Dynamic Games and Bargaining

Johan Stennek
Dynamic Games

• Logic of cartels
  – Idea: We agree to both charge high prices and share the market
  – Problem: Both have incentive to cheat
  – Solution: Threat to punish cheater tomorrow
  – Question: Will we really?
Dynamic Games

• Logic of negotiations
  – People continue haggling until they are satisfied
  – People with low time-cost (patient people) have strategic advantage
Dynamic Games

• **Common theme**
  
  – Often interaction takes place over time
  
  – If we wish to understand cartels and bargaining we must take the time-dimension into account
  
  – Normal form analysis and Nash equilibrium will lead us wrong
War & Peace I
(Non-credible threats)
War & Peace

- Two countries: East and West
- Fight over an island, currently part of East
- West may attack (land an army) or not
- East may defend or not (retreating over bridge)
- If war, both have 50% chance of winning
- Value of island = $V$; Cost of war = $C > V/2$
Now, let’s describe this situation as a “decision tree” with many “deciders”

Game Tree
(Extensive form game)
War & Peace

West

- Not attack
- Attack

0, 0, 0, ½, ½ - C,
War & Peace

West

Attack

East

Retreat

Defend

Not attack
War & Peace

First number is West's payoff

West

Not attack

Attack

East

Retreat

Defend

0, V

V, 0

½ V - C, ½ V - C
Q: How should we predict behavior?
War & Peace

West needs to predict East’s behavior before making its choice

West
- Not attack
- Attack

East
- Retreat
- Defend

0, V
V, 0
½ V - C, ½ V - C
Start from the end!

West
- Not attack
- Attack

East
- Retreat
- Defend

0, V
V, 0
½ V -C, ½ V -C
War & Peace

Start from the end!

West
- Not attack
- Attack

East
- Retreat
- Defend

0, V
V, 0
½ V -C, ½ V -C

Subgame
War & Peace

West

- Not attack
- Attack

East

- Retreat
- Defend

0, V

V, 0

½ V -C, ½ V -C
What will West do, given this prediction?
War & Peace

West
- Not attack
- Attack

East
- Retreat
- Defend

0, V
V, 0
½ V - C, ½ V - C
Unique prediction:
1. West attacks
2. East retreats

West

Not attack

Attack

East

Retreat

Defend

0, V

V, 0

½ V -C, ½ V -C
• Methodology
  – Represent order of moves = “game tree”
  – Procedure: Start analyzing last period, move backwards = “backwards induction”
• **Game Trees** (Decision tree with several “deciders”)
  – Nodes = Decisions
  – Branches = Actions
  – End-nodes = Outcomes

```
W
  /   \        0, V
  \   /        V, 0
   \ /  \       ½V-c, ⅔V-c
    V   E      Not attack
     / \   / \ Attack
  \   \ \Retreat
   \ V  Defend

```
• Extensive form = “game tree”
  – Players
  – Decisions players have to take
  – Actions available at each decision
  – Order of decisions
  – Payoff to all players for all possible outcomes
War & Peace

• Normal form
  – Always possible to reduce extensive form to normal form

• How?
  – Find (Players, Strategies, Payoffs) in the tree

• Player i’s strategy
  – A complete plan of action for player i
  – Specifies an action at every node belonging to i
War & Peace

• Strategies in War & Peace
  – West: Attack, Not
  – East: Defend, Retreat
Q: Compute Nash equilibria

<table>
<thead>
<tr>
<th></th>
<th>Defend</th>
<th>Retreat</th>
</tr>
</thead>
<tbody>
<tr>
<td>Attack</td>
<td>$\frac{1}{2} V - C, \frac{1}{2} V - C$</td>
<td>$V, 0$</td>
</tr>
<tr>
<td>Not</td>
<td>$0, V$</td>
<td>$0, V$</td>
</tr>
</tbody>
</table>

![Game tree diagram](image)

W: Not attack, attack, defend, retreat
E: V, 0, $\frac{1}{2} V - C, \frac{1}{2} V - C$
War & Peace

• Two Nash equilibria
  – Attack, Retreat
  – Not attack, Defend

  Unreasonable prediction

  East threatens to defend the island.
  And if West believes it, it does not attack.
  Then, East does not have to fight.

  But if West would attack, then East would retreat.
  Knowing this, West does not believe the threat.

  It is a non-credible threat
War & Peace

• Conclusion for game theory analysis
  – Need extensive form and backwards induction to get rid of non-reasonable Nash equilibria (non-credible threats).

• Conclusion for Generals (and others)
  – Threats (and promises) must be credible
War & Peace II
(Commitment)
War & Peace

• East reconsiders its position before West attacks
  – Gen. 1: “Burn bridge – makes retreat impossible!”
  – Gen. 2: “Then war – the worst possible outcome!”

• **Q:** How analyze?
  – Write up new extensive form game tree
  – Apply backwards induction
First number is West’s payoff

Q: Game tree?
Q: What method do we use to make prediction?
War & Peace

East

West

Not burn

Burn

Not attack

Attack

0, V

½ V -C, ½ V -C

0, V

½ V -C, ½ V -C

V, 0

Retreat

Defend
War & Peace

West
- Attack
- Not attack

East
- Burn
- Not burn

West
- Attack
- Not attack

East
- Retreat
- Defend

0, V
½ V - C, ½ V - C
0, V
V, 0
½ V - C, ½ V - C
War & Peace

Diagram of a game with East and West making decisions:

- East can choose to Burn or Not burn.
- West can choose to Attack or Not attack.
- If East Burns, West Attack results in $\frac{1}{2}V - C$, Not attack results in $0, V$.
- If East does Not burn, West Attack results in $V, 0$, Not attack results in $\frac{1}{2}V - C, \frac{1}{2}V - C$.

The diagram shows decision branches with outcomes and strategies.
War & Peace

- Attack
- Not attack

Decision Points:
- East
  - Burn
  - Not burn
  - West
    - Not attack
    - Attack
  - 0, V
    - ½ V -C, ½ V -C
- West
  - Attack
  - Not attack
  - Retreat
  - Defend
  - East
    - V, 0
    - ½ V -C, ½ V -C
    - 0, V
    - ½ V -C, ½ V -C
Equilibrium provides description of what every player will do at every decision node.
Also the decisions at the nodes that will never be reached are sensible decisions (East’s second decision)
At date 2, West makes different decisions, depending on what East did at date 1.
War & Peace

• Conclusion
  – East’s threat to defend made credible
  – Pre-commitment
• Two newspaper articles (in Swedish)
  – Pellnäs:
    • West needs new **credible** defense doctrine
    • We need to make clear to Putin when we will take the fight
  – Agrell:
    • We cannot use “game theory” to predict the behavior of countries (Russia) – they are not rational
Bargaining Bilateral & Market Power

Johan Stennek
Not included:

1. appendixes in lecture notes
2. Ch. 7.4
Bilateral Market Power

Example: Food Retailing
Food Retailing

• Food retailers are huge

<table>
<thead>
<tr>
<th>Company</th>
<th>Food Sales (US$mn)</th>
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</thead>
<tbody>
<tr>
<td>Wal-Mart</td>
<td>121 566</td>
</tr>
<tr>
<td>Carrefour</td>
<td>77 330</td>
</tr>
<tr>
<td>Ahold</td>
<td>72 414</td>
</tr>
<tr>
<td>Tesco</td>
<td>40 907</td>
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<tr>
<td>Kroger</td>
<td>39 320</td>
</tr>
<tr>
<td>Rewe</td>
<td>36 483</td>
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<tr>
<td>Aldi</td>
<td>36 189</td>
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<tr>
<td>Ito-Yokado</td>
<td>35 812</td>
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<tr>
<td>Metro Group ITM</td>
<td>34 700</td>
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</table>

\(\frac{1}{2}\) Swedish GDP
Food Retailing

- Retail markets are highly concentrated

<table>
<thead>
<tr>
<th>Kedja</th>
<th>Butiker (antal)</th>
<th>Butiksyta (kvm)</th>
<th>Omsättning (miljarder kr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Axfood</td>
<td>803 (24%)</td>
<td>625 855 (18%)</td>
<td>34,6 (18%)</td>
</tr>
<tr>
<td>Bergendahls</td>
<td>229 (7%)</td>
<td>328 196 (10%)</td>
<td>13,6 (7%)</td>
</tr>
<tr>
<td>Coop</td>
<td>730 (22%)</td>
<td>983 255 (29%)</td>
<td>41,4 (21%)</td>
</tr>
<tr>
<td>ICA</td>
<td>1 379 (41%)</td>
<td>1 240 602 (36%)</td>
<td>96,6 (50%)</td>
</tr>
<tr>
<td>Lidl</td>
<td>146 (4%)</td>
<td>170 767 (5%)</td>
<td>5,2 (3%)</td>
</tr>
<tr>
<td>Netto</td>
<td>105 (3%)</td>
<td>70 603 (2%)</td>
<td>3,0 (2%)</td>
</tr>
</tbody>
</table>
Food Retailing

• **Food manufacturers**
  – Some are huge:
    • Kraft Food, Nestle, Scan
    • Annual sales tenth of billions of Euros
  – Some are tiny:
    • local cheese
Food Retailing

• **Mutual dependence**
  – Some brands = Must have
    • ICA “must” sell Coke
    • Otherwise many families would shop at Coop
  – Some retailers = Must channel
    • Coke “must” sell via ICA to be active in Sweden
    • Probably large share of Coke’s sales in Sweden
  – Both would lose if ICA would not sell Coke
Food Retailing

• Mutual dependence
  – Manufacturers cannot dictate wholesale prices
  – Retailers cannot dictate wholesale prices

• Thus
  – They have to negotiate and agree

• In particular
  – Also retailers have market power
    = buyer power
Food Retailing

• Large retailers pay lower prices (= more buyer power)

<table>
<thead>
<tr>
<th>Retailer</th>
<th>Market Share (CC Table 5:3, p. 44)</th>
<th>Price (CC Table 5, p. 435)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tesco</td>
<td>24.6</td>
<td>100.0</td>
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<tr>
<td>Sainsbury</td>
<td>20.7</td>
<td>101.6</td>
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<td>Asda</td>
<td>13.4</td>
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<td>Somerfield</td>
<td>8.5</td>
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<td>Iceland</td>
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<td>Waitrose</td>
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<td>Netto</td>
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<tr>
<td>Budgens</td>
<td>0.4</td>
<td>111.1</td>
</tr>
</tbody>
</table>
Other examples

• Labor markets
  – Vårdförbundet vs Landsting

• Relation-specific investments
  – Car manufacturers vs producers of parts
Food Retailing

• **Questions**
  – How analyze bargaining in intermediate goods markets?
  – Why do large buyers get better prices?
Bilateral Monopoly
Bilateral Monopoly

- **Exogenous conditions**
  - One Seller: \( MC(q) \)
    - inverse supply if price taker
  - One Buyer: \( MV(q) \)
    - inverse demand if price taker
Bilateral Monopoly
Intuitive Analysis

• **Efficient quantity**
  – Complete information
  – Maximize the surplus to be shared

![Diagram showing economic concepts](image-url)
Bilateral Monopoly

Intuitive Analysis

- **Efficient quantity**
  - Complete information
  - Maximize the surplus to be shared

Efficiency from the point of view of the two firms = Same quantity as a vertically integrated firm would choose
• Problem
  – But what price?

• Only restrictions
  – Seller must cover his costs, \( C(q^*) \)
  – Buyer must not pay more than wtp, \( V(q^*) \)

=> Any split of \( S^* = V(q^*) - C(q^*) \) seems reasonable
Bilateral Monopoly
Intuitive Analysis

• Note
  – If someone demands “too much”
  – The other side will reject and make a counter-offer

• Problem
  – Haggling could go on forever
  – Gains from trade delayed

• Thus
  – Both sides have incentive to be reasonable
  – But, the party with less aversion to delay has strategic advantage
Bilateral Monopoly

Definitions

• Definitions
  – Efficient quantity: \( q^* \)
  – Walrasian price: \( p^w \)
  – Maximum bilateral surplus: \( S^* \)
Bilateral Monopoly

- **First important insight:**
  - Contract must specify both price and quantity, \((p, q)\)
  - Why?
- **Otherwise inefficient quantity**
  - If \(p > p^w\) then \(q < q^*\)
  - If \(p < p^w\) then \(q < q^*\)
  - Short side of the market decides
Extensive Form Bargaining

Ultimatum bargaining
Ultimatum bargaining

• One round of negotiations
  – One party, say seller, gets to propose a contract \((p, q)\)
  – Other party, say buyer, can accept or reject

• Outcome
  – If \((p, q)\) accepted, it is implemented
  – Otherwise game ends without agreement

• Payoffs
  – Buyer: \(V(q) - p q\) if agreement, zero otherwise
  – Seller: \(p q - C(q)\) if agreement, zero otherwise

• Perfect information
  – Backwards induction
Ultimatum bargaining

• **Time 2:** Buyer accepts or rejects proposed contract
  
  – **Q:** What would make buyer accept \((p, q)\)?
  
  – Buyer accepts \((p, q)\) iff \(V(q) – p \cdot q \geq 0\)

• **Time 1:** Seller proposes best contract that would be accepted
  
  – **Q:** How do we find the seller’s best contract?
  
  – \(\max_{p,q} \ p \cdot q – C(q)\) such that \(V(q) – p \cdot q \geq 0\)
Ultimatum bargaining

Seller's maximization problem
\[
\max_{p,q} \quad p \cdot q - C(q)
\]

subject to: \( V(q) - p \cdot q \geq 0 \)

Optimal price
Increase price until: \( p \cdot q = V(q) \) \hspace{5cm} \text{Seller takes whole surplus}

Optimal quantity
\[
\max_q V(q) - C(q)
\]

Must set q such that: \( MV(q) = MC(q) \) \hspace{5cm} \text{Efficient quantity}
Ultimatum bargaining

• **SPE of ultimatum bargaining game**
  – Unique equilibrium
  – There is agreement
  – Efficient quantity
  – Proposer takes the whole (maximal) surplus
Ultimatum bargaining

- Assume rest of lecture
  - Always efficient quantity
  - Surplus = 1
  - Player S gets share $\pi_S$
  - Player B gets share $\pi_B = 1 - \pi_S$

- Ultimatum game
  - $\pi_S = 1$
  - $\pi_B = 0$
Two rounds (T=2)
Two rounds \( (T=2) \)

- **Alternating offers**
  - Period 1
    - B proposes contract
    - S accepts or rejects
  - Period 2 (in case S rejected)
    - S proposes contract
    - B accepts or rejects

- **Perfect information**
  - No simultaneous moves
  - Players know what has happened before in the game

- **Solution concept**
  - Backwards induction (Subgame perfect equilibrium)
Two rounds \((T=2)\)

- **Player B is impatient**
  - €1 in period 2 is equally good as \(ε\delta_B\) in period 1
  - Where \(\delta_B < 1\) is B’s discount factor

- **Player S is impatient**
  - €1 in period 2 is equally good as \(ε\delta_S\) in period 1
  - Where \(\delta_S < 1\) is S’s discount factor
Two rounds \((T=2)\)

- **Period 1**
  - B proposes \((\pi_B^{T-1}, \pi_S^{T-1})\)
  - S accepts or rejects

- **Period 2** (in case S rejected)
  - S proposes \((\pi_B^T, \pi_S^T)\)
  - B accepts or rejects

- **Perfect information \(\Rightarrow\) Use BI**

Solve this game now!
Two rounds

• Period \( T = 2 \) (S bids) (What will happen in case S rejected?)
  - B accepts iff: \( \pi_B^T \geq 0 \)
  - S proposes: \( \pi_B^T = 0 \), \( \pi_S^T = 1 \)

• Period \( T-1 = 1 \) (B bids)
  - S accepts iff: \( \pi_S^{T-1} \geq \delta_S \pi_S^T = \delta_S < 1 \)
  - B proposes: \( \pi_B^{T-1} = 1 - \delta_S > 0 \), \( \pi_S^{T-1} = \delta_S \)

• Note
  - S willing to reduce his share to get an early agreement
  - Both players get part of surplus
  - B’s share determined by S’s impatience. If S very patient \( \pi_S \approx 1 \)
T rounds
T rounds

• Model
  – Large number of periods, T
  – Buyer and seller take turns to make offer
  – Common discount factor $\delta = \delta_B = \delta_S$
  – Subgame perfect equilibrium (ie start analysis in last period)
## T rounds

<table>
<thead>
<tr>
<th>Time</th>
<th>Bidder</th>
<th>$\pi_B$</th>
<th>$\pi_S$</th>
<th>Resp.</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>S</td>
<td>?</td>
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<td>rest</td>
<td>$\delta$</td>
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<td>T-1</td>
<td>B</td>
<td>1-$\delta$</td>
<td>$\delta$</td>
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<tr>
<td>T-1</td>
<td>B</td>
<td>1-(\delta)</td>
<td>(\delta)</td>
<td>yes</td>
</tr>
<tr>
<td>T-2</td>
<td>S</td>
<td>?</td>
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<td>B</td>
<td>1-(\delta)</td>
<td>(\delta)</td>
<td>yes</td>
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<tr>
<td>T-2</td>
<td>S</td>
<td>(\delta(1-\delta))</td>
<td>rest</td>
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<td>B</td>
<td>$1-\delta$</td>
<td>$\delta$</td>
<td>yes</td>
</tr>
<tr>
<td>T-2</td>
<td>S</td>
<td>$\delta(1-\delta)$</td>
<td>$1-\delta(1-\delta)$</td>
<td>yes</td>
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<td>B</td>
<td>1-δ</td>
<td>δ</td>
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</tr>
<tr>
<td>T-2</td>
<td>S</td>
<td>$\delta(1-\delta)$</td>
<td>1-δ(1-δ)</td>
<td>yes</td>
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multiply
### T rounds

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<td>B</td>
<td>1-$\delta$</td>
<td>$\delta$</td>
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<tr>
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<tr>
<td>T-1</td>
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<td>1-δ</td>
<td>δ</td>
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<td>T-2</td>
<td>S</td>
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<td>1-δ+δ²</td>
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<tr>
<td>T-3</td>
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<tr>
<td>T-3</td>
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<td>rest</td>
<td>δ(1-δ+δ²)</td>
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<td>T-3</td>
<td>B</td>
<td>1-$\delta$(1-$\delta$+$\delta^2$)</td>
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<td>B</td>
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<td>T-4</td>
<td>S</td>
<td>$\delta$(1-$\delta$+$\delta^2$-$\delta^3$)</td>
<td>rest</td>
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<td>(\delta)</td>
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<td>T-2</td>
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<td>(\delta-\delta^2)</td>
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<td>1</td>
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<td>1-δ+δ^2-δ^3+δ^4-…+δ^{T-1}</td>
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\[
\pi_B = \delta - \delta^2 + \delta^3 - \delta^4 + \ldots - \delta^{T-1}
\]

\[
\pi_S = 1 - \delta + \delta^2 - \delta^3 + \delta^4 - \ldots + \delta^{T-1}
\]
T rounds

Geometric series

\[ \pi_B = \delta - \delta^2 + \delta^3 - \delta^4 + \ldots - \delta^{T-1} \]

\[ \pi_S = 1 - \delta + \delta^2 - \delta^3 + \delta^4 - \ldots + \delta^{T-1} \]
T rounds

S's share

$$\pi_s = 1 - \delta + \delta^2 - \delta^3 + \delta^4 - \ldots + \delta^{T-1}$$
T rounds

S's share
\[ \pi_s = 1 - \delta + \delta^2 - \delta^3 + \delta^4 - \ldots + \delta^{T-1} \]

Multiply
\[ \delta \pi_s = \delta - \delta^2 + \delta^3 - \delta^4 + \delta^5 - \ldots + \delta^T \]
T rounds

S's share
\[ \pi_S = 1 - \delta + \delta^2 - \delta^3 + \delta^4 - \ldots + \delta^{T-1} \]

Multiply
\[ \delta \pi_S = \delta - \delta^2 + \delta^3 - \delta^4 + \delta^5 - \ldots + \delta^T \]

Add
\[ \pi_S + \delta \pi_S = 1 + \delta^T \]
T rounds

S's share
\[ \pi_S = 1 - \delta + \delta^2 - \delta^3 + \delta^4 - \ldots + \delta^{T-1} \]

Multiply
\[ \delta \pi_S = \delta - \delta^2 + \delta^3 - \delta^4 + \delta^5 - \ldots + \delta^T \]

Add
\[ \pi_S + \delta \pi_S = 1 + \delta^T \]

Solve
\[ \pi_S = \frac{1 + \delta^T}{1 + \delta} \]
T rounds

Equilibrium shares with T periods

\[ \pi_s = \frac{1}{1+\delta} (1 + \delta^T) \]

\[ \pi_B = \frac{\delta}{1+\delta} (1 - \delta^{T-1}) \]
T rounds

Equilibrium shares with T periods

\[ \pi_s = \frac{1}{1 + \delta} (1 + \delta^T) \]

\[ \pi_B = \frac{\delta}{1 + \delta} (1 - \delta^{T-1}) \]

S has advantage of making last bid

\[ 1 + \delta^T > 1 - \delta^{T-1} \]

To confirm this, solve model where
- B makes last bid
- S makes first bid
T rounds

Equilibrium shares with T periods

\[ \pi_s = \frac{1}{1 + \delta} (1 + \delta^T) \]

\[ \pi_B = \frac{\delta}{1 + \delta} (1 - \delta^{T-1}) \]

S has advantage of making last bid

\[ 1 + \delta^T > 1 - \delta^{T-1} \]  

Disappears if T very large
T rounds

Equilibrium shares with $T \approx \infty$ periods

$$\pi_S = \frac{1}{1 + \delta}$$

$$\pi_B = \frac{\delta}{1 + \delta}$$
T rounds

Equilibrium shares with $T \approx \infty$ periods

$$\pi_S = \frac{1}{1+\delta}$$

$$\pi_B = \frac{\delta}{1+\delta}$$

S has advantage of making first bid

$$\frac{1}{1+\delta} > \frac{\delta}{1+\delta}$$

To confirm this, solve model where
- B makes first bid
T rounds

Equilibrium shares with $T \approx \infty$ periods

$$\pi_S = \frac{1}{1 + \delta}$$

$$\pi_B = \frac{\delta}{1 + \delta}$$

S has advantage of making first bid

$$\frac{1}{1 + \delta} > \frac{\delta}{1 + \delta}$$

First bidder’s advantage disappears if $\delta \approx 1$
T rounds

Equilibrium shares with $T \approx \infty$ periods and very patient players ($\delta \approx 1$)

$$\pi_S = \frac{1}{2}$$

$$\pi_B = \frac{1}{2}$$
Difference in Patience

Equilibrium shares with $T \approx \infty$ periods and different discount factors

$$\pi_s = \frac{1 - \delta_B}{1 - \delta_s \delta_B}$$

$$\pi_B = \frac{1 - \delta_s}{1 - \delta_s \delta_B} \delta_B$$

(Easy to show using same method as above)
Difference in Patience

• Recall \( \delta_i = e^{-r_i \Delta} \)
  
  – \( r_i \) = continuous-time discount factor
  
  – \( \Delta \) = length of time period

• Then, as \( \Delta \rightarrow 0 \):
  
  – \( \pi_s = \frac{1 - \delta_B}{1 - \delta_s \delta_B} \approx \frac{r_B}{r_s + r_B} \)
  
  – Using l’Hopital’s rule
Conclusions

• Exists unique equilibrium (SPE)
• There is agreement
• Agreement is immediate
• Efficient agreement (here: quantity)
• Split of surplus (price) determined by:
  • Relative patience
  • Right to make last bid gives advantage (if $T < \infty$)
  • Right to make first bid gives advantage (if $\delta < 1$)
Implications for Bilateral Monopoly
Implications for Bilateral Monopoly

- Equal splitting

\[ \Pi_s = \Pi_B \]

\[ p \cdot q - C(q) = V(q) - p \cdot q \]

\[ 2 \cdot p \cdot q = V(q) + C(q) \]

\[ p = \frac{1}{2} \left[ \frac{V(q)}{q} + \frac{C(q)}{q} \right] \]
Implications for Bilateral Monopoly

- Equal splitting

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Implications for Bilateral Monopoly

• Equal splitting

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\[ 2 \cdot p \cdot q = V(q) + C(q) \]

\[ p = \frac{1}{2} \left[ \frac{V(q)}{q} + \frac{C(q)}{q} \right] \]

The firms share the Retailer’s revenues and the Manufacturer’s costs equally.
Nash Bargaining Solution
-- A Reduced Form Model
Nash Bargaining Solution

- Extensive form bargaining model
  - Intuitive
  - But tedious

- Nash bargaining solution
  - Less intuitive
  - But easier to find the same outcome
Nash Bargaining Solution

• Three steps
  1. Describe bargaining situation
  2. Define Nash product
  3. Maximize Nash product
Nash Bargaining Solution

• Step 1: Describe bargaining situation
  1. Who are the two players?
  2. What contracts can they agree upon?
  3. What payoff would they get from every possible contract?
  4. What payoff do they have before agreement?
  5. What is their relative patience (= bargaining power)
Nash Bargaining Solution
Example 1: Bilateral monopoly

• Step 1: Describe the bargaining situation
  – Players: Manufacturer and Retailer
  – Contracts: \((T, q)\)
  – Payoffs:
    • Retailer: \(\pi_R(T, q) = V(q) - T\)
    • Manufacturer: \(\pi_M(T, q) = T - C(q)\)
  – Payoff if there is no agreement (while negotiating)
    • Retailer: \(\tilde{\pi}_R = 0\)
    • Manufacturer: \(\tilde{\pi}_M = 0\)
  – Same patience \(\Rightarrow\) same bargaining power

\(T = \text{total price for } q \text{ units.}\)
Nash Bargaining Solution

Example 1: Bilateral monopoly

• Step 2: Set up Nash product

\[ N(T,q) = \left[ \pi_R(T,q) - \tilde{\pi}_R \right] \cdot \left[ \pi_M(T,q) - \tilde{\pi}_M \right] \]

Retailer’s profit from contract  
Manufacturer’s profit from contract
Nash Bargaining Solution

Example 1: Bilateral monopoly

- Step 2: Set up Nash product

\[ N(T,q) = \left[ \pi_R(T,q) - \tilde{\pi}_R \right] \cdot \left[ \pi_M(T,q) - \tilde{\pi}_M \right] \]

Retailer’s extra profit from contract

Manufacturer’s extra profit from contract
Nash Bargaining Solution

Example 1: Bilateral monopoly

• Step 2: Set up Nash product

\[ N(T,q) = \left[ \pi_R(T,q) - \tilde{\pi}_R \right] \cdot \left[ \pi_M(T,q) - \tilde{\pi}_M \right] \]

Nash product
- Product of payoff increases

Depends on contract
Nash Bargaining Solution

Example 1: Bilateral monopoly

- Step 2: Set up Nash product

\[ N(T, q) = \left[ \pi_R(T, q) - \tilde{\pi}_R \right] \cdot \left[ \pi_M(T, q) - \tilde{\pi}_M \right] \]

Claim:
The contract \((T, q)\) maximizing \(N\) is the same contract that the parties would agree upon in an extensive form bargaining game!
Nash Bargaining Solution

Example 1: Bilateral monopoly

Step 2: Set up Nash product

\[ N(T,q) = \left[ \pi_R(T,q) - \bar{\pi}_R \right] \cdot \left[ \pi_M(T,q) - \bar{\pi}_M \right] \]

\[ N(T,q) = \left[ V(q) - T \right] \cdot \left[ T - C(q) \right] \]
Nash Bargaining Solution

Example 1: Bilateral monopoly

• Maximize Nash product

\[ N(T, q) = [V(q) - T] \cdot [T - C(q)] \]

\[ \frac{\partial N}{\partial T} = -[T - C(q)] + [V(q) - T] = 0 \]

Equal profits = Equal split of surplus
Nash Bargaining Solution

Example 1: Bilateral monopoly

- Maximize Nash product

\[
N(T, q) = [V(q) - T] \cdot [T - C(q)]
\]

\[
\frac{\partial N}{\partial T} = -[T - C(q)] + [V(q) - T] = 0 \quad \Rightarrow \quad T = \frac{1}{2} [V(q) + C(q)]
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Example 1: Bilateral monopoly

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\[
\Rightarrow \quad p = \frac{1}{2} \left[ \frac{V(q)}{q} + \frac{C(q)}{q} \right]
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Nash Bargaining Solution

Example 1: Bilateral monopoly

• Maximize Nash product

\[ N(T, q) = [V(q) - T] \cdot [T - C(q)] \]

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\[ \frac{\partial N}{\partial q} = V'(q) \cdot [T - C(q)] - C'(q) \cdot [V(q) - T] = 0 \quad \Rightarrow \quad V'(q) = C'(q) \]
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Example 1: Bilateral monopoly

• Maximize Nash product

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Nash Bargaining Solution

Example 1: Bilateral monopoly

• Conclusion
  – Maximizing Nash product is easy way to find equilibrium
  – Efficient quantity
  – Price splits surplus equally
Nash Bargaining Solution

• With different bargaining power

\[ N(T, q) = \left[ \pi_R(T, q) - \tilde{\pi}_R \right]^\beta \cdot \left[ \pi_M(T, q) - \tilde{\pi}_M \right]^{1-\beta} \]

Exponents determined by relative patience