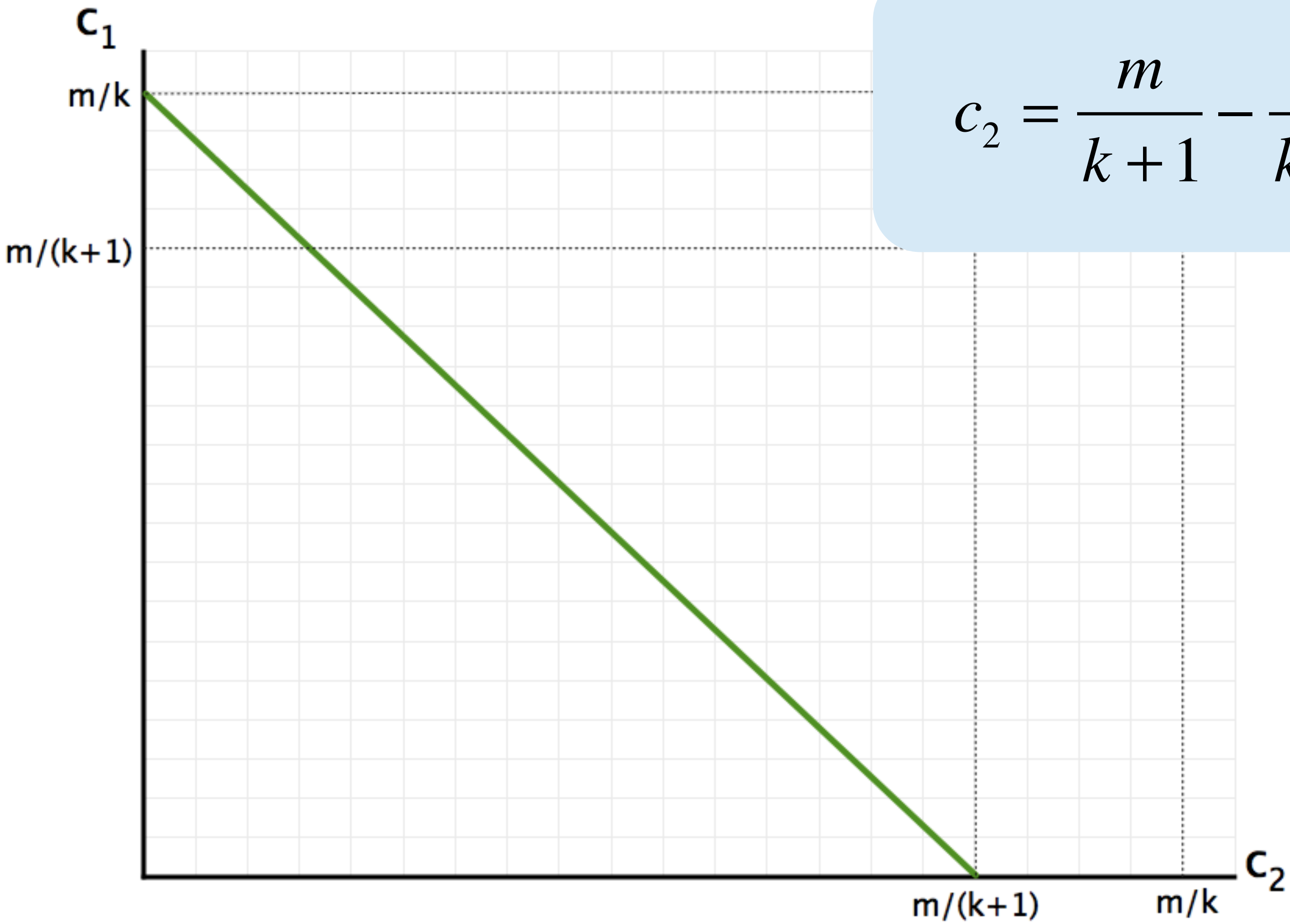
Voluntary user contributions

- Similarly: User 2’s best-reply function
  - Describes user 2’s best choice for every possible choice that user 1 could make

$$c_2 = \frac{m}{k+1} - \frac{k}{k+1} \cdot c_1$$

  - User 2 contributes less, the more the other user contributes

Voluntary user contributions

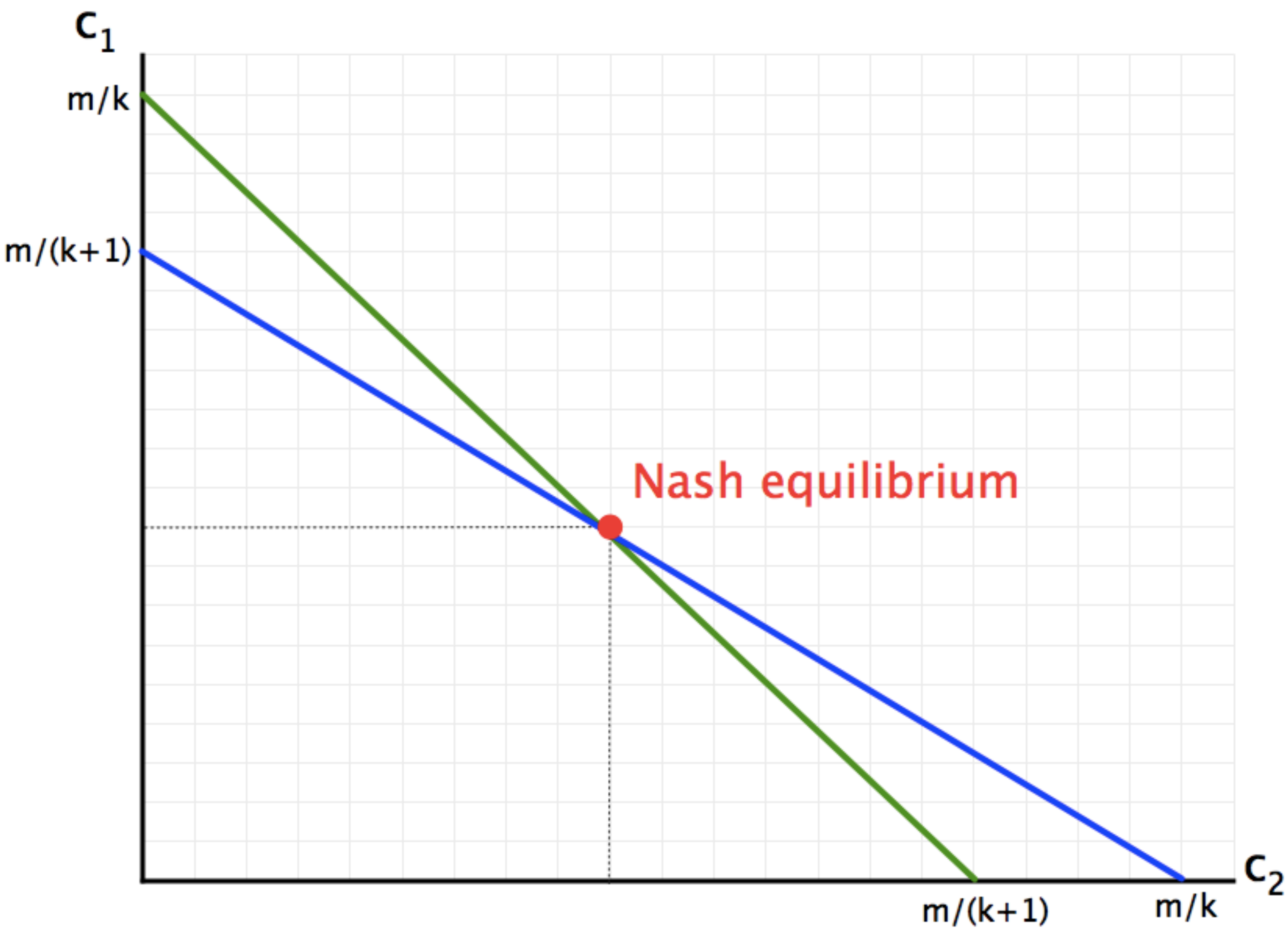


$$c_2 = \frac{m}{k+1} - \frac{k}{k+1} \cdot c_1$$

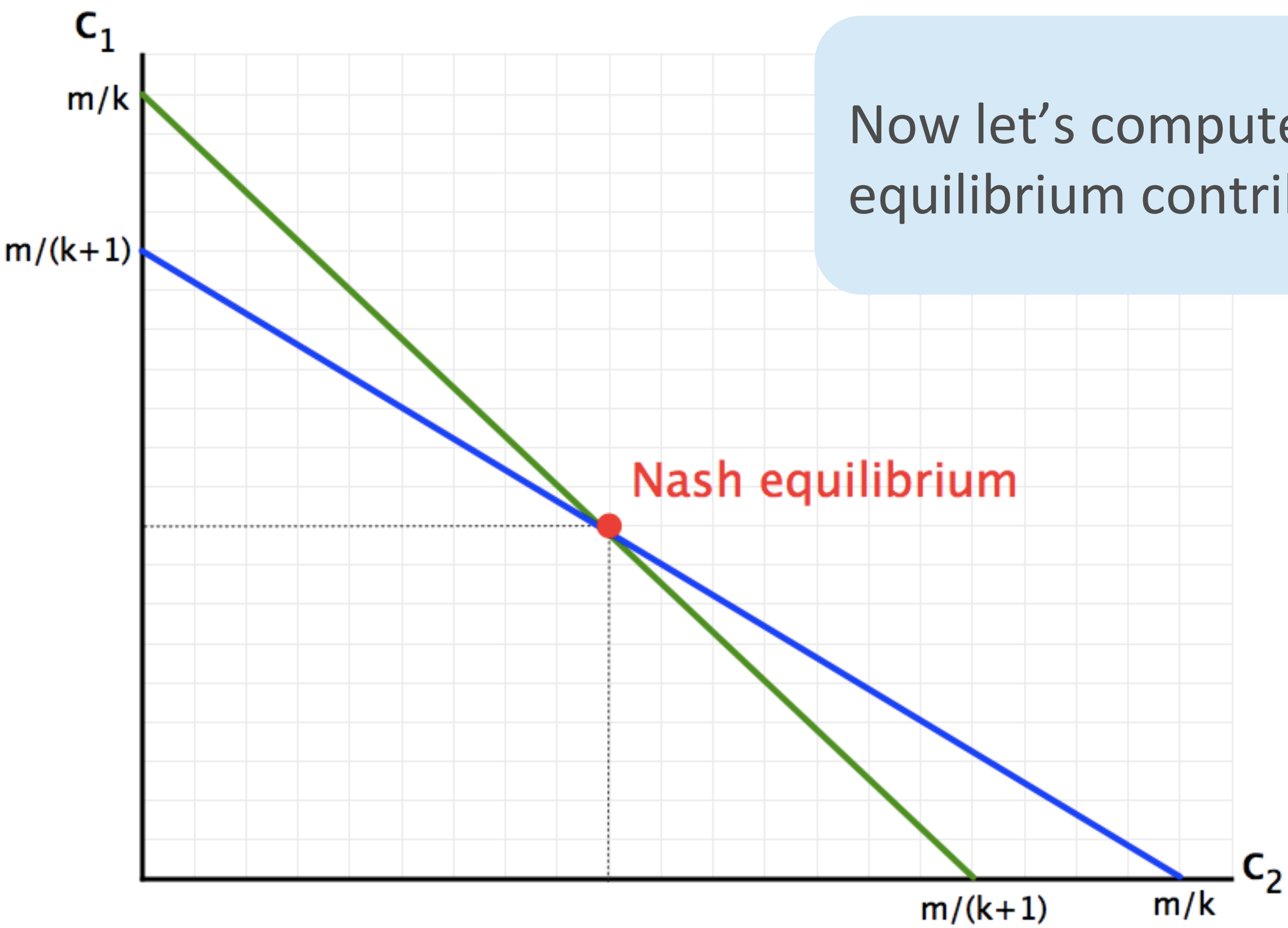
Voluntary user contributions

- Nash equilibrium
  - Both users do as good as they can, given what the other does
  - Both must be on their best-reply curves

Voluntary user contributions



Voluntary user contributions



Now let's compute equilibrium contributions

Voluntary user contributions

- Nash equilibrium
  - Both users do as good as they can, given what the other does
- System of two “best-reply” equations

$$c_1 = \frac{m}{k+1} - \frac{k}{k+1} \cdot c_2$$

$$c_2 = \frac{m}{k+1} - \frac{k}{k+1} \cdot c_1$$

- Next step
  - Solve for two contributions
  - (Substitute one into the other, and then back)

Voluntary user contributions

- Substitute one into the other

$$c_1 = \frac{m}{k+1} - \frac{k}{k+1} \cdot \left\{ \frac{m}{k+1} - \frac{k}{k+1} \cdot c_1 \right\}$$

One equation in one unknown!

Voluntary user contributions

- Substitute one into the other

$$c_1 = \frac{m}{k+1} - \frac{k}{k+1} \cdot \left\{ \frac{m}{k+1} - \frac{k}{k+1} \cdot c_1 \right\}$$

$$(k+1)^2 \cdot c_1 = (k+1) \cdot m - k \cdot [m - k \cdot c_1]$$

$$\left[(k+1)^2 - k^2\right] \cdot c_1 = m$$

$$\left[(k+1) - k\right]\left[(k+1) + k\right] \cdot c_1 = m$$

$$c_1 = \frac{m}{2k+1}$$

Voluntary user contributions

- And back

$$c_2 = \frac{m}{k+1} - \frac{k}{k+1} \cdot \left\{ \frac{m}{2k+1} \right\}$$

Voluntary user contributions

- And back

$$c_2 = \frac{m}{k+1} - \frac{k}{k+1} \cdot \left\{ \frac{m}{2k+1} \right\}$$

$$c_2 = \frac{m}{k+1} \left[1 - \frac{k}{2k+1} \right]$$

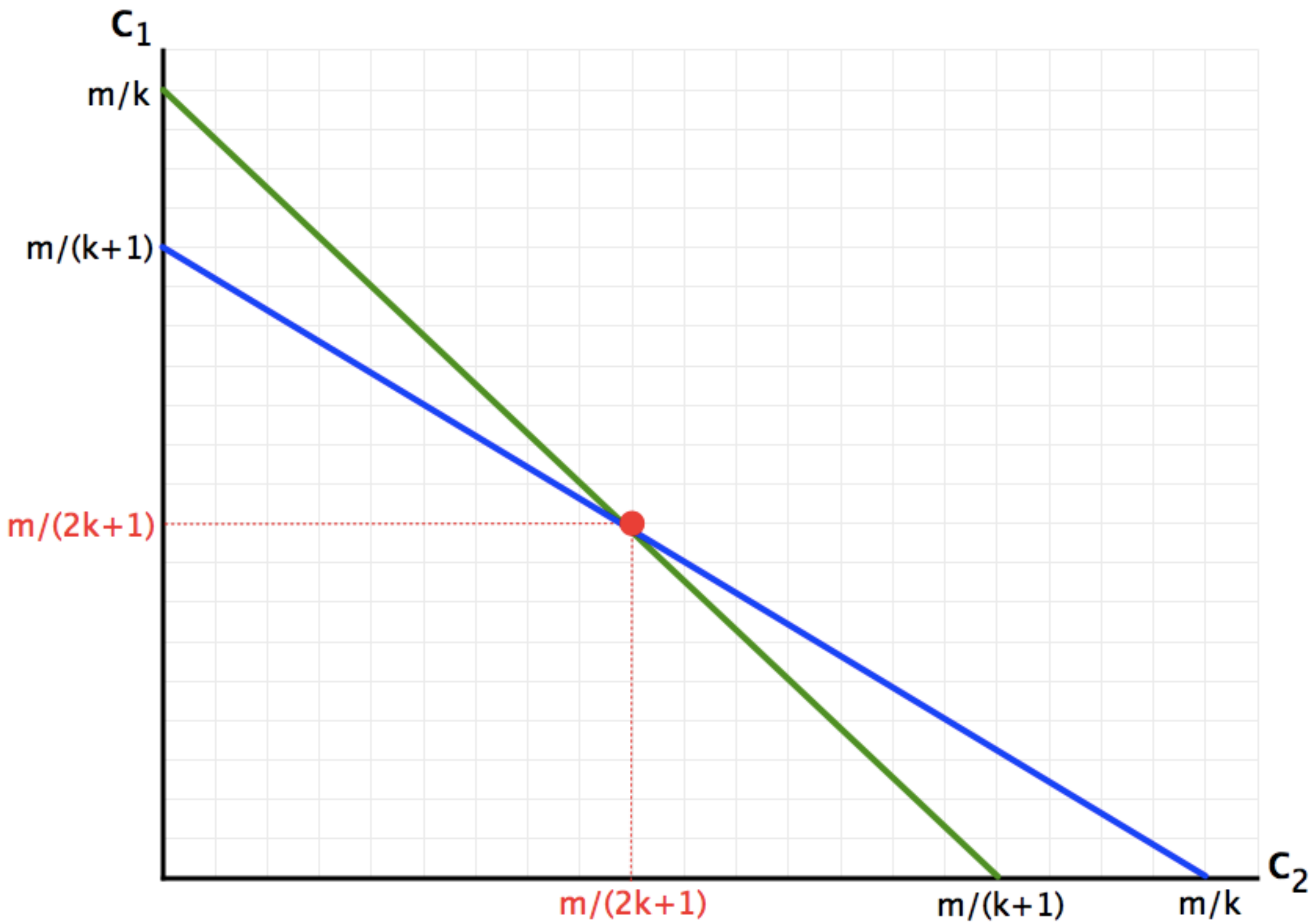
$$c_2 = \frac{m}{k+1} \left[\frac{2k+1}{2k+1} - \frac{k}{2k+1} \right]$$

$$c_2 = \frac{m}{2k+1}$$

Voluntary user contributions

- Equilibrium contributions
  - If both users contribute m/(2k+1) both maximize their utilities, given what the other user contributes

Voluntary user contributions



Voluntary user contributions

- Summary
  - Public goods are characterized by
    - Non-rival consumption
    - Nor-excludable consumption
  - Firms don't provide public goods, since they cannot charge
  - Users' decisions to voluntary contribute to a public good are interdependent
  - In Nash equilibrium, all users make the contributions that maximize their utilities, given what other users contribute
  - People contribute less, the more other users contribute

Efficiency

1. Characterize voluntary contributions
2. Characterize efficient contributions

Voluntary user contributions

- To interpret the voluntary contributions, let's look at the first-order conditions again

Voluntary user contributions

- Person 1's decision problem

$$\max_{c_1} U_1 = (m - c_1)^{0.5} + \left(\frac{c_1 + c_2}{k}\right)^{0.5}$$

- Person 1's first-order condition

$$\frac{dU_1}{dc_1} = -\frac{1}{2} \cdot (m - c_1)^{-0.5} + \frac{1}{2} \cdot \left(\frac{c_1 + c_2}{k}\right)^{-0.5} \cdot \frac{1}{k} = 0$$

$$= -\frac{\partial U_1}{\partial q_1}$$

Voluntary user contributions

- Person 1's decision problem

$$\max_{c_1} U_1 = (m - c_1)^{0.5} + \left(\frac{c_1 + c_2}{k}\right)^{0.5}$$

- Person 1's first-order condition

$$\frac{dU_1}{dc_1} = -\frac{1}{2}\cdot(m - c_1)^{-0.5} + \frac{1}{2}\cdot\left(\frac{c_1 + c_2}{k}\right)^{-0.5}\cdot\frac{1}{k} = 0$$

$$= \frac{\partial U_1}{\partial g}$$

Voluntary user contributions

- Interpret first-order condition

$$\frac{dU_1}{dc_1} = -\frac{1}{2}\cdot\left(m - c_1\right)^{-0.5} + \frac{1}{2}\cdot\left(\frac{c_1 + c_2}{k}\right)^{-0.5}\cdot\frac{1}{k} = 0$$

$$\frac{dU_1}{dc_1} = -\frac{\partial U_1}{\partial q_1} + \frac{\partial U_1}{\partial g}\cdot\frac{1}{k} = 0$$

Voluntary user contributions

- Interpret first-order condition

$$\frac{dU_1}{dc_1} = -\frac{1}{2} \cdot \left(m - c_1\right)^{-0.5} + \frac{1}{2} \cdot \left(\frac{c_1 + c_2}{k}\right)^{-0.5} \cdot \frac{1}{k} = 0$$

$$\frac{dU_1}{dc_1} = -\frac{\partial U_1}{\partial q_1} + \frac{\partial U_1}{\partial g} \cdot \frac{1}{k} = 0 \qquad \Leftrightarrow \qquad \frac{\partial U_1}{\partial g} = k \cdot \frac{\partial U_1}{\partial q_1}$$

Voluntary user contributions

- Interpret first-order condition

$$\frac{dU_1}{dc_1} = -\frac{1}{2} \cdot (m - c_1)^{-0.5} + \frac{1}{2} \cdot \left(\frac{c_1 + c_2}{k} \right)^{-0.5} \cdot \frac{1}{k} = 0$$

$$\frac{dU_1}{dc_1} = -\frac{\partial U_1}{\partial q_1} + \frac{\partial U_1}{\partial g} \cdot \frac{1}{k} = 0 \quad \Leftrightarrow \quad \frac{\partial U_1}{\partial g} = k \cdot \frac{\partial U_1}{\partial q_1} \quad \Leftrightarrow \quad \frac{\partial U_1 / \partial g}{\partial U_1 / \partial q_1} = k$$

Voluntary user contributions

- Interpret first order condition

Q: Interpretation?

$$\frac{dU_1}{dc_1} = -\frac{1}{2}\cdot\left(m - c_1\right)^{-0\ldots}$$

$$\frac{dU_1}{dc_1} = -\frac{\partial U_1}{\partial q_1} + \frac{\partial U_1}{\partial g}\cdot\frac{1}{k} = 0 \qquad \Leftrightarrow \qquad \frac{\partial U_1}{\partial g} = k\cdot\frac{\partial U_1}{\partial q_1} \qquad \Leftrightarrow \qquad \frac{\partial U_1 / \partial g}{\partial U_1 / \partial q_1} = k$$

Voluntary user contributions

- Interpret first order condition

Q: Interpretation?

$$\frac{dU_1}{dc_1} = -\frac{1}{2}\cdot\left(m - c_1\right)^{-0}$$

$$\frac{dU_1}{dc_1} = -\frac{\partial U_1}{\partial q_1} + \frac{\partial U_1}{\partial g}\cdot\frac{1}{k} = 0 \quad \Leftrightarrow \quad \frac{\partial U_1}{\partial g} = k\cdot\frac{\partial U_1}{\partial q_1} \quad \Leftrightarrow \quad \frac{\partial U_1/\partial g}{\partial U_1/\partial q_1} = k$$

$$MRS^1_{g,q_1} = -\frac{\partial U_1/\partial g}{\partial U_1/\partial q_1}$$

Voluntary user contributions

- Interpret first order condition

Q: Interpretation?

$$\frac{dU_1}{dc_1} = -\frac{1}{2}\cdot\left(m - c_1\right)^{-0\ldots}$$

$$\frac{dU_1}{dc_1} = -\frac{\partial U_1}{\partial q_1} + \frac{\partial U_1}{\partial g}\cdot\frac{1}{k} = 0 \quad\Leftrightarrow\quad \frac{\partial U_1}{\partial g} = k\cdot\frac{\partial U_1}{\partial q_1} \quad\Leftrightarrow\quad \frac{\partial U_1/\partial g}{\partial U_1/\partial q_1} = k$$

$$MC_g = k \qquad MRT_{g,q} = -k$$

Voluntary user contributions

- Interpret first-order condition

$$\frac{dU_1}{dc_1} = -\frac{1}{2} \cdot (m - c_1)^{-0}$$

Q: Interpretation?

$$\frac{dU_1}{dc_1} = -\frac{\partial U_1}{\partial q_1} + \frac{\partial U_1}{\partial g} \cdot \frac{1}{k} = 0 \quad \Leftrightarrow \quad \frac{\partial U_1}{\partial g} = k \cdot \frac{\partial U_1}{\partial q_1} \quad \Leftrightarrow \quad \frac{\partial U_1 / \partial g}{\partial U_1 / \partial q_1} = k$$

$$MRS^1_{g,q_1} = MRT_{g,q_1}$$

Voluntary user contributions

- Interpretation of first-order condition
  - Each user voluntarily contributes to a public good to equalize his *marginal willingness to pay* to the *marginal cost* of the public good

Efficient contributions

- Q: Define Pareto efficiency in this context
  - The two contributions c_1 and c_2 are Pareto efficient if it is impossible to find two other contributions k_1 and k_2 that increases the utility of one of the donors without reducing the utility of the other

Efficient contributions

- Find Pareto efficient contributions
  - Maximize utility of one donor, given the utility of the other

$$L = U_1(q_1, g) + \lambda \cdot \left[U_2(q_2, g) - \bar{U}_2 \right] + \mu \cdot \left[\frac{(m - q_1) + (m - q_2)}{k} - g \right]$$

Maximize person 1's utility

Efficient contributions

- Find Pareto efficient contributions
  - Maximize utility of one donor, given the utility of the other

$$L = U_1(q_1, g) + \lambda \cdot \left[U_2(q_2, g) - \bar{U}_2 \right] + \mu \cdot \left[\frac{(m - q_1) + (m - q_2)}{k} - g \right]$$

Person 2's utility should be at least $\bar{U}_2$

Efficient contributions

- Find Pareto efficient contributions
  - Maximize utility of one donor, given the utility of the other

$$L = U_1(q_1, g) + \lambda \cdot \left[U_2(q_2, g) - \bar{U}_2\right] + \mu \cdot \left[\frac{(m - q_1) + (m - q_2)}{k} - g\right]$$

Resource constraint (budgets & production function)

Efficient contributions

- Find Pareto efficient contributions
  - Maximize utility of one donor, given the utility of the other

$$L = U_1(q_1, g) + \lambda \cdot \left[U_2(q_2, g) - \bar{U}_2 \right] + \mu \cdot \left[\frac{(m - q_1) + (m - q_2)}{k} - g \right]$$

Lagrange multipliers

Efficient contributions

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- First-order conditions

$$\frac{dL}{dq_1} = \frac{\partial U_1}{\partial q_1} - \mu \cdot \tfrac{1}{k} = 0$$

$$\frac{dL}{dq_2} = \lambda \cdot \frac{\partial U_2}{\partial q_2} - \mu \cdot \tfrac{1}{k} = 0$$

$$\frac{dL}{dg} = \frac{\partial U_1}{\partial g} + \lambda \cdot \frac{\partial U_2}{\partial g} - \mu = 0$$

Efficient contributions

- First-order conditions

$$\frac{dL}{dq_1} = \frac{\partial U_1}{\partial q_1} - \mu \cdot \tfrac{1}{k} = 0 \qquad \Rightarrow \qquad \mu = \frac{\partial U_1}{\partial q_1} \cdot k$$

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Efficient contributions

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$$\frac{dL}{dg} = \frac{\partial U_1}{\partial g} + \lambda \cdot \frac{\partial U_2}{\partial g} - \mu = 0$$

Efficient contributions

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Efficient contributions

- First-order conditions

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$$\frac{dL}{dg} = \frac{\partial U_1}{\partial g} + \lambda \cdot \frac{\partial U_2}{\partial g} - \mu = 0$$

$$\frac{\partial U_1}{\partial g}\left(\frac{\partial U_1}{\partial q_1}\right)^{-1} + \left(\frac{\partial U_2}{\partial q_2}\right)^{-1} \cdot \frac{\partial U_2}{\partial g} = k$$

Efficient contributions

- Optimal choice of public good

$$\frac{\partial U_1}{\partial g}\left(\frac{\partial U_1}{\partial q_1}\right)^{-1}+\left(\frac{\partial U_2}{\partial q_2}\right)^{-1}\cdot\frac{\partial U_2}{\partial g}=k$$

Q: Interpretation?

Efficient contributions

- Optimal choice of public good

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Q: Interpretation?

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$$\frac{\partial U_1}{\partial g}\left(\frac{\partial U_1}{\partial q_1}\right)^{-1} + \left(\frac{\partial U_2}{\partial q_2}\right)^{-1} \cdot \frac{\partial U_2}{\partial g} = k$$

Q: Interpretation?

$$MRS^1_{g,q_1} = -\frac{\partial U_1 / \partial g}{\partial U_1 / \partial q_1}$$

Efficient contributions

- Optimal choice of public good

$$\frac{\partial U_1}{\partial g}\left(\frac{\partial U_1}{\partial q_1}\right)^{-1} + \left(\frac{\partial U_2}{\partial q_2}\right)^{-1} \cdot \frac{\partial U_2}{\partial g} = k$$

Q: Interpretation?

Efficient contributions

- Optimal choice of public good

$$\frac{\partial U_1}{\partial g}\left(\frac{\partial U_1}{\partial q_1}\right)^{-1} + \left(\frac{\partial U_2}{\partial q_2}\right)^{-1} \cdot \frac{\partial U_2}{\partial g} = k$$

Q: Interpretation?

$$MRS^2_{g,q_2} = -\frac{\partial U_2 / \partial g}{\partial U_2 / \partial q_2}$$

Efficient contributions

- Optimal choice of public good

$$\frac{\partial U_1}{\partial g}\left(\frac{\partial U_1}{\partial q_1}\right)^{-1}+\left(\frac{\partial U_2}{\partial q_2}\right)^{-1}\cdot\frac{\partial U_2}{\partial g}=k$$

Q: Interpretation?

Efficient contributions

- Optimal choice of public good

$$\frac{\partial U_1}{\partial g}\left(\frac{\partial U_1}{\partial q_1}\right)^{-1}+\left(\frac{\partial U_2}{\partial q_2}\right)^{-1}\cdot\frac{\partial U_2}{\partial g}=k$$

Q: Interpretation?

$$MC_g = k \qquad MRT_{g,q} = -k$$

Efficient contributions

- Optimal choice of public good

$$\frac{\partial U_1}{\partial g}\left(\frac{\partial U_1}{\partial q_1}\right)^{-1} + \left(\frac{\partial U_2}{\partial q_2}\right)^{-1} \cdot \frac{\partial U_2}{\partial g} = k$$

$$MRS^1_{g,q_1} + MRS^2_{g,q_2} = MRT_{g,q}$$

Efficient contributions

- Optimal choice of public good

$$\frac{\partial U_1}{\partial g}\left(\frac{\partial U_1}{\partial q_1}\right)^{-1} + \left(\frac{\partial U_2}{\partial q_2}\right)^{-1} \cdot \frac{\partial U_2}{\partial g} = k$$

$$MRS^1_{g,q_1} + MRS^2_{g,q_2} = MRT_{g,q}$$

Produce public good to equalize marginal cost to the *sum* of the users' marginal valuations

"Samuelson rule"

Efficient contributions

- For completeness

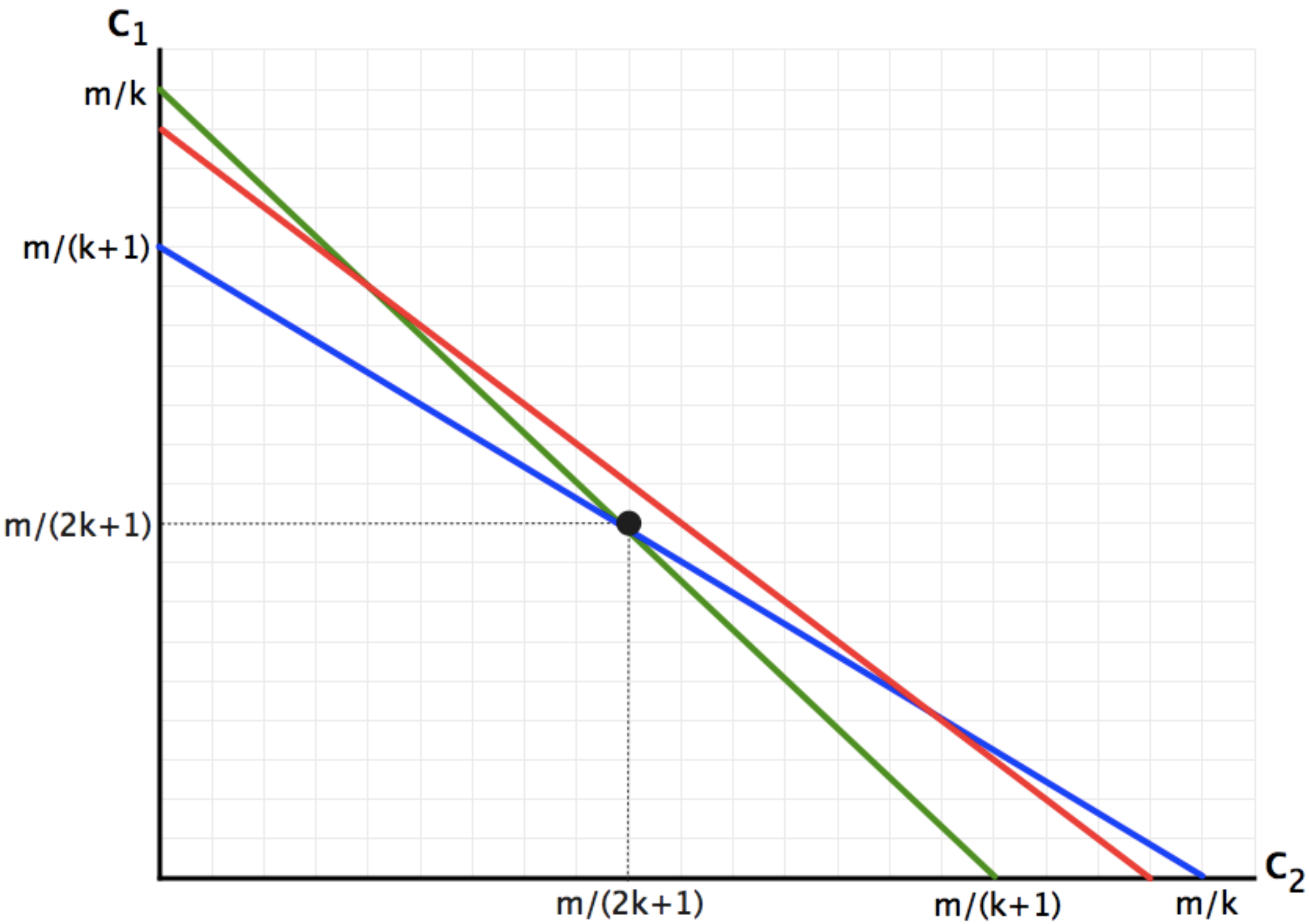
$$\max_{c=c_1=c_2} U_1 + U_2 = 2 \cdot \left[(m-c)^{0.5} + \left(\frac{2c}{k} \right)^{0.5} \right]$$

- First-order condition

$$\frac{d(U_1+U_2)}{dc} = -\tfrac{1}{2} \cdot (m-c)^{-0.5} + \tfrac{1}{2} \cdot \left(\frac{2c}{k} \right)^{-0.5} \cdot \tfrac{2}{k} = 0$$

$$\Rightarrow \qquad c = \frac{m}{\tfrac{k}{2}+1}$$

Efficient contributions



Efficient contributions

- Voluntary user provision
  - $MRS^1_{g,q_1} = MRS^2_{g,q_2} = MRT_{g,q}$
  - Each user contributes to equalize marginal revenue to marginal cost
- Efficient provision (Samuelson rule)
  - $MRS^1_{g,q_1} + MRS^2_{g,q_2} = MRT_{g,q}$
  - Marginal cost should be equal to *sum* of marginal valuations

Efficient contributions

- Conclusion
  - Voluntary user contributions to public goods are too small from an efficiency point of view.
  - Efficient contributions are based on the sum of all users' willingness to pay
  - Voluntary contributions are based on individual willingness to pay.
    - Users don't care about other people gaining from their contributions

Public provision

Public provision

- Market failure
  - Firms don't provide public goods, since they cannot charge
  - Users voluntarily contribute too little, since they only think about their own benefit
- Possible policy intervention
  - Government provides public good

Public provision

- Policy problem 1: Financing
  - Production must be financed
  - Taxes create distortions
  - Need modified Samuelson rule to account for cost of taxation

Public provision

- Policy problem 2: Information
  - Government doesn't know people's valuations
  - "Guestimates"
  - Electoral competition?
  - Elicitation: Can we ask people what their valuations are?

Elicitation

- Q: Can we ask people about their valuations?
  - Problem 1: A user may have an incentive to **overstate** his valuation, if he does not have to pay according to valuation
  - Problem 2: A user may have an incentive to **understate** his valuation, if he has to pay according to valuation (free-riding)

Elicitation

- Model
  - Indivisible public good, either produced or not
  - Two users, A and B
  - Valuations V_A and V_B are private information
  - Cost of production: C
  - Efficiency: Produce iff $V_A + V_B \geq C$ (Samuelson rule)
  - Problem: nobody knows $V_A + V_B$

Groves-Clark mechanism

- Ask users what their valuations are
  - A states valuation W_A
  - B states valuation W_B
- Allocation rule
  - If $W_A + W_B \geq C$ the good is produced
  - User A pays $P_A = C - W_B$
  - User B pays $P_B = C - W_A$

Groves-Clark mechanism

- Recall definition: truth-telling is NE if
  - It is in A's best interest to tell the truth, if B tells the truth
  - It is in B's best interest to tell the truth, if A tells the truth

Groves-Clark mechanism

- Claim: truth-telling is NE
  - Proof: Not part of this course

Proof

Groves-Clark mechanism

- Assume B tells truth. Is it in A's best interest to do so?
  - Problem: A does not know $P_A = C - V_B$
  - Consider case: $P_A = C – V_B \leq V_A$
    - Then A wants the good to be provided
    - It will if he reports $W_A = V_A$
  - Consider case: $P_A = C – V_B > V_A$
    - Then A does not want the good to be provided.
    - It will not if he reports $W_A = V_A$

Alternative argument

Groves-Clark mechanism

- If A reports $W_A < V_A$
  - Production whenever: $P_A = C - V_B < W_A$
    - Great since $P_A < V_A$
    - But A did not gain anything, since P_A is independent of W_A
  - No production whenever: $P_A = C - V_B > W_A$
    - Loss if $P_A = C - V_B < V_A$
- Thus: A does has no incentive to **understate** V_A

Groves-Clark mechanism

- If A reports $W_A > V_A$
  - Production whenever: $P_A = C - V_B < W_A$
    - Loss if $P_A = C - V_B > V_A$
  - No production whenever: $P_A = C - V_B > W_A$
    - Great since $P_A > V_A$
    - But A did not gain anything
- Thus: A has no incentive to **overstate** V_A

Groves-Clark mechanism

- Conclusion: truth-telling is a NE
- In fact, incentives to tell truth are very strong
  - A tells the truth independent of what B does
  - B tells the truth independent of what A does
- Intuition
  - A's price ($P_A = C - W_B$) does not depend on what he says

Groves-Clark mechanism

- Government budget
  - $G = P_A + P_B - C$
  - $= (C - V_B) + (C - V_A) - C$
  - $= C - V_B - V_A < 0$

Groves-Clark mechanism

- Groves-Clark mechanism
  - Implements the Samuelson rule
  - Contributes to the financing of the public good
  - But requires additional funding
- Implementation?
  - Difficult for people to understand
  - Costly to ask 9 million Swedes about every public good

Impossibility

- There does not exist a mechanism that achieves all the following (Mailath & Postlewaite, ReStud, 1990)
  - **Efficiency**: Implements the Samuelson rule
  - **Budget balance**
  - **Participation**: People always pay less than their valuations
  - **Truthful revelation:** All users state their true valuations
  - **Information**: Users are not required to know others' valuations

Impossibility

- Conclusion
  - Political intervention can typically not *eliminate* market failure (due to public goods)
  - Main problem: Information dispersed

Impossibility

- But
  - If
    - Public good very valuable compared to tax distortions
    - People have similar valuations
    - This is “obvious”
  - Then
    - Political intervention may increase welfare to many people without reducing it much to many others

Why public policy?

Why Politics?

- Economic explanation
  - “We” are not satisfied with the market outcome
  - “We” wish to improve welfare of somebody
- Two possibilities
  - By reducing welfare of somebody else => Redistribution
  - Without reducing welfare of somebody else => Correcting market failure

Why Politics?

- Q: What are the causes of market failure?
  - **Information**
    - Consumers don't know the quality of goods
    - Employers don't know the productivity of job applicants
    - Firm owners cannot induce managements to maximize profits
  - **Market power**
    - Firms charge prices above marginal cost
    - Employers and unions negotiate wages
  - **Externalities**
    - Pollution
    - Traffic congestion
  - **Public goods**
    - National security (Defense)
    - Law and order (Police, courts)
  - **Bounded rationality** (This is controversial)
    - Smoking, obesity
    - Choosing pension funds

Why Politics?

- Economic analysis of public policy
  - Always start by defining the political goal
    - Efficiency: What is the market failure?
    - Distribution: From whom to whom?
  - Next ask what interventions achieve the goal
  - Don't forget political limitations
    - Authorities lack information
    - Interventions change peoples incentives in unintended ways (e.g. tax distortions)

Recommended reading

- Cowell, Microeconomics, pp284-250, 448-449,451-458
- Gravelle and Rees, Microeconomics, Chapter 14, pp 326-334
- Hindriks and Myles (2006), Intermediate Public Economics, Chap 5
- Varian, Microeconomic Analysis, Chapter 23, pp 414-431
- Harvey S. Rosen and Ted Gayer: Public Finance, 8th Edition, McGraw-Hill, 2008, Ch. 4.