School of Business, Economics and Law
GÖTEBORG UNIVERSITY

Dynamic Games and Bargaining

Johan Stennek

Dynamic Games

- Logic of cartels
  - Idea: We agree to both charge high prices and share the market
  - Problem: Both have incentive to cheat
  - Solution: Threat to punish cheater tomorrow
  - Question: Will we really?

Dynamic Games

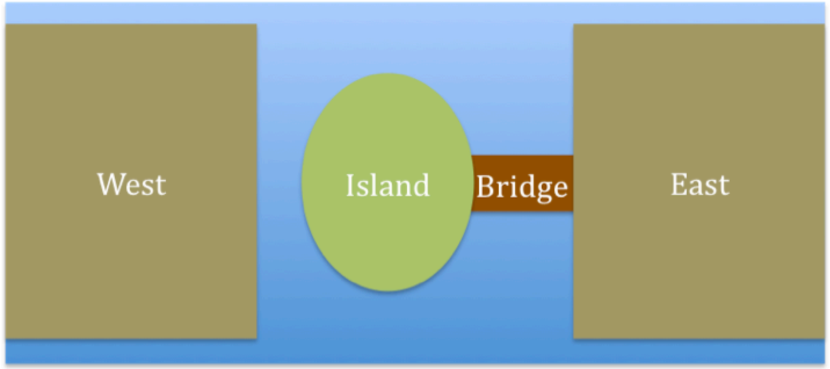
- Logic of negotiations
  - People continue haggling until they are satisfied
  - People with low time-cost (patient people) have strategic advantage

Dynamic Games

- Common theme
  - Often interaction takes place over time
  - If we wish to understand cartels and bargaining we must take the time-dimension into account
  - Normal form analysis and Nash equilibrium will lead us wrong

War & Peace I
(Non-credible threats)

War & Peace



- Two countries: East and West
- Fight over an island, currently part of East
- West may attack (land an army) or not
- East may defend or not (retreating over bridge)
- If war, both have 50% chance of winning
- Value of island = V; Cost of war = $C > V/2$

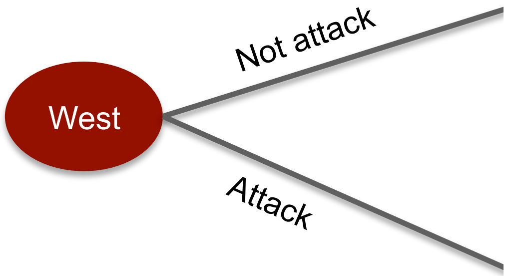
War & Peace

Now, let's describe this situation as a "decision tree" with many "deciders"

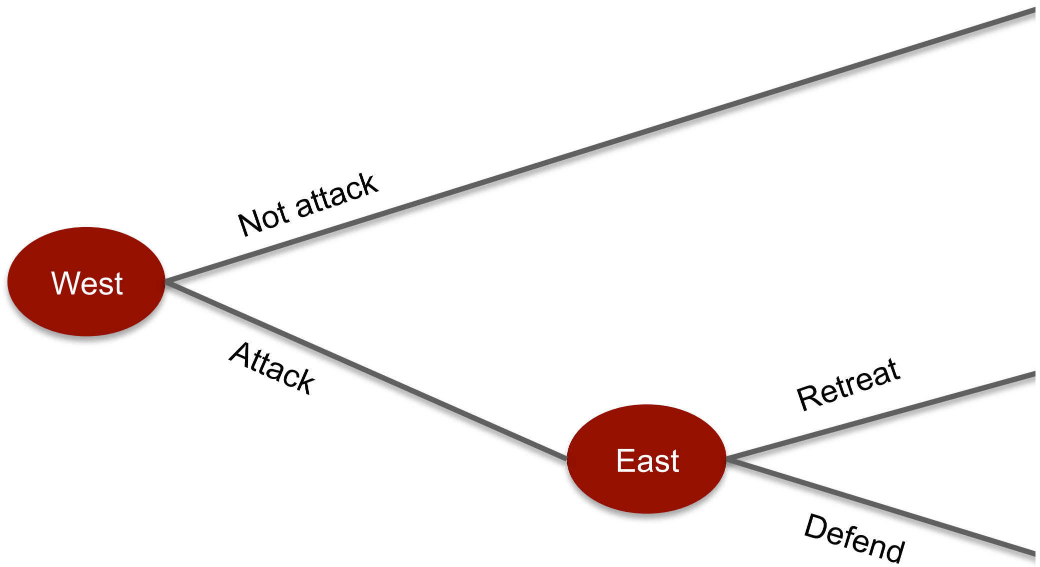
Game Tree

(Extensive form game)

War & Peace

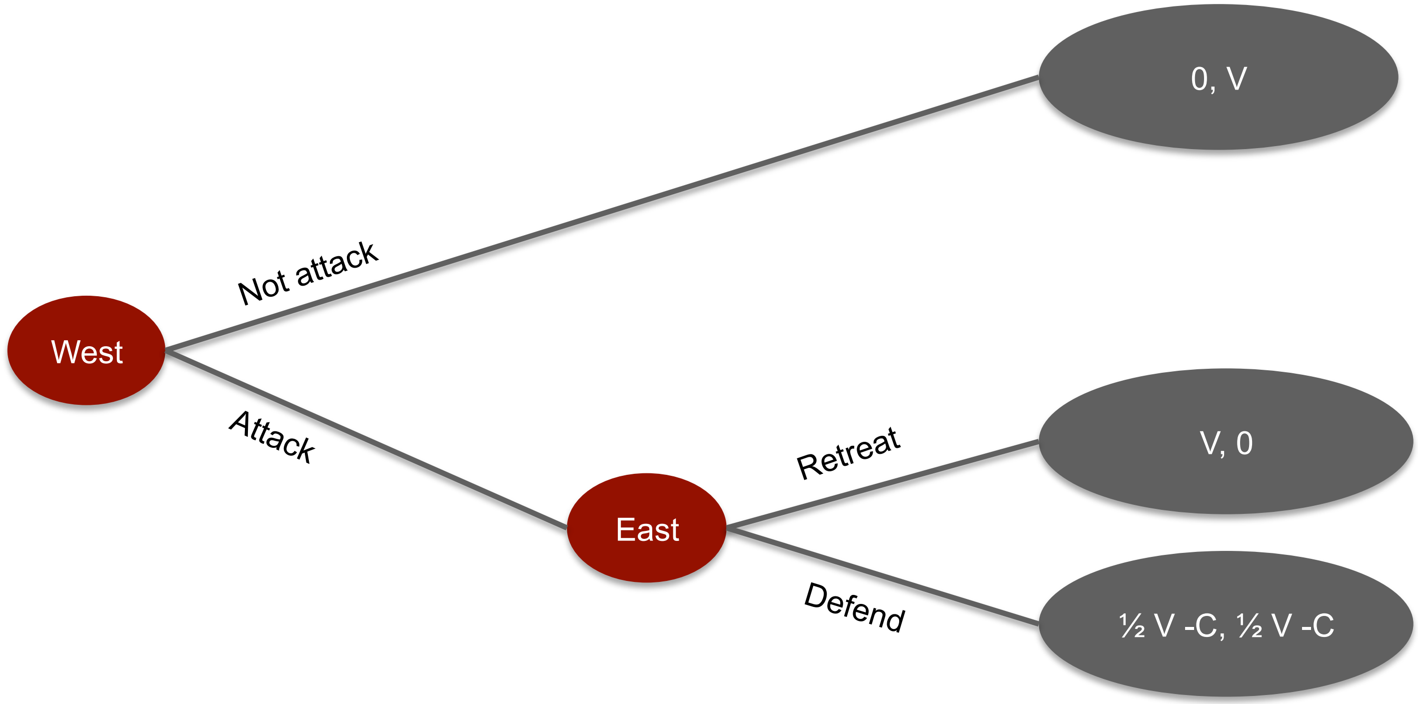


War & Peace



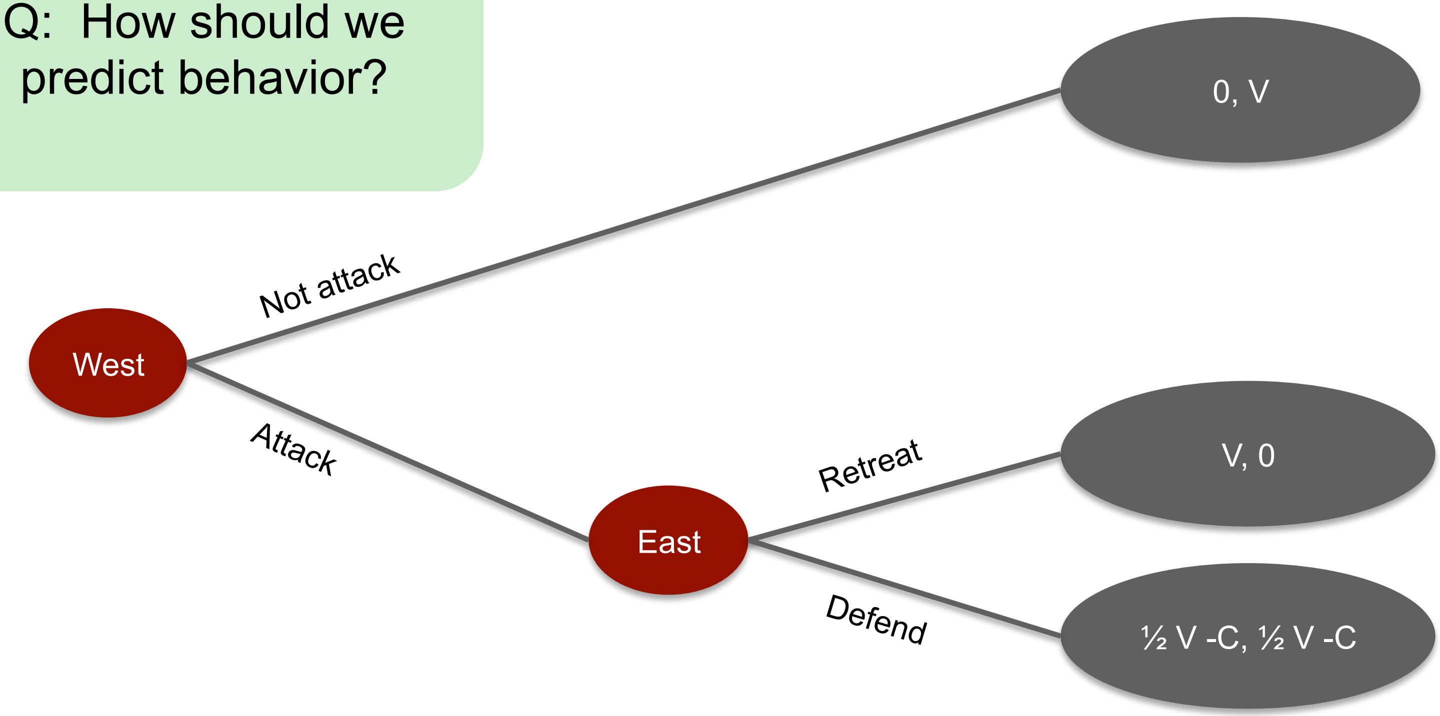
War & Peace

First number is West's payoff



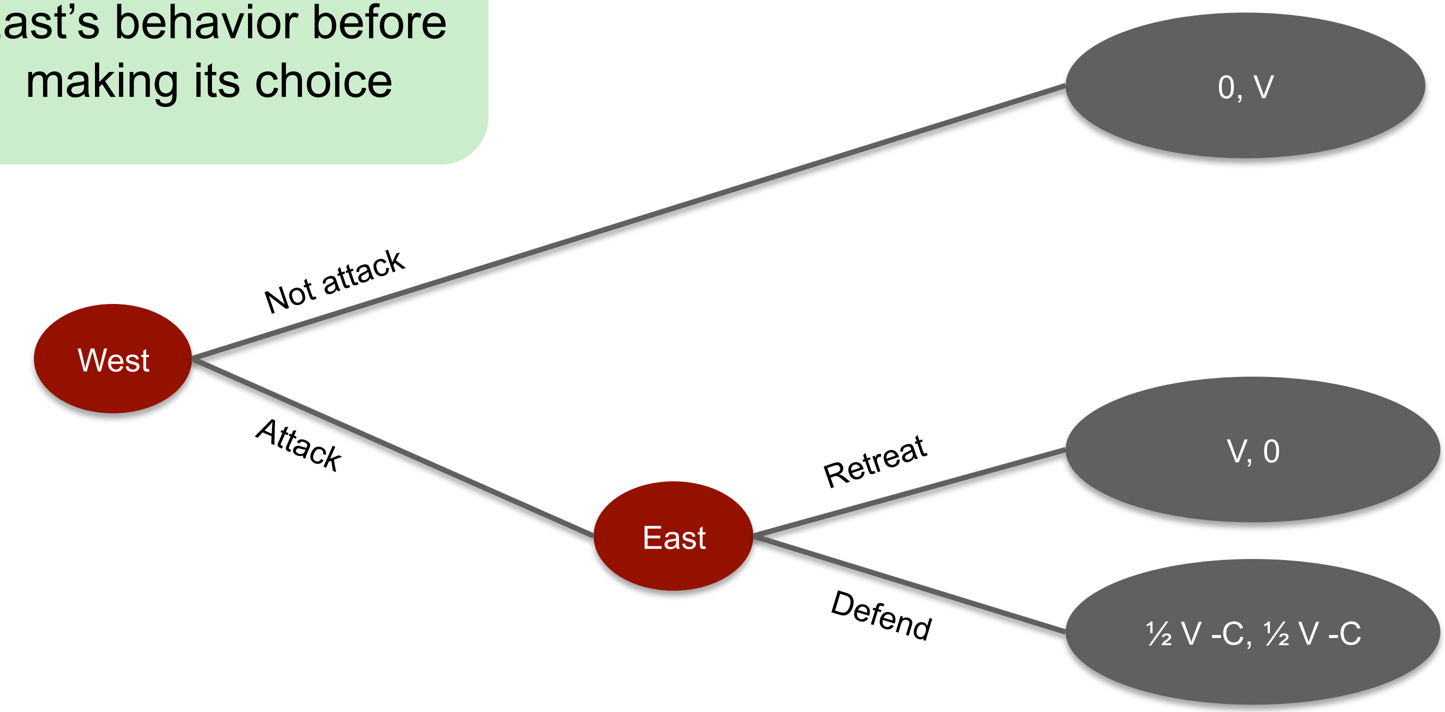
War & Peace

Q: How should we predict behavior?



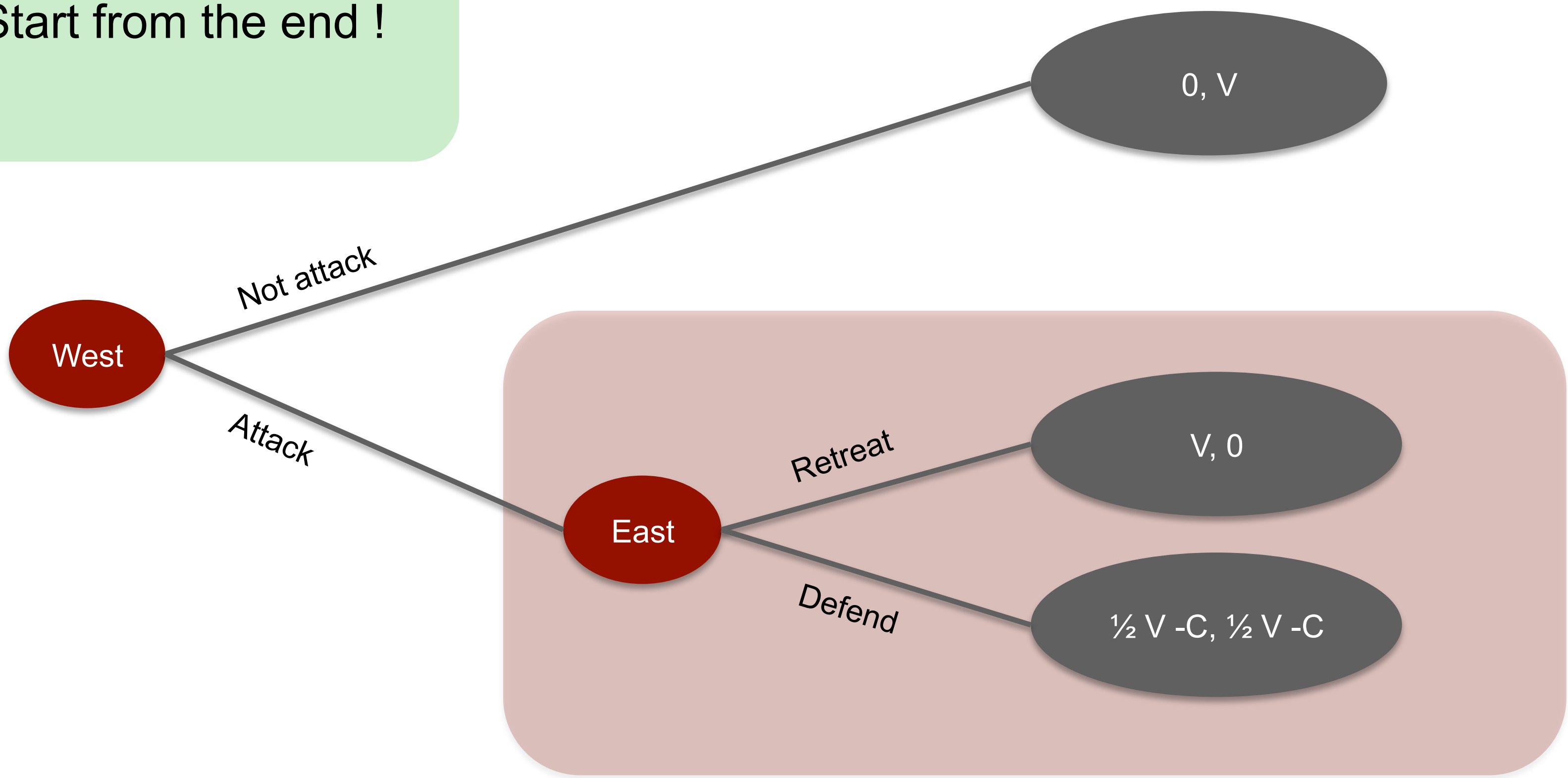
War & Peace

West needs to predict East's behavior before making its choice



War & Peace

Start from the end !



War & Peace

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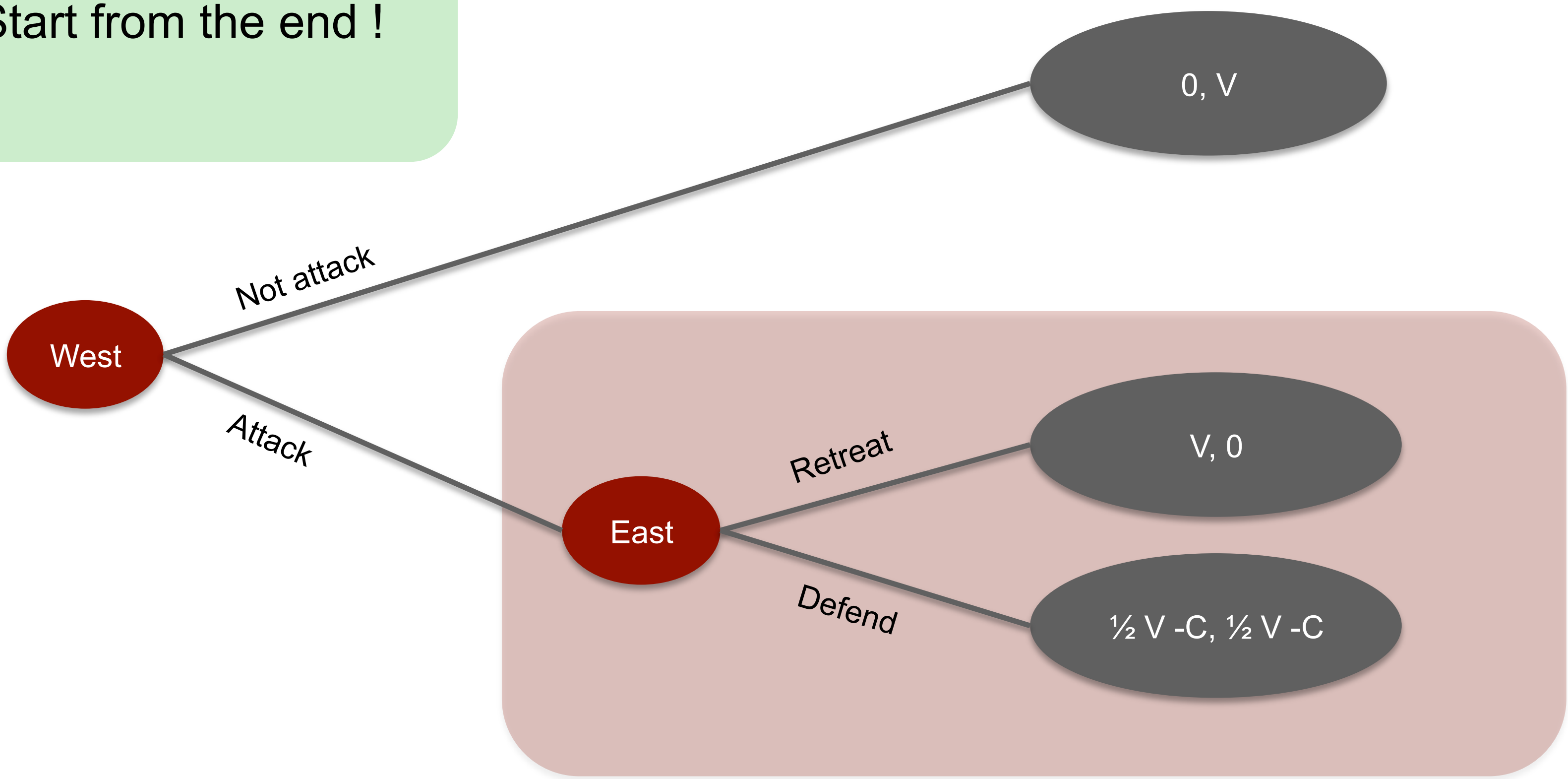
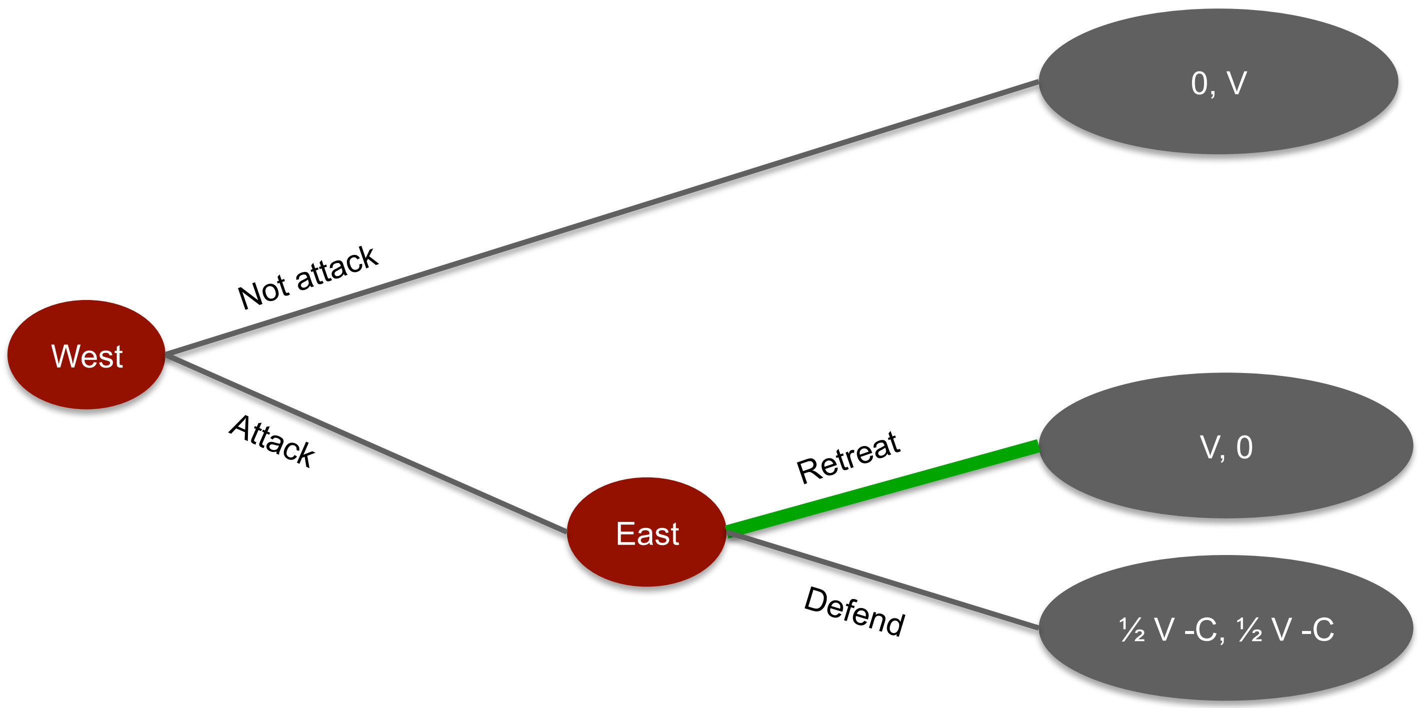


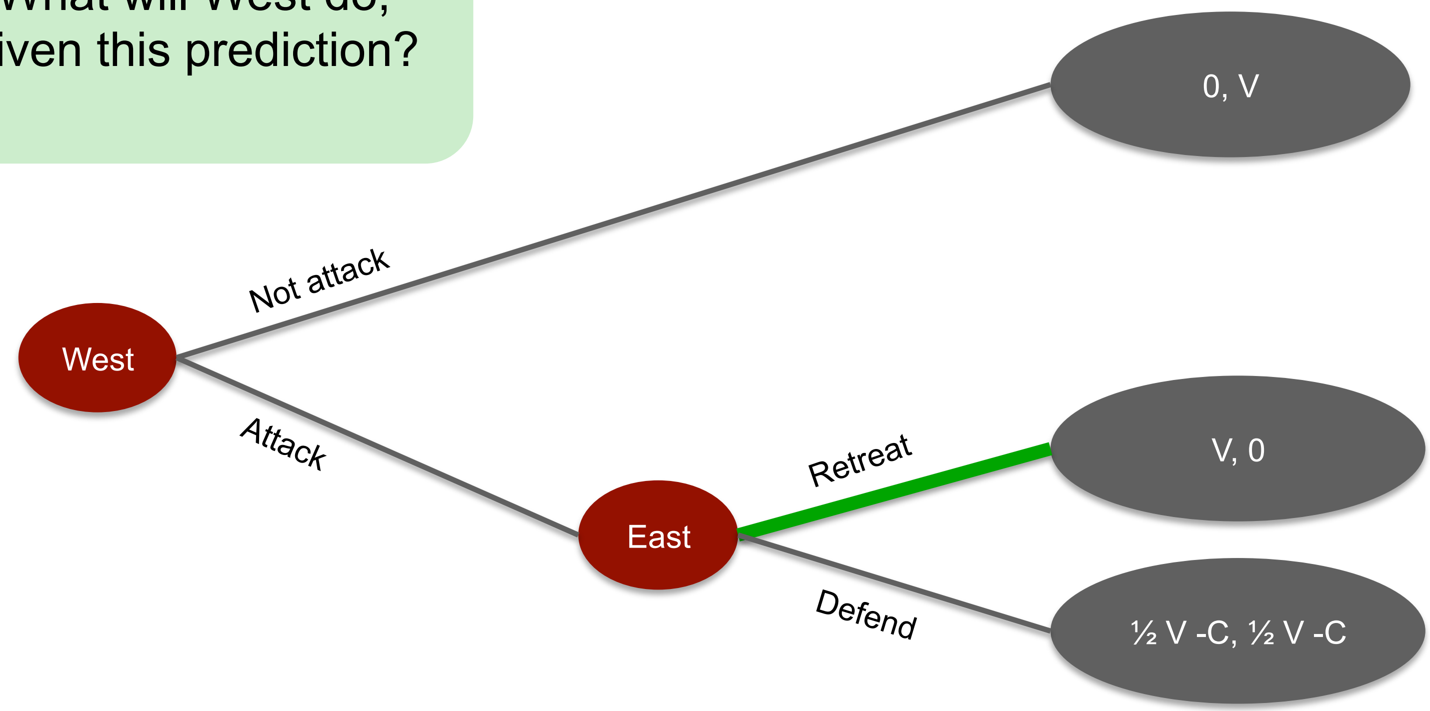
Subgame

War & Peace

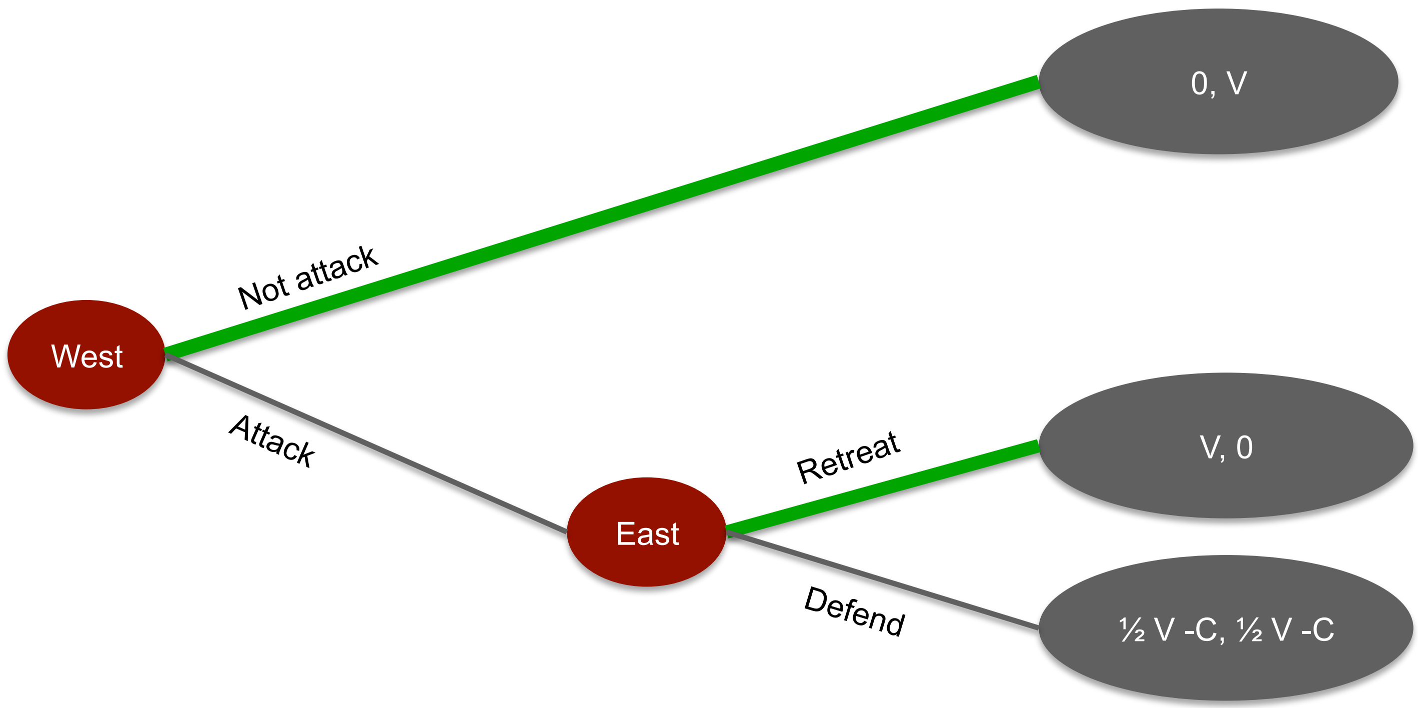


War & Peace

What will West do, given this prediction?

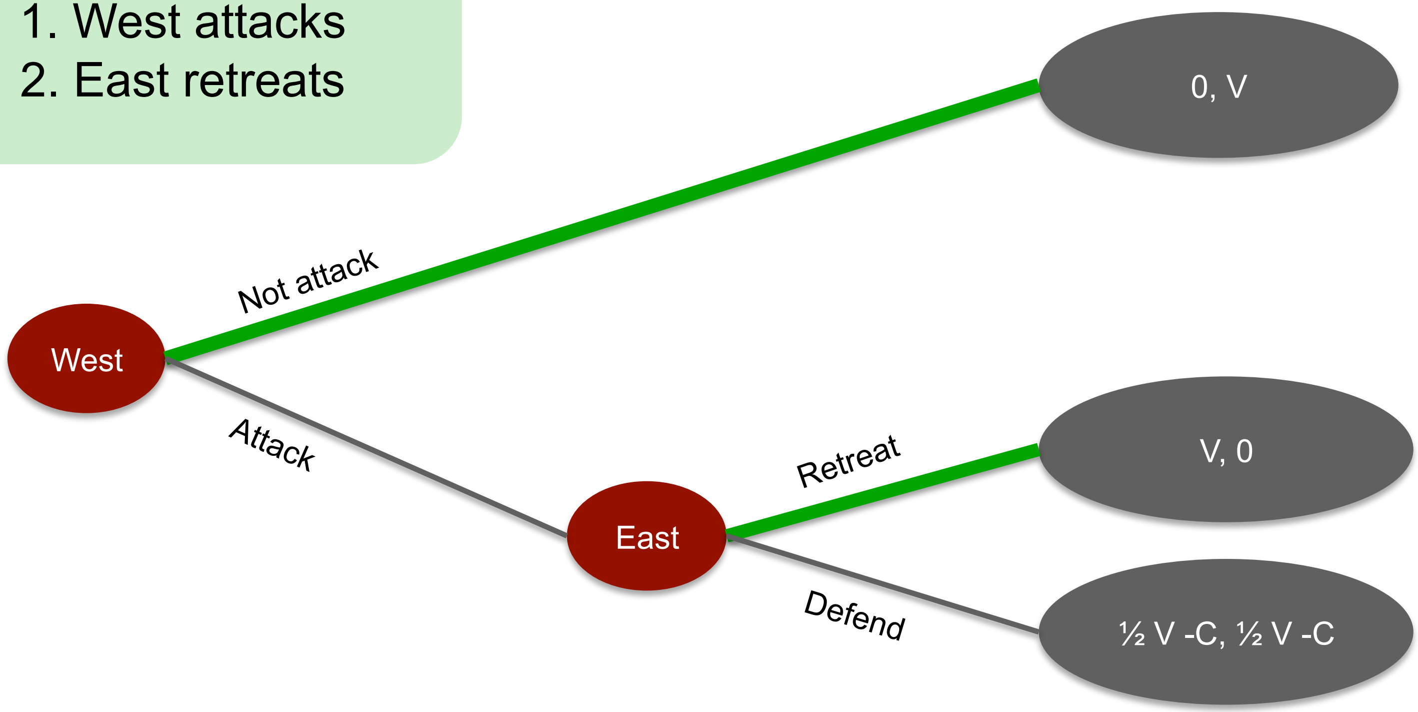


War & Peace



War & Peace

Unique prediction:
1. West attacks
2. East retreats

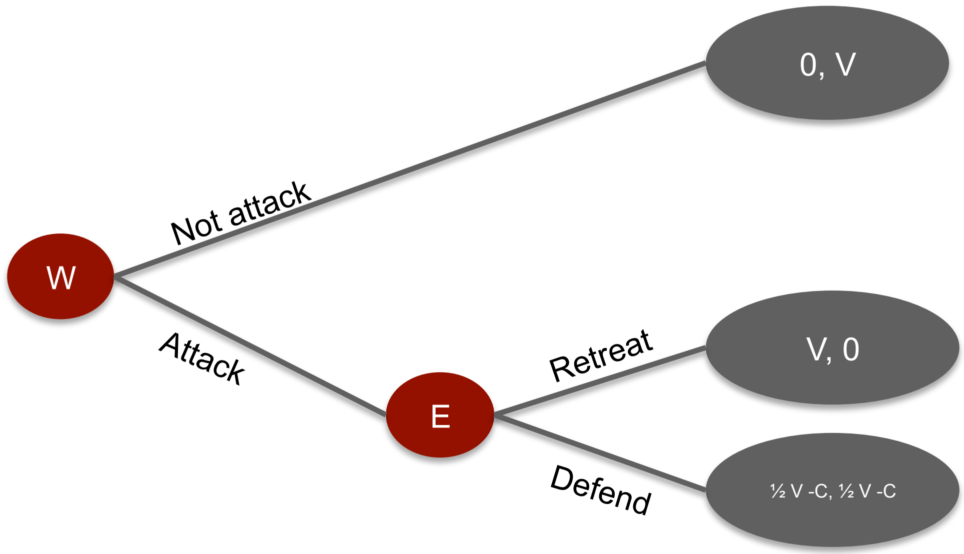


War & Peace

- Methodology
  - Represent order of moves
    = “*game tree*”
  - Procedure:
    Start analyzing last period, move backwards
    = “*backwards induction*”

War & Peace

- Game Trees (Decision tree with several “deciders”)
  - – Nodes = Decisions
  - – Branches = Actions
  - – End-nodes = Outcomes



War & Peace

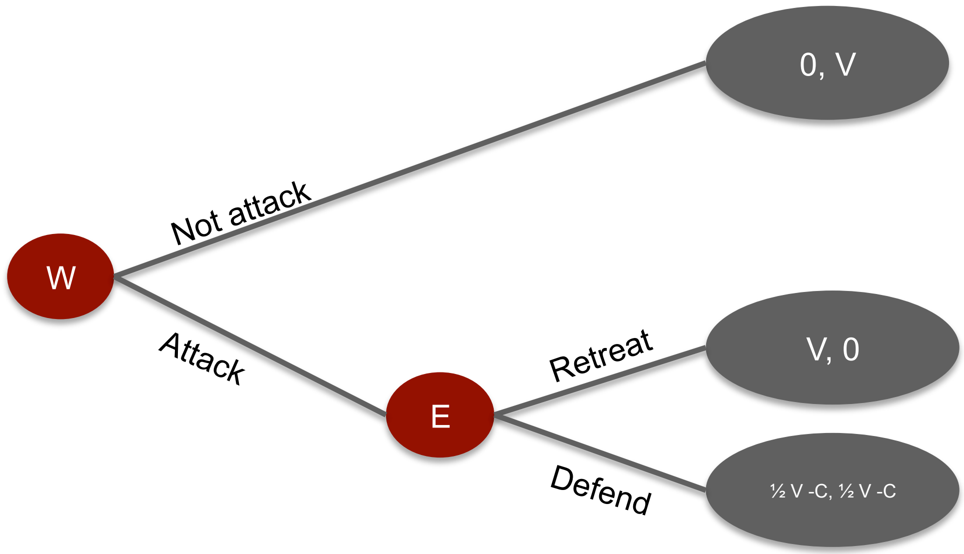
- Extensive form = “game tree”
  - Players
  - Decisions players have to take
  - Actions available at each decision
  - Order of decisions
  - Payoff to all players for all possible outcomes

War & Peace

- Normal form
  - Always possible to reduce extensive form to normal form
- How?
  - Find (Players, Strategies, Payoffs) in the tree
- Player i's strategy
  - A *complete plan of action* for player i
  - Specifies an action at every node belonging to i

War & Peace

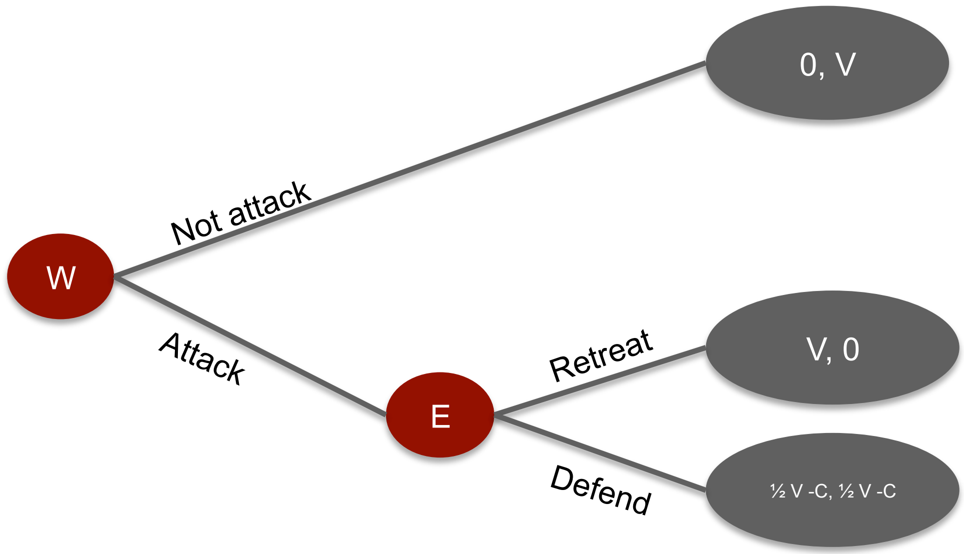
- Strategies in War & Peace
  - West: Attack, Not
  - East: Defend, Retreat



War & Peace

Q: Compute Nash equilibria

	Defend	Retreat
Attack	½ V – C, ½ V – C	V, 0
Not	0, V	0, V



War & Peace

- Two Nash equilibria
  - Attack, Retreat
  - Not attack, Defend

Same as backwards induction

Unreasonable prediction

East threatens to defend the island.
And if West believes it, it does not attack.
Then, East does not have to fight.

But if West would attack, then East would retreat.
Knowing this, West does not believe the threat.

It is a non-credible threat

War & Peace

- Conclusion for game theory analysis
  - Need extensive form and backwards induction to get rid of non-reasonable Nash equilibria (non-credible threats).
- Conclusion for Generals (and others)
  - Threats (and promises) must be *credible*

War & Peace II

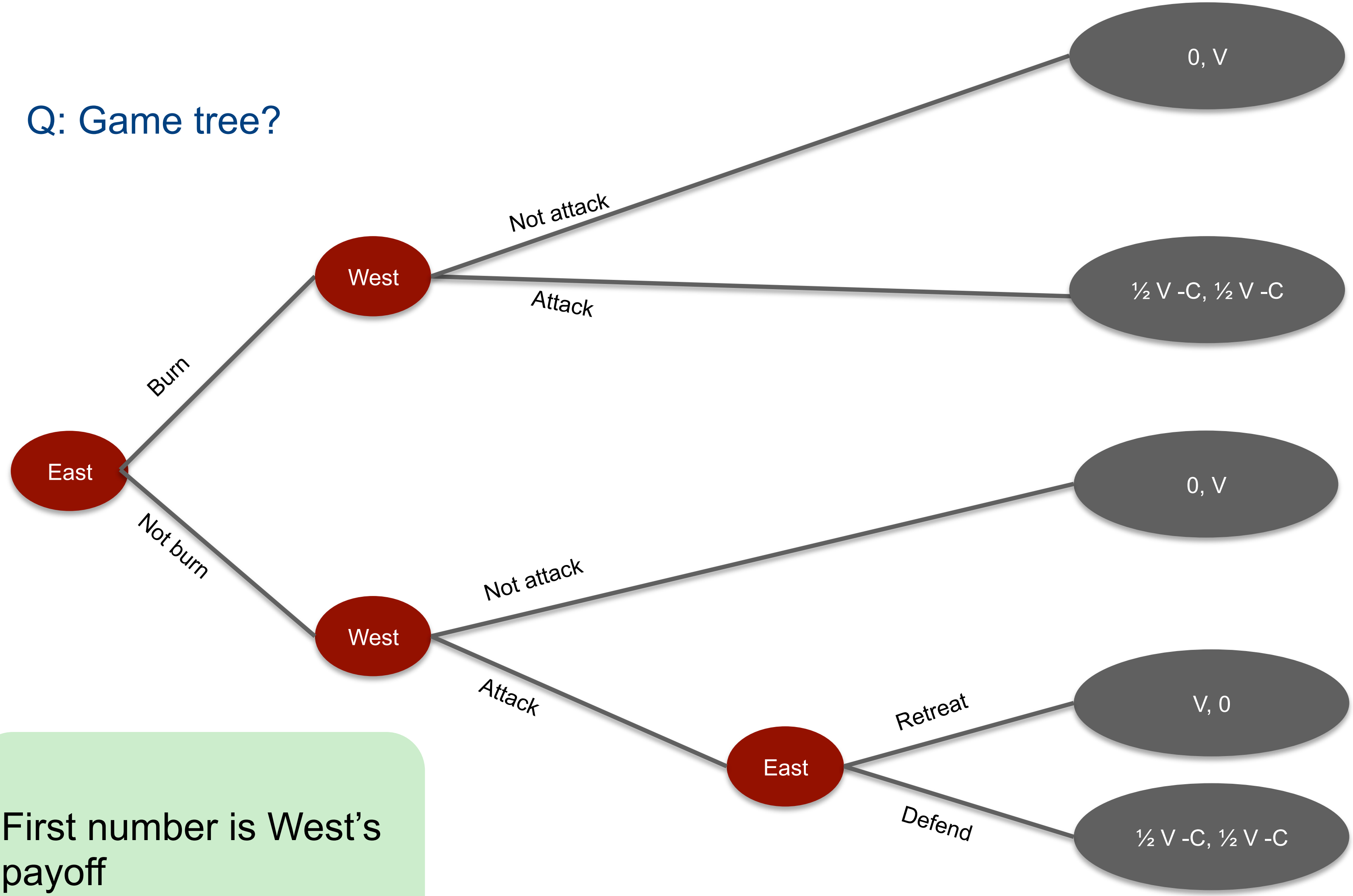
(Commitment)

War & Peace

- East reconsiders its position before West attacks
  - Gen. 1: "Burn bridge – makes retreat impossible!"
  - Gen. 2: "Then war – the worst possible outcome!"
- Q: How analyze?
  - Write up new extensive form game tree
  - Apply backwards induction

War & Peace

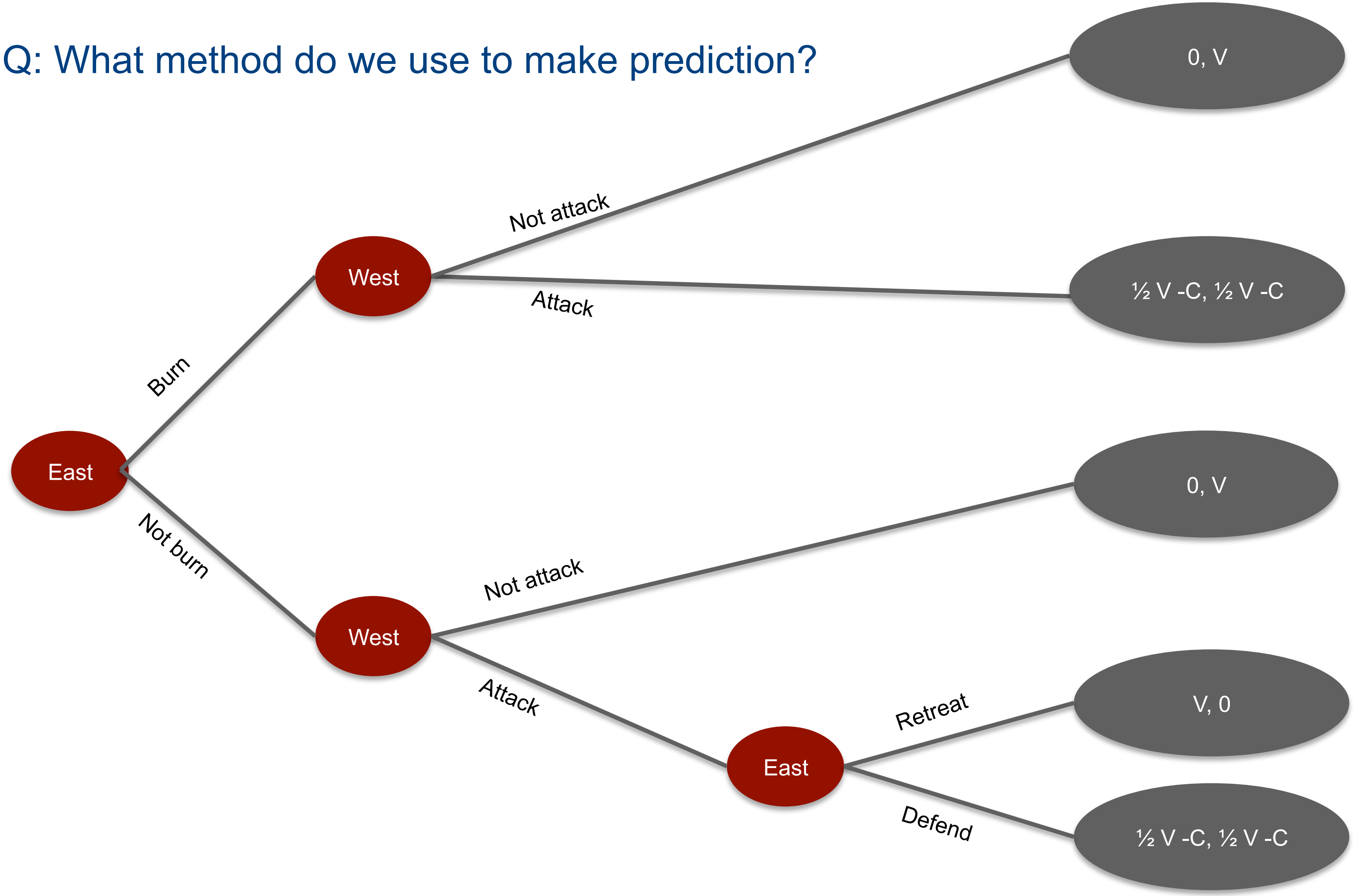
Q: Game tree?



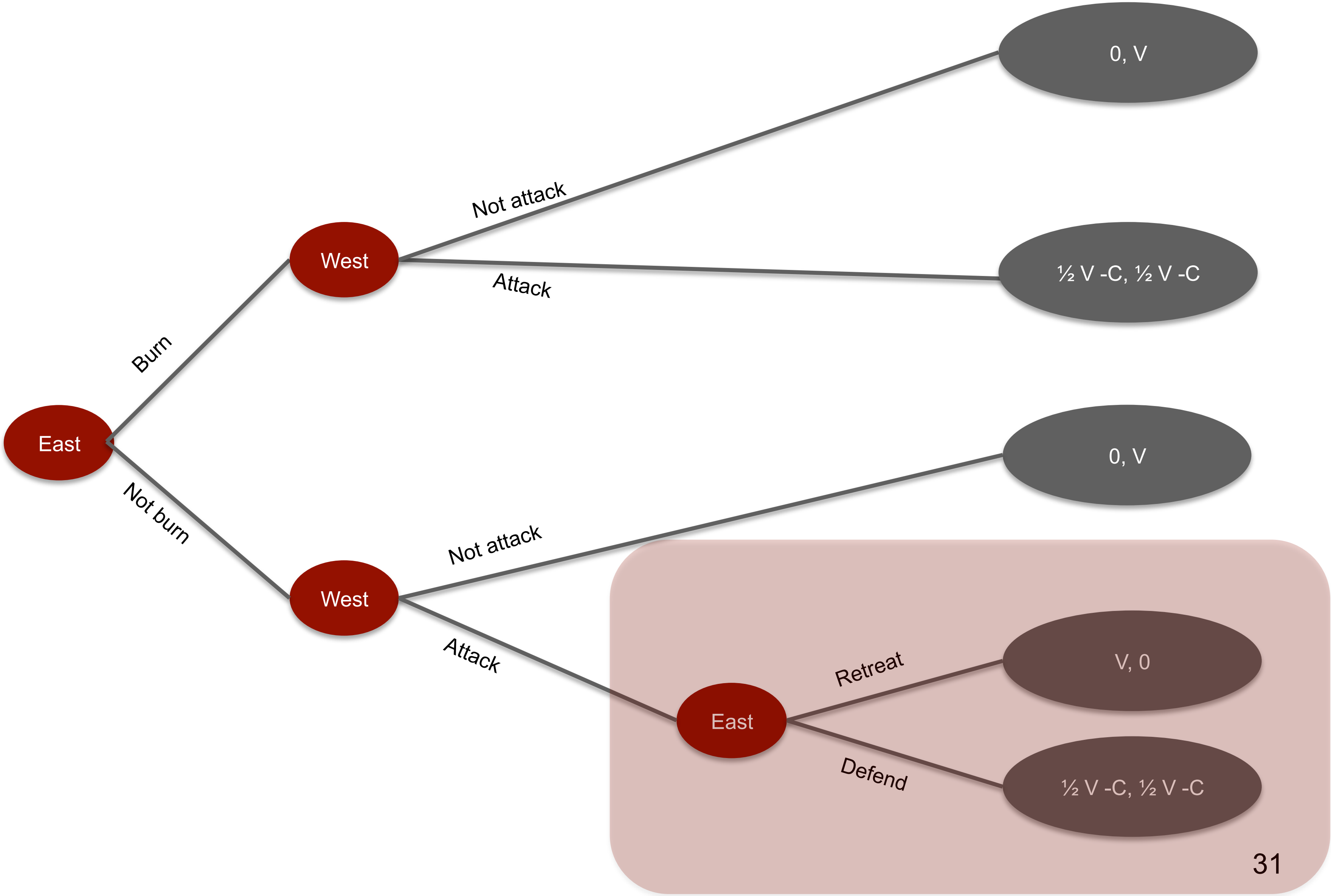
First number is West’s payoff

War & Peace

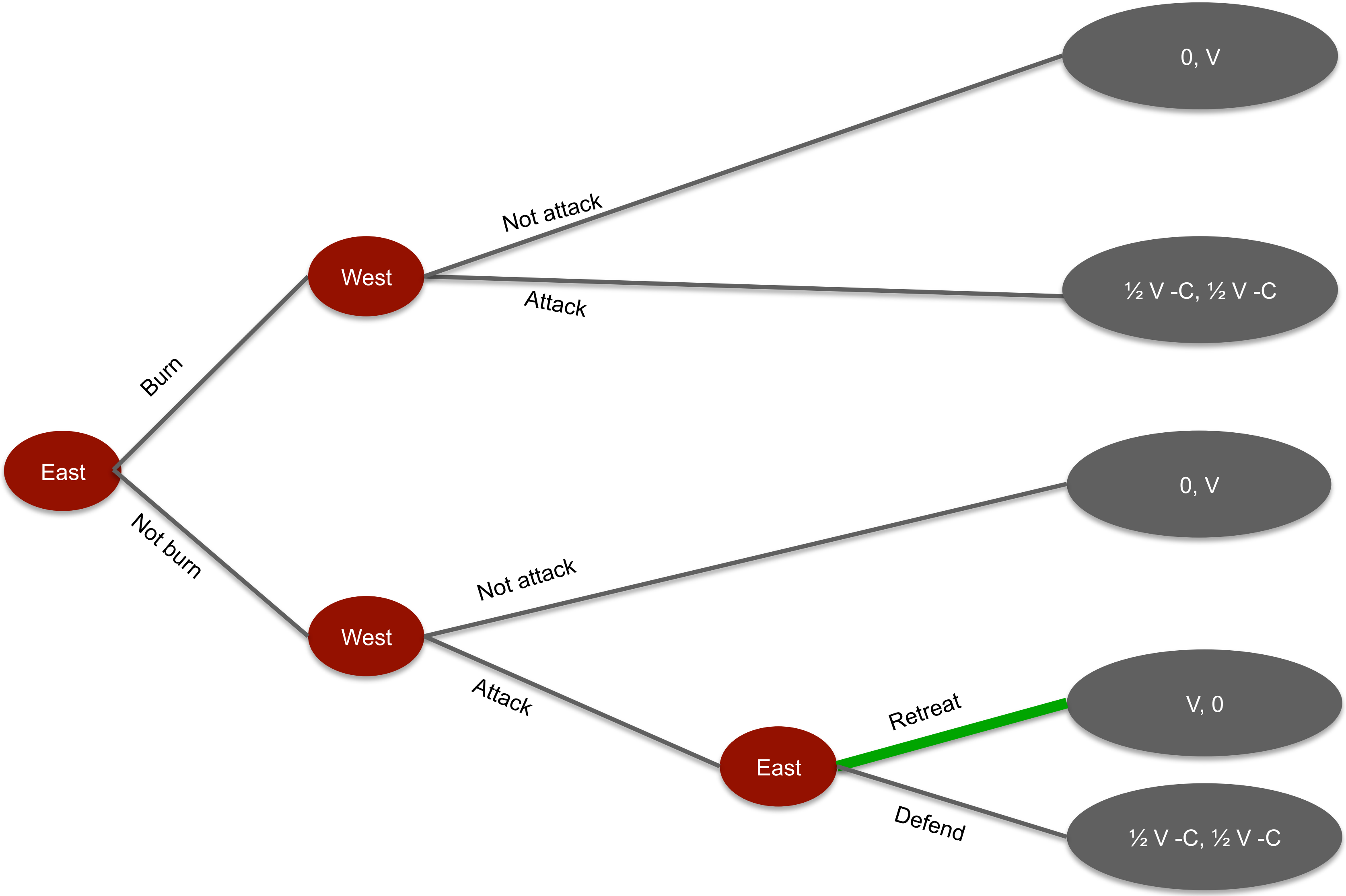
Q: What method do we use to make prediction?



War & Peace


East
Burn
Not burn
West
Not attack
Attack
0, V
½ V -C, ½ V -C
West
Not attack
Attack
0, V
East
Retreat
Defend
V, 0
½ V -C, ½ V -C

War & Peace



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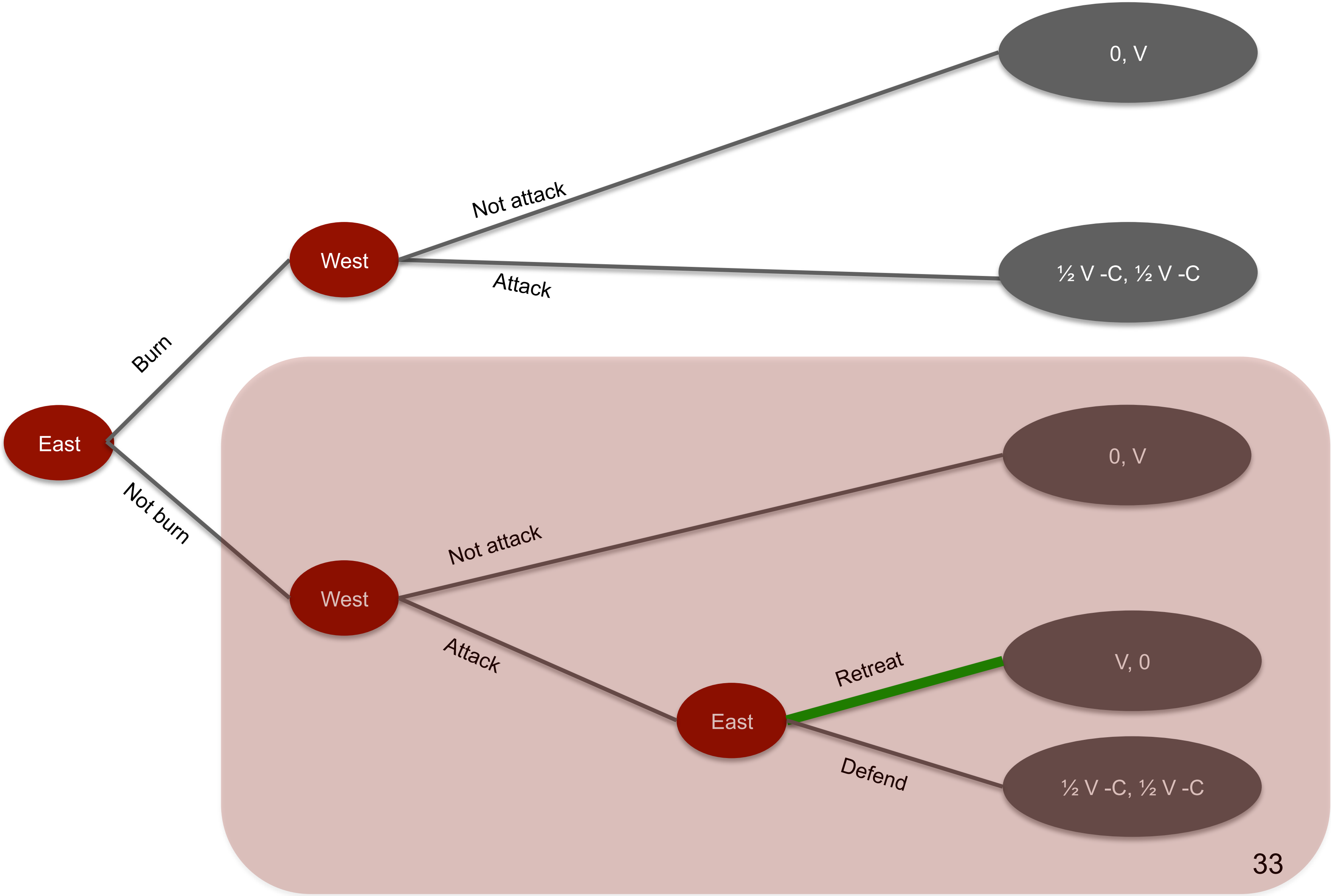
War & Peace



0, V
½ V -C, ½ V -C
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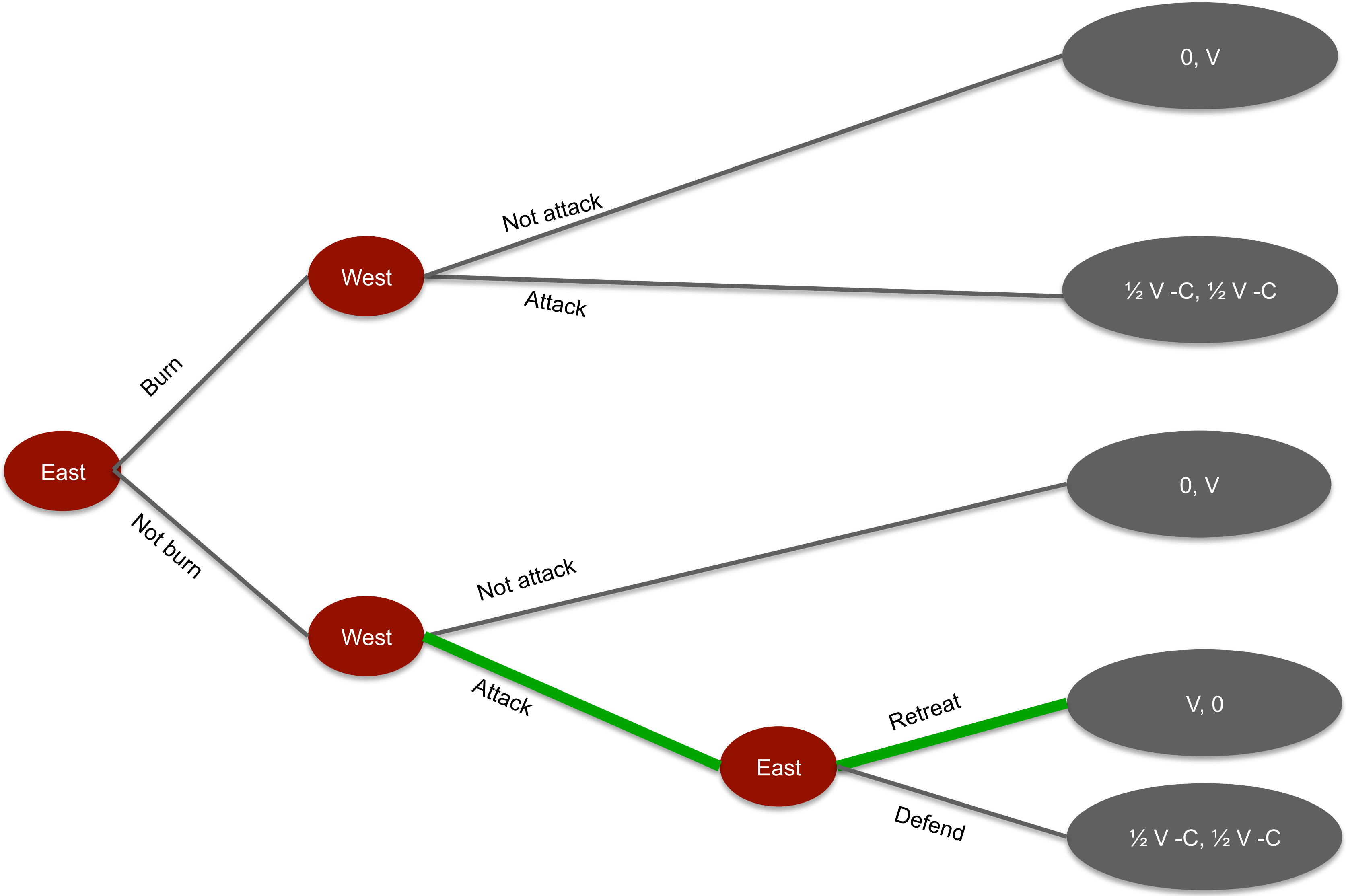
War & Peace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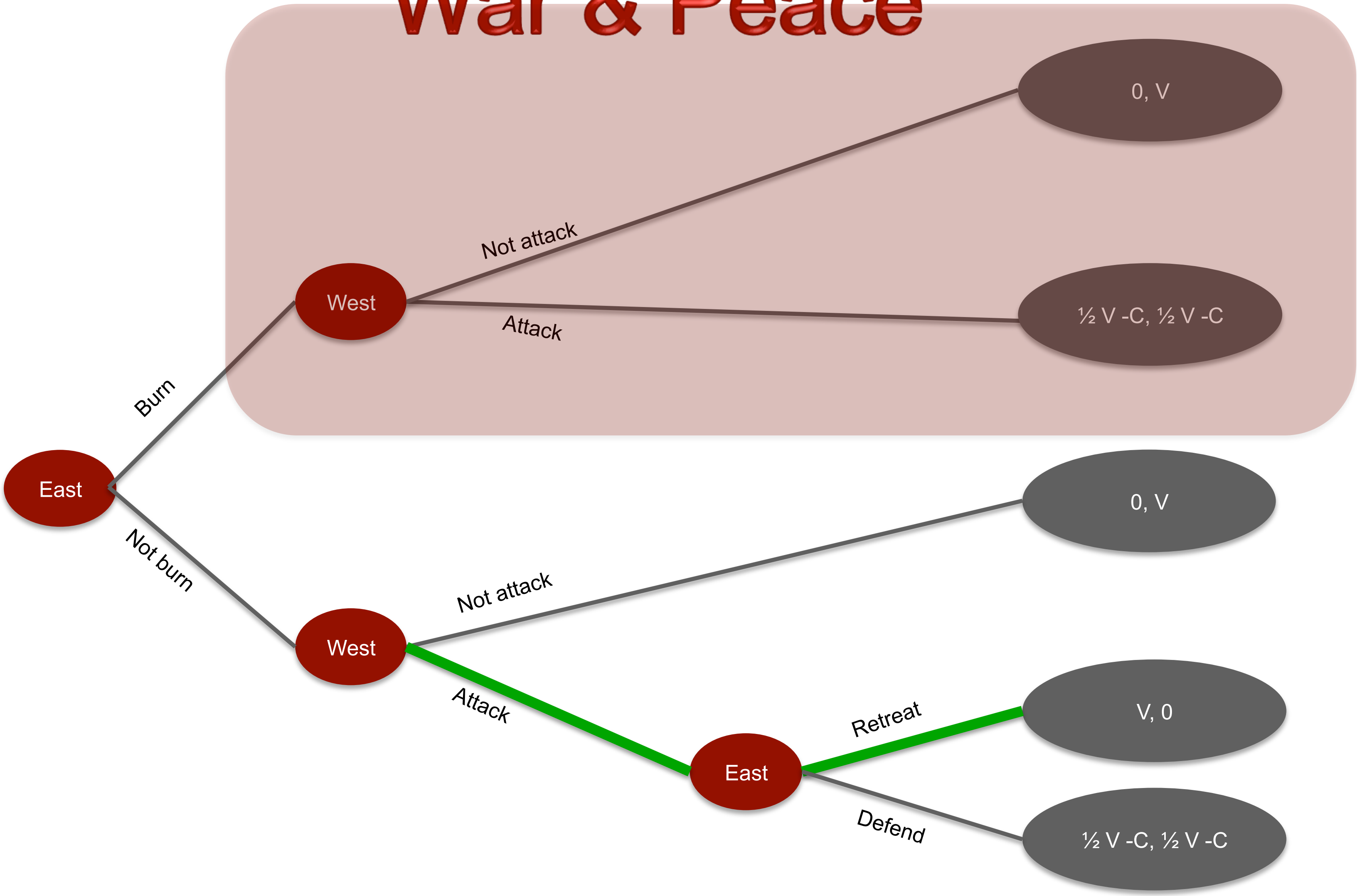
War & Peace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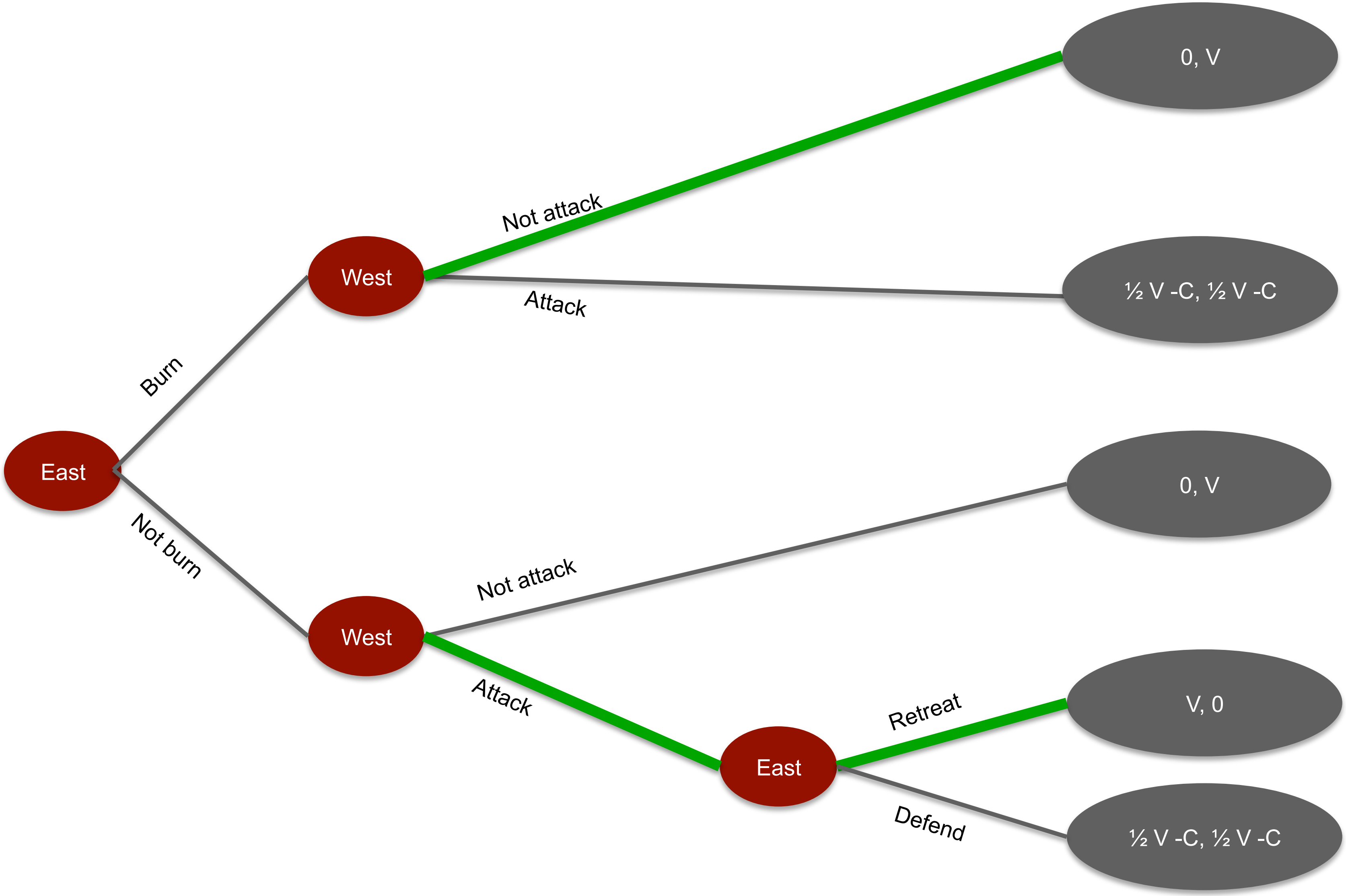
War & Peace



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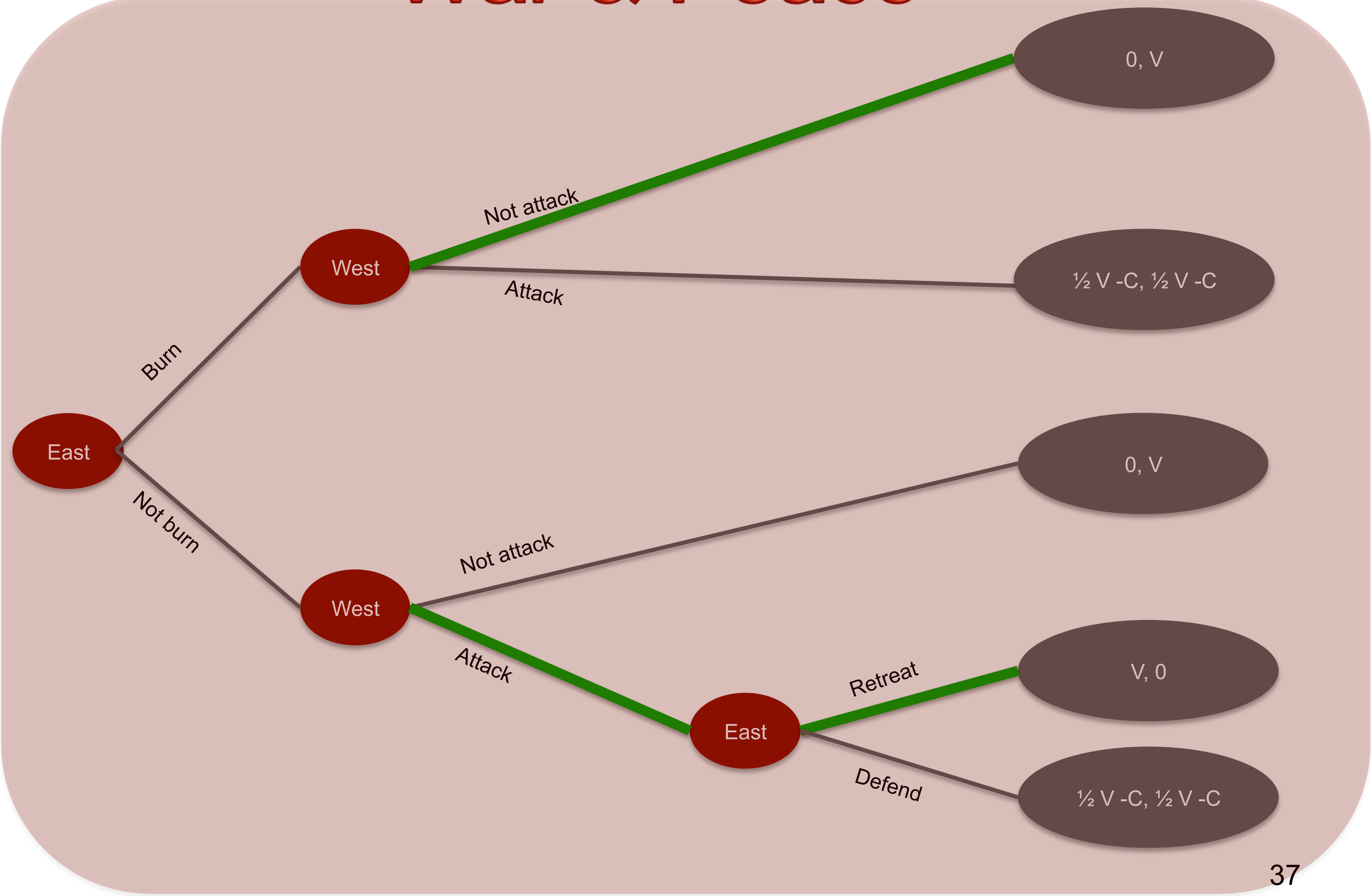
War & Peace



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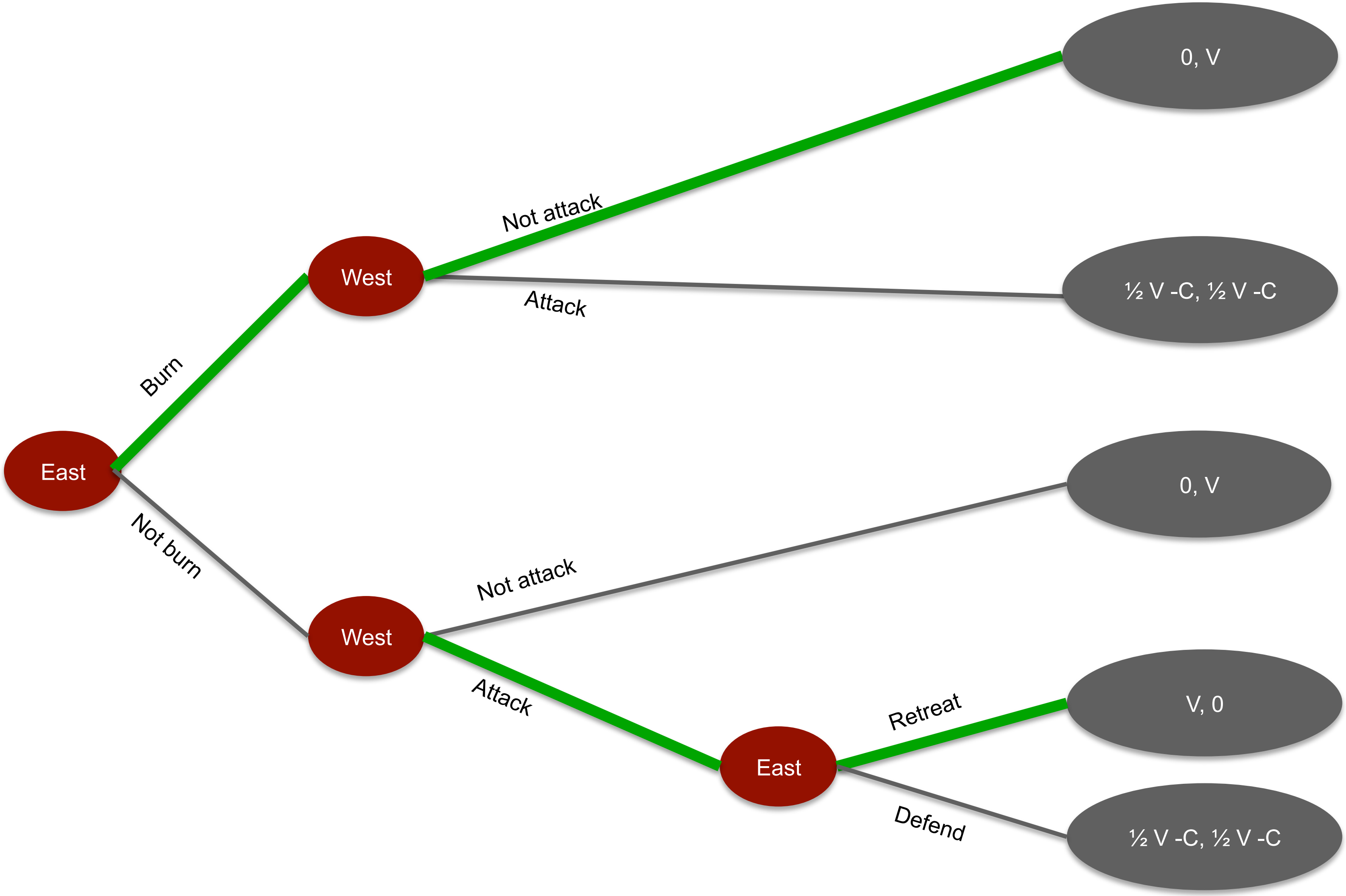
War & Peace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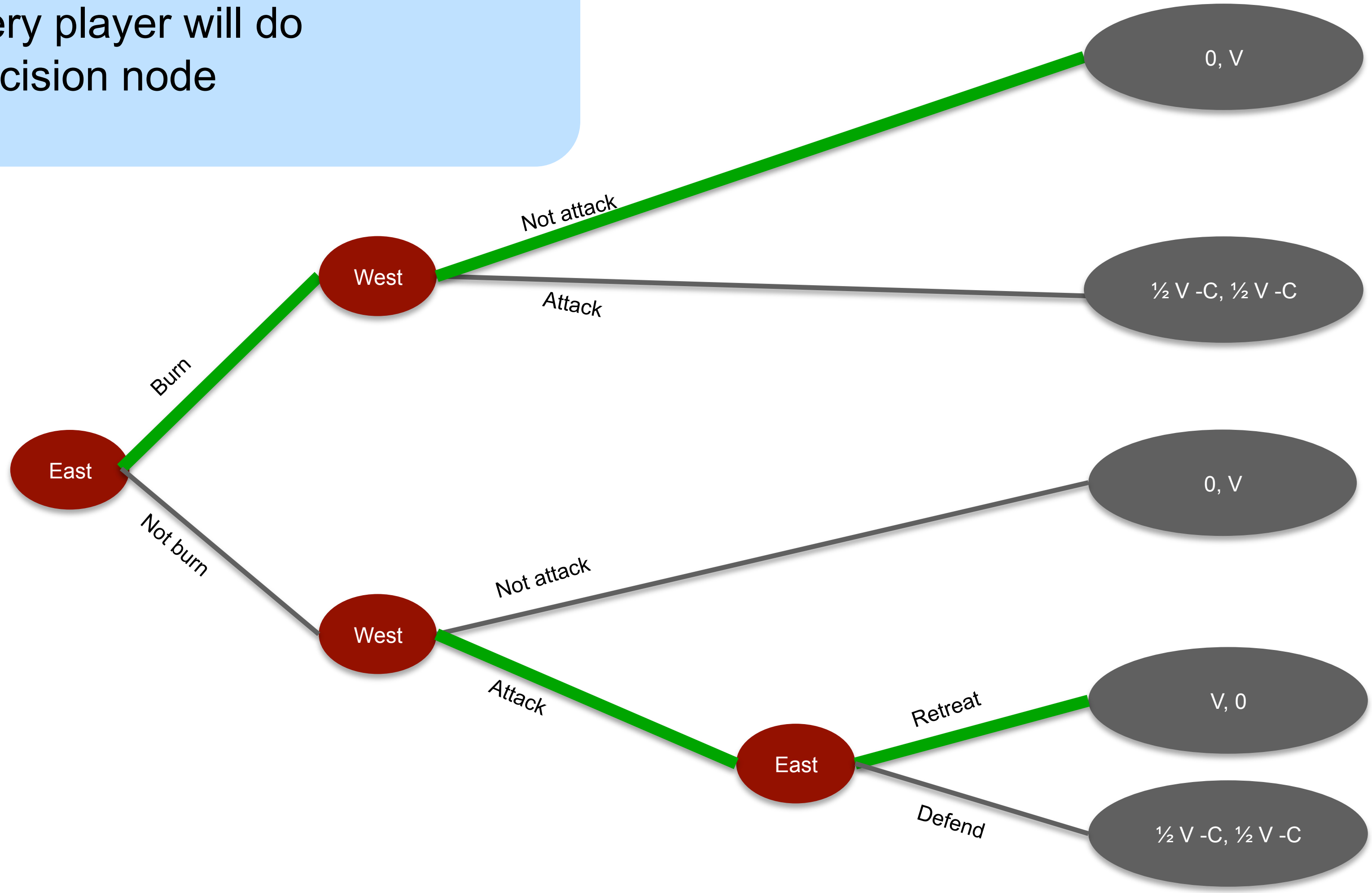
Retreat
Defend
V, 0
½ V -C, ½ V -C

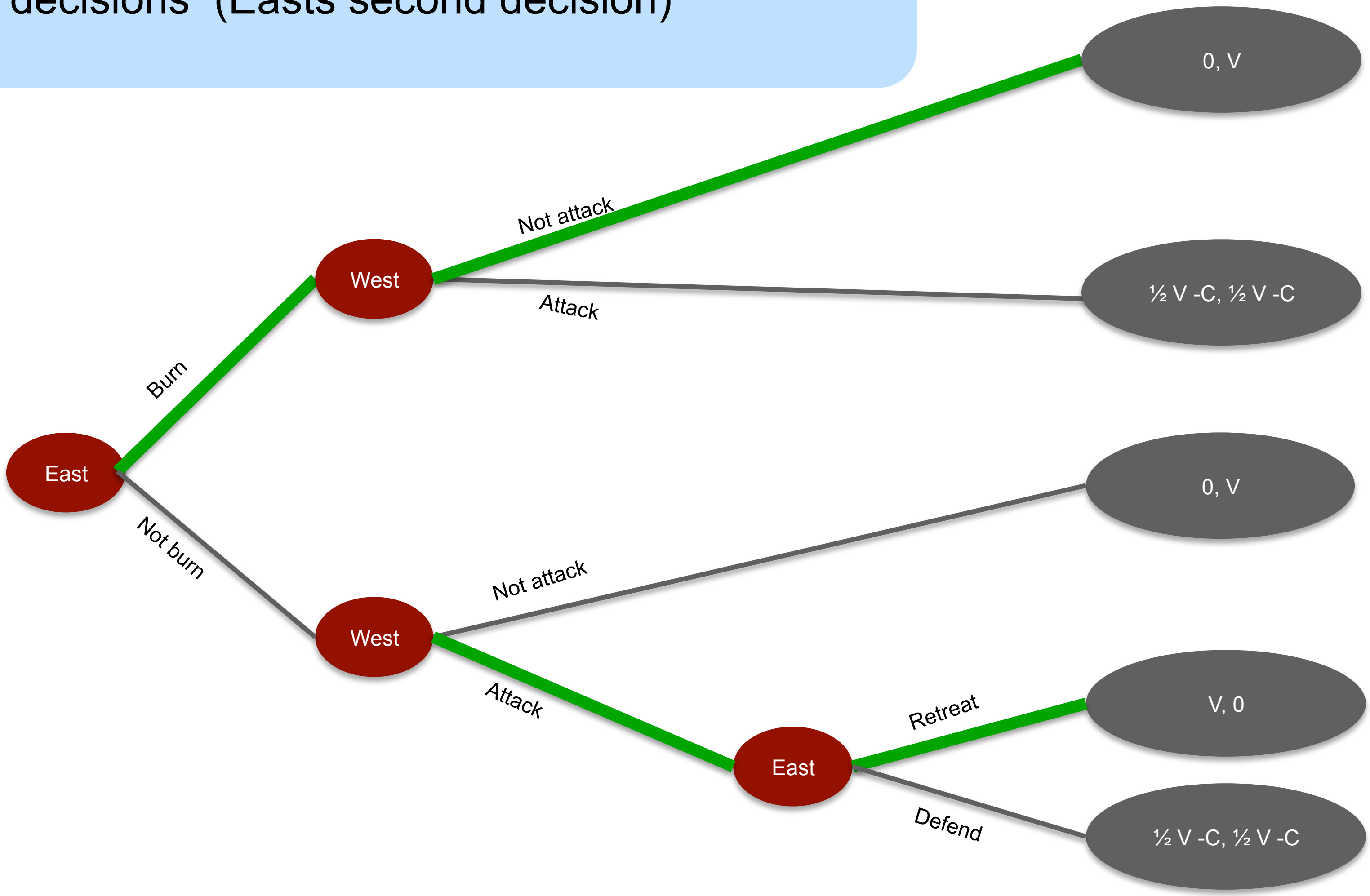
War & Peace

Equilibrium provides description
of what every player will do
at every decision node



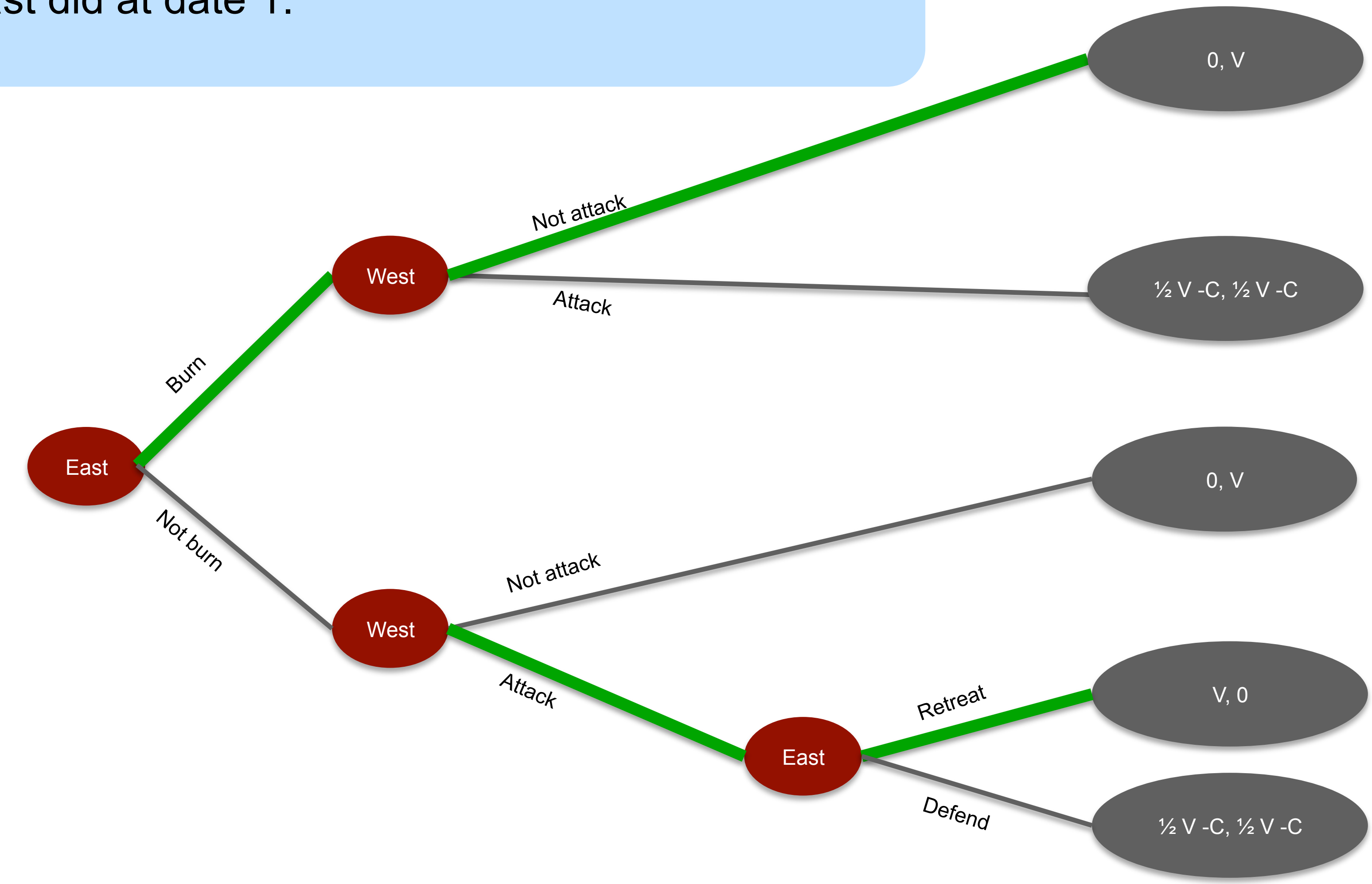
War & Peace

Also the decisions at the nodes that will never be reached are sensible decisions (Easts second decision)



War & Peace

At date 2, West makes different decisions, depending on what East did at date 1.



War & Peace

- Conclusion
  – East’s threat to defend made credible
  – Pre-commitment

War & Peace

- Two newspaper articles (in Swedish)
  - Pellnäs:
    - West needs new ***credible*** defense doctrine
    - We need to make clear to Putin when we will take the fight
  - Agrell:
    - We cannot use "game theory" to predict the behavior of countries (Russia) – they are not rational

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Bargaining Bilateral & Market Power

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Not included:

1. appendixes in lecture notes
2. Ch. 7.4

Bilateral Market Power

Example: Food Retailing

Food Retailing

- Food retailers are huge

The world's largest food retailers in 2003	
Company	**Food Sales (US$mn)**
Wal-Mart	121 566
Carrefour	77 330
Ahold	72 414
Tesco	40 907
Kroger	39 320
Rewe	36 483
Aldi	36 189
Ito-Yokado	35 812
Metro Group ITM	34 700

½ Swedish GDP

Food Retailing

- Retail markets are highly concentrated

Tabell 1a. Dagligvarukedjornas andel av den svenska marknaden			
Kedja	**Butiker (antal)**	**Butiksyta (kvm)**	**Omsättning (miljarder kr)**
Axfood	803 (24%)	625 855 (18%)	34,6 (18%)
Bergendahls	229 (7%)	328 196 (10%)	13,6 (7%)
Coop	730 (22%)	983 255 (29%)	41,4 (21%)
ICA	1 379 (41%)	1 240 602 (36%)	96,6 (50%)
Lidl	146 (4%)	170 767 (5%)	5,2 (3%)
Netto	105 (3%)	70 603 (2%)	3,0 (2%)

Food Retailing

- Food manufacturers
  - Some are huge:
    - Kraft Food, Nestle, Scan
    - Annual sales tenth of billions of Euros
  - Some are tiny:
    - local cheese

Food Retailing

- Mutual dependence
  - Some brands = Must have
    - ICA “must” sell Coke
    - Otherwise many families would shop at Coop
  - Some retailers = Must channel
    - Coke “must” sell via ICA to be active in Sweden
    - Probably large share of Coke’s sales in Sweden
  - Both would lose if ICA would not sell Coke

Food Retailing

- Mutual dependence
  - Manufacturers cannot dictate wholesale prices
  - Retailers cannot dictate wholesale prices
- Thus
  - They have to negotiate and agree
- In particular
  - Also retailers have market power
    = buyer power

Food Retailing

- Large retailers pay lower prices (= more buyer power)

Retailer	Market Share (CC Table 5:3, p. 44)	Price (CC Table 5, p. 435)
Tesco	24.6	100.0
Sainsbury	20.7	101.6
Asda	13.4	102.3
Somerfield	8.5	103.0
Safeway	12.5	103.1
Morrison	4.3	104.6
Iceland	0.1	105.3
Waitrose	3.3	109.4
Booth	0.1	109.5
Netto	0.5	110.1
Budgens	0.4	111.1

Other examples

- Labor markets
  - Vårdförbundet vs Landsting
- Relation-specific investments
  - Car manufacturers vs producers of parts

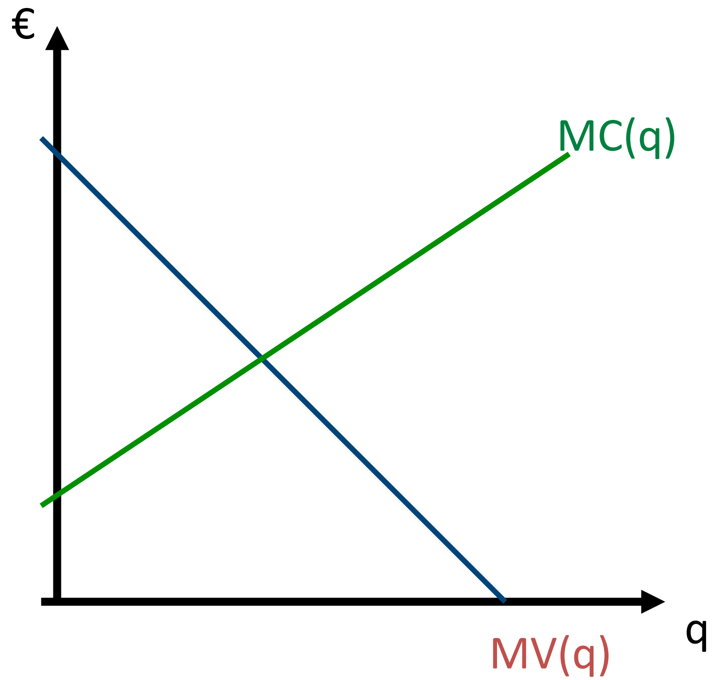
Food Retailing

- Questions
  - How analyze bargaining in intermediate goods markets?
  - Why do large buyers get better prices?

Bilateral Monopoly

Bilateral Monopoly

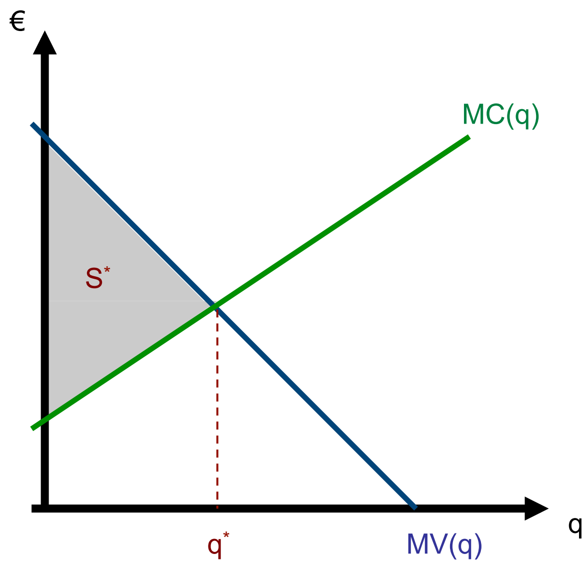
- Exogenous conditions
  - One Seller: MC(q)
    = inverse supply if price taker
  - One Buyer: MV(q)
    = inverse demand if price taker



Bilateral Monopoly

Intuitive Analysis

- Efficient quantity
  - Complete information
  - Maximize the surplus to be shared



Bilateral Monopoly
Intuitive Analysis

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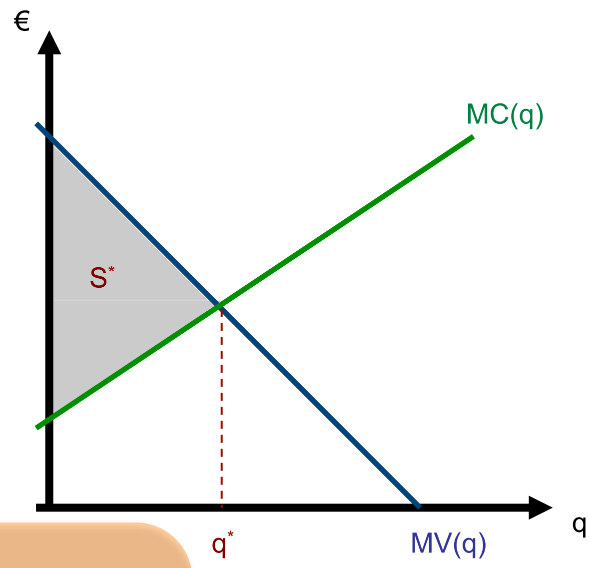


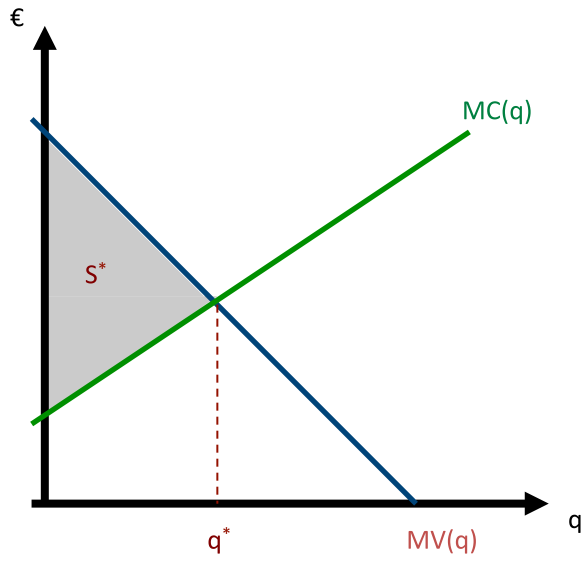
Efficiency from the point of view of the two firms = Same quantity as a vertically integrated firm would choose

Bilateral Monopoly

Intuitive Analysis

- Problem
  - But what price?

- Only restrictions
  - Seller must cover his costs, $C(q^*)$
  - Buyer must not pay more than wtp, $V(q^*)$



=> Any split of $S^* = V(q^*) - C(q^*)$ seems reasonable

Bilateral Monopoly

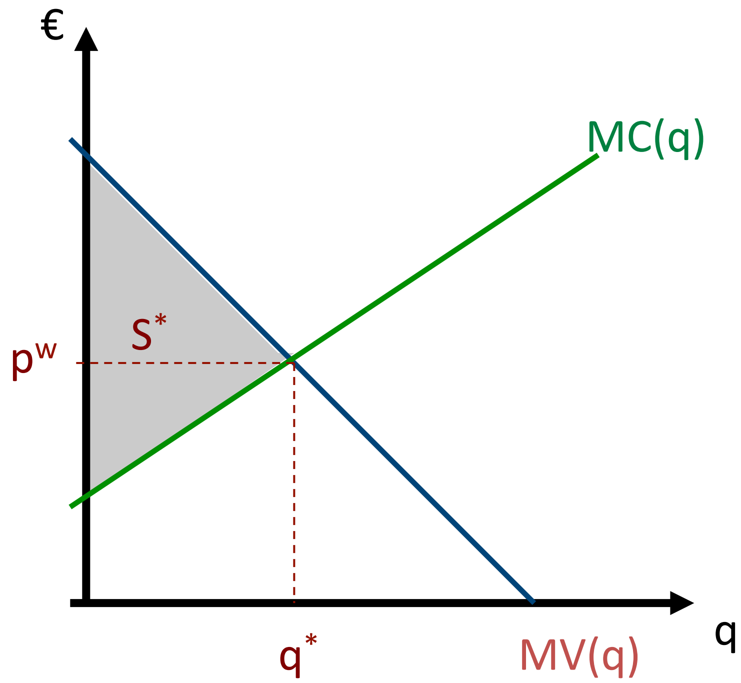
Intuitive Analysis

- Note
  - If someone demands “too much”
  - The other side will reject and make a counter-offer
- Problem
  - Haggling could go on forever
  - Gains from trade delayed
- Thus
  - Both sides have incentive to be reasonable
  - But, the party with less aversion to delay has strategic advantage

Bilateral Monopoly

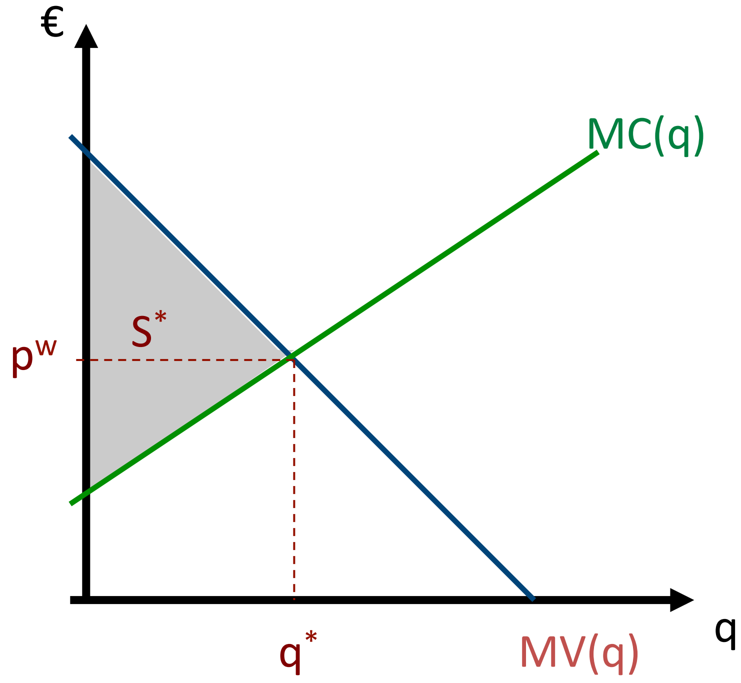
Definitions

- Definitions
  - Efficient quantity: q^*
  - Walrasian price: p^w
  - Maximum bilateral surplus: S^*



Bilateral Monopoly

- First important insight:
  - Contract must specify both price and quantity, (p, q)
  - Q: Why?
- Otherwise inefficient quantity
  - If $p > p^w$ then $q < q^*$
  - If $p < p^w$ then $q < q^*$
  - Short side of the market decides



Extensive Form Bargaining

Ultimatum bargaining

Ultimatum bargaining

Solve this game now!

- One round of negotiations
  - One party, say seller, gets to propose a contract (p, q)
  - Other party, say buyer, can accept or reject
- Outcome
  - If (p, q) accepted, it is implemented
  - Otherwise game ends without agreement
- Payoffs
  - Buyer: $V(q) - p\,q$ if agreement, zero otherwise
  - Seller: $p\,q - C(q)$ if agreement, zero otherwise
- Perfect information
  - Backwards induction

Ultimatum bargaining

- Time 2: Buyer accepts or rejects proposed contract
  - Q: What would make buyer accept (p, q)?
  - Buyer accepts (p, q) iff $V(q) - p\,q \geq 0$

- Time 1: Seller proposes best contract that would be accepted
  - Q: How do we find the seller's best contract?
  - $\max_{p,q}\ p\,q - C(q)$ such that $V(q) - p\,q \geq 0$

Ultimatum bargaining

Seller's maximization problem

$$\max_{p,q} \quad p \cdot q - C(q)$$

$$st: \quad V(q) - p \cdot q \geq 0$$

Optimal price

Increase price until: $p \cdot q = V(q)$

Seller takes whole surplus

Optimal quantity

$$\max_q V(q) - C(q)$$

Must set q such that: $MV(q) = MC(q)$

Efficient quantity

Ultimatum bargaining

- SPE of ultimatum bargaining game
  - Unique equilibrium
  - There is agreement
  - Efficient quantity
  - Proposer takes the whole (maximal) surplus

Ultimatum bargaining

- Assume rest of lecture
  - Always efficient quantity
  - Surplus = 1
  - Player S gets share π_S
  - Player B gets share $\pi_B = 1 - \pi_S$
- Ultimatum game
  - $\pi_S = 1$
  - $\pi_B = 0$

Two rounds (T=2)

Two rounds (T=2)

- Alternating offers
  - Period 1
    - B proposes contract
    - S accepts or rejects
  - Period 2 (in case S rejected)
    - S proposes contract
    - B accepts or rejects
- Perfect information
  - No simultaneous moves
  - Players know what has happened before in the game
- Solution concept
  - Backwards induction (Subgame perfect equilibrium)

Two rounds (T=2)

- Player B is impatient
  - €1 in period 2 is equally good as €δ_B in period 1
  - Where $\delta_B < 1$ is B's discount factor
- Player S is impatient
  - €1 in period 2 is equally good as €δ_S in period 1
  - Where $\delta_S < 1$ is S's discount factor

Two rounds (T=2)

Solve this game now!

- Period 1
  - B proposes $\left(\pi_B^{T-1}, \pi_S^{T-1}\right)$
  - S accepts or rejects
- Period 2 (in case S rejected)
  - S proposes $\left(\pi_B^{T}, \pi_S^{T}\right)$
  - B accepts or rejects
- Perfect information => Use BI

Two rounds

- Period T = 2 (S bids) (What will happen in case S rejected?)
  - B accepts iff: $\pi_B^T \geq 0$
  - S proposes: $\pi_B^T = 0$ $\pi_S^T = 1$
- Period T-1 = 1 (B bids)
  - S accepts iff: $\pi_S^{T-1} \geq \delta_S \pi_S^T$ $= \delta_S < 1$
  - B proposes: $\pi_B^{T-1} = 1 - \delta_S > 0$ $\pi_S^{T-1} = \delta_S$
- Note
  - S willing to reduce his share to get an early agreement
  - Both players get part of surplus
  - B's share determined by S's impatience. If S very patient $\pi_S \approx 1$

T rounds

T rounds

- Model
  - Large number of periods, T
  - Buyer and seller take turns to make offer
  - Common discount factor $\delta = \delta_B = \delta_S$
  - Subgame perfect equilibrium (ie start analysis in last period)

T rounds

Time	Bidder	π_B	π_S	Resp.
T	S	?	?	?

T rounds

Time	Bidder	π_B	π_S	Resp.
T	S	0	1	yes

T rounds

Time	Bidder	π_B	π_S	Resp.
T	S	0	1	yes
T-1	B	?	?	?

T rounds

Time	Bidder	π_B	π_S	Resp.
T	S	0	1	yes
T-1	B	rest	δ	yes

T rounds

Time	Bidder	π_B	π_S	Resp.
T	S	0	1	yes
T-1	B	1-δ	δ	yes

T rounds

Time	Bidder	π_B	π_S	Resp.
T	S	0	1	yes
T-1	B	1-δ	δ	yes
T-2	S	?	?	?

T rounds

Time	Bidder	π_B	π_S	Resp.
T	S	0	1	yes
T-1	B	1-δ	δ	yes
T-2	S	δ(1-δ)	rest	yes

T rounds

Time	Bidder	π_B	π_S	Resp.
T	S	0	1	yes
T-1	B	$1-\delta$	δ	yes
T-2	S	$\delta(1-\delta)$	$1-\delta(1-\delta)$	yes

T rounds

Time	Bidder	π_B	π_S	Resp.
T	S	0	1	yes
T-1	B	$1-\delta$	δ	yes
T-2	S	$\delta(1-\delta)$	$1-\delta(1-\delta)$	yes

multiply

T rounds

Time	Bidder	π_B	π_S	Resp.
T	S	0	1	yes
T-1	B	$1-\delta$	δ	yes
T-2	S	$\delta-\delta^2$	$1-\delta+\delta^2$	yes

T rounds

Time	Bidder	π_B	π_S	Resp.
T	S	0	1	yes
T-1	B	$1-\delta$	δ	yes
T-2	S	$\delta-\delta^2$	$1-\delta+\delta^2$	yes
T-3	B	?	?	?

T rounds

Time	Bidder	π_B	π_S	Resp.
T	S	0	1	yes
T-1	B	$1-\delta$	δ	yes
T-2	S	$\delta-\delta^2$	$1-\delta+\delta^2$	yes
T-3	B	rest	$\delta(1-\delta+\delta^2)$	yes

T rounds

Time	Bidder	π_B	π_S	Resp.
T	S	0	1	yes
T-1	B	$1-\delta$	δ	yes
T-2	S	$\delta-\delta^2$	$1-\delta+\delta^2$	yes
T-3	B	$1-\delta(1-\delta+\delta^2)$	$\delta(1-\delta+\delta^2)$	yes

T rounds

Time	Bidder	π_B	π_S	Resp.
T	S	0	1	yes
T-1	B	$1-\delta$	δ	yes
T-2	S	$\delta-\delta^2$	$1-\delta+\delta^2$	yes
T-3	B	$1-\delta+\delta^2-\delta^3$	$\delta-\delta^2+\delta^3$	yes

T rounds

Time	Bidder	π_B	π_S	Resp.
T	S	0	1	yes
T-1	B	$1-\delta$	δ	yes
T-2	S	$\delta-\delta^2$	$1-\delta+\delta^2$	yes
T-3	B	$1-\delta+\delta^2-\delta^3$	$\delta-\delta^2+\delta^3$	yes
T-4	S	$\delta(1-\delta+\delta^2-\delta^3)$	rest	yes

T rounds

Time	Bidder	π_B	π_S	Resp.
T	S	0	1	yes
T-1	B	$1-\delta$	δ	yes
T-2	S	$\delta-\delta^2$	$1-\delta+\delta^2$	yes
T-3	B	$1-\delta+\delta^2-\delta^3$	$\delta-\delta^2+\delta^3$	yes
T-4	S	$\delta(1-\delta+\delta^2-\delta^3)$	$1-\delta(1-\delta+\delta^2-\delta^3)$	yes

T rounds

Time	Bidder	π_B	π_S	Resp.
T	S	0	1	yes
T-1	B	$1-\delta$	δ	yes
T-2	S	$\delta-\delta^2$	$1-\delta+\delta^2$	yes
T-3	B	$1-\delta+\delta^2-\delta^3$	$\delta-\delta^2+\delta^3$	yes
T-4	S	$\delta-\delta^2+\delta^3-\delta^4$	$1-\delta+\delta^2-\delta^3+\delta^4$	yes

T rounds

Time	Bidder	π_B	π_S	Resp.
T	S	0	1	yes
T-1	B	$1-\delta$	δ	yes
T-2	S	$\delta-\delta^2$	$1-\delta+\delta^2$	yes
T-3	B	$1-\delta+\delta^2-\delta^3$	$\delta-\delta^2+\delta^3$	yes
T-4	S	$\delta-\delta^2+\delta^3-\delta^4$	$1-\delta+\delta^2-\delta^3+\delta^4$	yes
…	…	…	…	…
1	S	$\delta-\delta^2+\delta^3-\delta^4+\ldots-\delta^{T-1}$	$1-\delta+\delta^2-\delta^3+\delta^4-\ldots+\delta^{T-1}$	yes

T rounds

Time	Bidder	π_B	π_S	Resp.
T	S	0	1	yes
T-1	B	$1-\delta$	δ	yes
T-2	S	$\delta-\delta^2$	$1-\delta+\delta^2$	yes
T-3	B	$1-\delta+\delta^2-\delta^3$	$\delta-\delta^2+\delta^3$	yes
T-4	S	$\delta-\delta^2+\delta^3-\delta^4$	$1-\delta+\delta^2-\delta^3+\delta^4$	yes
…	…	…	…	…
1	S	$\delta-\delta^2+\delta^3-\delta^4+\ldots-\delta^{T-1}$	$1-\delta+\delta^2-\delta^3+\delta^4-\ldots+\delta^{T-1}$	yes

$$\pi_B = \delta - \delta^2 + \delta^3 - \delta^4 + \ldots - \delta^{T-1}$$

$$\pi_S = 1 - \delta + \delta^2 - \delta^3 + \delta^4 - \ldots + \delta^{T-1}$$

T rounds

Geometric series

$$\pi_B = \delta - \delta^2 + \delta^3 - \delta^4 + ... - \delta^{T-1}$$

$$\pi_S = 1 - \delta + \delta^2 - \delta^3 + \delta^4 - ... + \delta^{T-1}$$

T rounds

S's share

$$\pi_S = 1 - \delta + \delta^2 - \delta^3 + \delta^4 - ... + \delta^{T-1}$$

T rounds

S's share

$$\pi_S = 1 - \delta + \delta^2 - \delta^3 + \delta^4 - ... + \delta^{T-1}$$

Multiply

$$\delta\pi_S = \delta - \delta^2 + \delta^3 - \delta^4 + \delta^5 - ... + \delta^T$$

T rounds

S's share

$$\pi_S = 1 - \delta + \delta^2 - \delta^3 + \delta^4 - \ldots + \delta^{T-1}$$

Multiply

$$\delta\pi_S = \delta - \delta^2 + \delta^3 - \delta^4 + \delta^5 - \ldots + \delta^T$$

Add

$$\pi_S + \delta\pi_S = 1 + \delta^T$$

T rounds

S's share

$$\pi_S = 1 - \delta + \delta^2 - \delta^3 + \delta^4 - ... + \delta^{T-1}$$

Multiply

$$\delta\pi_S = \delta - \delta^2 + \delta^3 - \delta^4 + \delta^5 - ... + \delta^T$$

Add

$$\pi_S + \delta\pi_S = 1 + \delta^T$$

Solve

$$\pi_S = \frac{1+\delta^T}{1+\delta}$$

T rounds

Equilibrium shares with T periods

$$\pi_S = \frac{1}{1+\delta}\left(1+\delta^T\right)$$

$$\pi_B = \frac{\delta}{1+\delta}\left(1-\delta^{T-1}\right)$$

T rounds

Equilibrium shares with T periods

$$\pi_S = \frac{1}{1+\delta}\left(1+\delta^T\right)$$

$$\pi_B = \frac{\delta}{1+\delta}\left(1-\delta^{T-1}\right)$$

S has advantage of making last bid

$$1+\delta^T > 1-\delta^{T-1}$$

To confirm this, solve model where

- B makes last bid
- S makes first bid

T rounds

Equilibrium shares with T periods

$$\pi_S = \frac{1}{1+\delta}\left(1+\delta^T\right)$$

$$\pi_B = \frac{\delta}{1+\delta}\left(1-\delta^{T-1}\right)$$

S has advantage of making last bid

$1+\delta^T > 1-\delta^{T-1}$ ← Disappears if T very large

T rounds

Equilibrium shares with T ≈ ∞ periods

$$\pi_S = \frac{1}{1+\delta}$$

$$\pi_B = \frac{\delta}{1+\delta}$$

T rounds

Equilibrium shares with T ≈ ∞ periods

$$\pi_S = \frac{1}{1+\delta}$$

$$\pi_B = \frac{\delta}{1+\delta}$$

S has advantage of making first bid

$$\frac{1}{1+\delta} > \frac{\delta}{1+\delta}$$

To confirm this, solve model where
- B makes first bid

T rounds

Equilibrium shares with T ≈ ∞ periods

$$\pi_S = \frac{1}{1+\delta}$$

$$\pi_B = \frac{\delta}{1+\delta}$$

S has advantage of making first bid

$$\frac{1}{1+\delta} > \frac{\delta}{1+\delta}$$

← First bidder's advantage disappears if δ ≈ 1

T rounds

Equilibrium shares with T $\approx \infty$ periods and very patient players ($\delta \approx 1$)

$$\pi_S = \frac{1}{2}$$

$$\pi_B = \frac{1}{2}$$

Difference in Patience

Equilibrium shares with T ≈ ∞ periods and different discount factors

$$\pi_S = \frac{1-\delta_B}{1-\delta_S\delta_B}$$

$$\pi_B = \frac{1-\delta_S}{1-\delta_S\delta_B}\delta_B$$

(Easy to show using same method as above)

Difference in Patience

- Recall $\delta_i = e^{-r_i \Delta}$
  - r_i = continous-time discount factor
  - Δ = length of time period
- Then, as Δ ➔ 0:
  - $$\pi_S = \frac{1-\delta_B}{1-\delta_S \delta_B} \approx \frac{r_B}{r_S + r_B}$$
  - Using l'Hopital's rule

Conclusions

- Exists unique equilibrium (SPE)
- There is agreement
- Agreement is immediate
- Efficient agreement (here: quantity)
- Split of surplus (price) determined by:
  - Relative patience
  - Right to make last bid gives advantage (if $T < \infty$)
  - Right to make first bid gives advantage (if $\delta < 1$)

Implications for Bilateral Monopoly

Implications for Bilateral Monopoly

- Equal splitting

$$\Pi_S = \Pi_B$$

$$p \cdot q - C(q) = V(q) - p \cdot q$$

$$2 \cdot p \cdot q = V(q) + C(q)$$

$$p = \frac{1}{2}\left[\frac{V(q)}{q} + \frac{C(q)}{q}\right]$$

Implications for Bilateral Monopoly

- Equal splitting

$$\Pi_S = \Pi_B$$

$$p \cdot q - C(q) = V(q) - p \cdot q$$

$$2 \cdot p \cdot q = V(q) + C(q)$$

$$p = \frac{1}{2}\left[\frac{V(q)}{q} + \frac{C(q)}{q}\right]$$

Retailer's
average revenues

Implications for Bilateral Monopoly

- Equal splitting

$$\Pi_S = \Pi_B$$

$$p \cdot q - C(q) = V(q) - p \cdot q$$

$$2 \cdot p \cdot q = V(q) + C(q)$$

$$p = \frac{1}{2}\left[\frac{V(q)}{q} + \frac{C(q)}{q}\right]$$

Manufacturer's average costs

Implications for Bilateral Monopoly

- Equal splitting

$$\Pi_S = \Pi_B$$

$$p \cdot q - C(q) = V(q) - p \cdot q$$

$$2 \cdot p \cdot q = V(q) + C(q)$$

$$p = \frac{1}{2}\left[\frac{V(q)}{q} + \frac{C(q)}{q}\right]$$

The firms share
the Retailer's revenues
and
the Manufacturer's costs
equally

Nash Bargaining Solution

-- A Reduced Form Model

Nash Bargaining Solution

- Extensive form bargaining model
  - Intuitive
  - But tedious
- Nash bargaining solution
  - Less intuitive
  - But easier to find the <u>same</u> outcome

Nash Bargaining Solution

- Three steps
  1. Describe bargaining situation
  2. Define Nash product
  3. Maximize Nash product

Nash Bargaining Solution

- Step 1: Describe bargaining situation

1. Who are the two players?
2. What contracts can they agree upon?
3. What payoff would they get from every possible contract?
4. What payoff do they have before agreement?
5. What is their relative patience (= bargaining power)

Nash Bargaining Solution

Example 1: Bilateral monopoly

- Step 1: Describe the bargaining situation
  - Players: Manufacturer and Retailer
  - Contracts: (T, q)

    T = total price for q units.
  - Payoffs:
    - Retailer: $\pi_R(T,q) = V(q) - T$
    - Manufacturer: $\pi_M(T,q) = T - C(q)$
  - Payoff if there is no agreement (while negotiating)
    - Retailer: $\tilde{\pi}_R = 0$
    - Manufacturer: $\tilde{\pi}_M = 0$
  - Same patience => same bargaining power

Nash Bargaining Solution

Example 1: Bilateral monopoly

- Step 2: Set up Nash product

$$N(T,q)=\left[\pi_R(T,q)-\tilde{\pi}_R\right]\cdot\left[\pi_M(T,q)-\tilde{\pi}_M\right]$$

Retailer's profit from contract

Manufacturer's profit from contract

Nash Bargaining Solution

Example 1: Bilateral monopoly

- Step 2: Set up Nash product

$$N(T,q)=\left[\pi_R(T,q)-\tilde{\pi}_R\right]\cdot\left[\pi_M(T,q)-\tilde{\pi}_M\right]$$

Retailer's extra profit from contract

Manufacturer's extra profit from contract

Nash Bargaining Solution

Example 1: Bilateral monopoly

- Step 2: Set up Nash product

$$N(T,q)=\left[\pi_R(T,q)-\tilde{\pi}_R\right]\cdot\left[\pi_M(T,q)-\tilde{\pi}_M\right]$$

Nash product
-Product of payoff increases

Depends on contract

Nash Bargaining Solution

Example 1: Bilateral monopoly

- Step 2: Set up Nash product

$$N(T,q) = \left[\pi_R(T,q) - \tilde{\pi}_R\right] \cdot \left[\pi_M(T,q) - \tilde{\pi}_M\right]$$

Claim:
The contract (T, q) maximizing N is the same contract that the parties would agree upon in an extensive form bargaining game!

Nash Bargaining Solution

Example 1: **B**ilateral monopoly

- Step 2: Set up Nash product

$$N(T,q) = \left[\pi_R(T,q) - \tilde{\pi}_R\right] \cdot \left[\pi_M(T,q) - \tilde{\pi}_M\right]$$

$$N(T,q) = \left[V(q) - T\right] \cdot \left[T - C(q)\right]$$

Nash Bargaining Solution

Example 1: Bilateral monopoly

- Maximize Nash product

$$N(T,q)=\left[V(q)-T\right]\cdot\left[T-C(q)\right]$$

$$\frac{\partial N}{\partial T}=-\left[T-C(q)\right]+\left[V(q)-T\right]=0$$

Equal profits = Equal split of surplus

Nash Bargaining Solution

Example 1: Bilateral monopoly

- Maximize Nash product

$$N(T,q) = [V(q) - T] \cdot [T - C(q)]$$

$$\frac{\partial N}{\partial T} = -[T - C(q)] + [V(q) - T] = 0 \qquad \Rightarrow \qquad T = \tfrac{1}{2}[V(q) + C(q)]$$

Nash Bargaining Solution

Example 1: Bilateral monopoly

- Maximize Nash product

$$N(T,q)=\left[V(q)-T\right]\cdot\left[T-C(q)\right]$$

$$\frac{\partial N}{\partial T}=-\left[T-C(q)\right]+\left[V(q)-T\right]=0 \qquad \Rightarrow \qquad T=\tfrac{1}{2}\left[V(q)+C(q)\right]$$

$$\Rightarrow \qquad p=\tfrac{1}{2}\left[\frac{V(q)}{q}+\frac{C(q)}{q}\right]$$

Convert to price per unit.

Nash Bargaining Solution

Example 1: Bilateral monopoly

- Maximize Nash product

$$N(T,q) = [V(q) - T] \cdot [T - C(q)]$$

$$\frac{\partial N}{\partial T} = -[T - C(q)] + [V(q) - T] = 0$$

$$\frac{\partial N}{\partial q} = V'(q) \cdot [T - C(q)] - C'(q) \cdot [V(q) - T] = 0 \qquad \Rightarrow \qquad V'(q) = C'(q)$$

Nash Bargaining Solution

Example 1: Bilateral monopoly

- Maximize Nash product

$$N(T,q)=[V(q)-T]\cdot[T-C(q)]$$

$$\frac{\partial N}{\partial T}=-[T-C(q)]+[V(q)-T]=0$$

$$\frac{\partial N}{\partial q}=V'(q)\cdot[T-C(q)]-C'(q)\cdot[V(q)-T]=0 \qquad \Rightarrow \qquad V'(q)=C'(q)$$

Efficiency

Nash Bargaining Solution

Example 1: Bilateral monopoly

- Conclusion
  - Maximizing Nash product is easy way to find equilibrium
  - Efficient quantity
  - Price splits surplus equally

Nash Bargaining Solution

- With different bargaining power

$$N(T,q) = \left[\pi_R(T,q) - \tilde{\pi}_R\right]^{\beta} \cdot \left[\pi_M(T,q) - \tilde{\pi}_M\right]^{1-\beta}$$

Exponents determined by relative patience