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Static Games

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Interdependent decisions

- Food retailing
  - ICA:s optimal price depends on Coop:s price
  - Coop: optimal price depends on ICA:s price
- How analyze?

Interdependent decisions

- Theory of interdependent decision making
  (a.k.a Game Theory)
  - How should we expect people to behave when the outcome depends on several persons actions?

Prisoners' Dilemma

Prisoners' Dilemma

- Police arrest two suspects
  - Enough evidence for short conviction (1 month)
  - More evidence needed for long conviction (10 months)
- Can the prisoners be made to confess?
  - Prosecutor asks prisoners independently to "rat"
    = provide information
  - Offering a rebate on the sentence

Prisoners' Dilemma

- Sentences after rebates:
  - If both "clam"
    - both get 1 month
  - If one person "rats"
    - the betrayer goes free
    - the other gets 10 months
  - If both "rat"
    - both get 4 months

Prisoners' Dilemma

- Prisoners put in separate cells
  - Simultaneous decisions

Prisoners' Dilemma

An outcome matrix summarizes the game:

		Prisoner 2	
		Clam	Rat
Prisoner 1	Clam	1, 1	10, 0
	Rat	0, 10	4, 4

If prisoner 1 rats and prisoner 2 clams:

- Prisoner 1 goes free
- Prisoner 2 gets 10 months

Prisoners' Dilemma

An outcome matrix summarizes the game:

		Prisoner 2	
		Clam	Rat
Prisoner 1	Clam	1, 1	10, 0
	Rat	0, 10	4, 4

Complete information

- Both prisoners know all facts

Q: Assume you are prisoner 1

- What would you do?

Prisoners' Dilemma

An outcome matrix summarizes the game:

		Prisoner 2	
		Clam	Rat
Prisoner 1	Clam	1, 1	10, 0
	Rat	0, 10	4, 4

If you only care for the other:

- Clam!

If you are selfish:

- Rat!

Prisoners' Dilemma

An outcome matrix summarizes the game:

		Prisoner 2	
		Clam	Rat
Prisoner 1	Clam	1, 1	10, 0
	Rat	0, 10	4, 4

We need to know people's preferences to predict how they will behave!

Prisoners' Dilemma 1

- Alternative representation
  - Utility = 10 - #months
- Payoff matrix

	Clam	Rat
Clam	9, 9	0, 10
Rat	10, 0	6, 6

Selfish

- Prisoners only care about their own sentence

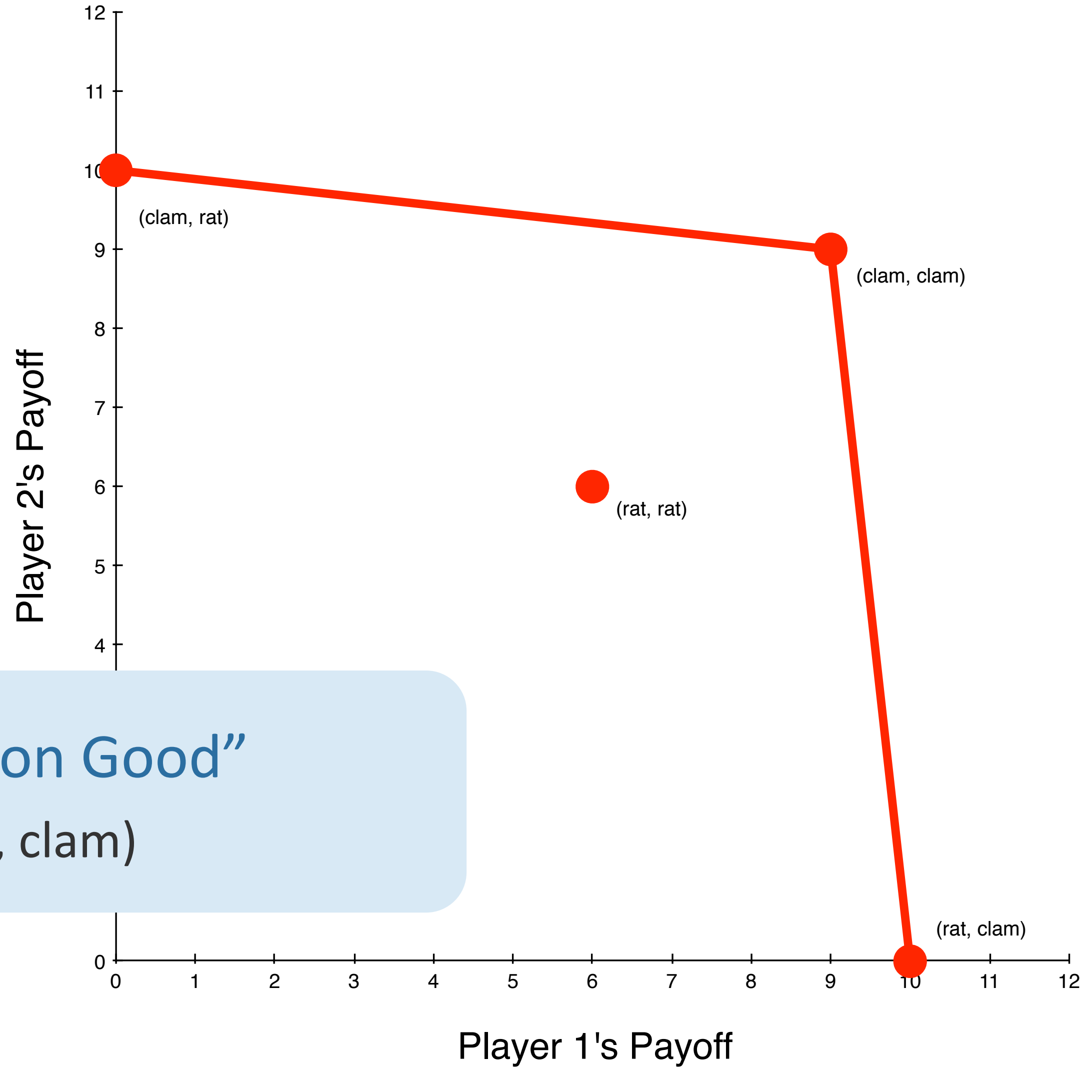
Convention

- Player 1 is row player

Complete information

- Both prisoners know all facts

Prisoners' Dilemma 1



"Common Good"

- (clam, clam)

Prisoners' Dilemma 1

- Assume they agreed to clam
  - Will they honor the agreement?

Prisoners' Dilemma 1

- Best-reply function
  - Simple procedure to predict behavior

Prisoners' Dilemma 1

- Player 1
  - Q: what is player 1's best choice if 2 would clam?
  - A: to rat

	Clam	Rat
Clam	9, 9	0, 10
Rat	10, 0	6, 6

Best reply
= utility maximizing choice for a given behavior by the other

Prisoners' Dilemma 1

- Player 1
  - Q: what is player 1's best choice if 2 would rat?
  - A: to rat

	Clam	Rat
Clam	9, 9	0, 10
Rat	10, 0	6, 6

Best reply
= utility maximizing choice for a given behavior by the other

Prisoners' Dilemma 1

- Player 1:s best reply ***function***
  - IF player 2 clams, THEN player 1:s best reply is to rat
  - IF player 2 rats, THEN player 1:s best reply is to rat

	Clam	Rat
Clam	9, 9	0, 10
Rat	<u>10</u>, 0	<u>6</u>, 6

Best reply
= utility maximizing choice for **a given** behavior by the other

Best reply function
= rule assigning best choice for **every possible** behavior by the other

Prisoners' Dilemma 1

- Here player 1's best-reply function says
  - Rat, independent of what the other player does

Notice: Rat is a strictly dominating strategy.

Definition: A strategy is ***strictly dominating*** if
- it is strictly better than all other strategies,
- independent of what other people do.

Notice: Very rare

Prisoners' Dilemma 1

- Here player 1's best-reply function says
  - Rat, independent of what the other player does

Notice: Clam is a strictly dominated strategy.

if

ly

One should never play a strictly dominated strategy

Notice: Quite common.

Prisoners' Dilemma 1

- Player 2
  - Q: what is player 2's best choice if 1 would clam?
  - A: to rat

	Clam	Rat
Clam	9, 9	0, 10
Rat	10, 0	6, 6

Prisoners' Dilemma 1

- Player 2
  - Q: what is player 2's best choice if 1 would rat?
  - A: to rat

	Clam	Rat
Clam	9, 9	0, 10
Rat	10, 0	6, 6

Prisoners' Dilemma 1

- Player 2:s best reply ***function***
  - IF player 1 clams, THEN player 2:s best reply is to rat
  - IF player 1 rats, THEN player 2:s best reply is to rat

	Clam	Rat
Clam	9, 9	0, <u>10</u>
Rat	10, 0	6, <u>6</u>

Prisoners' Dilemma 1

- Here player 2's best-reply function says
  - Rat, independent of what the other player does
- Conclusion
  - Both will rat

Prisoners' Dilemma 1

- Important insights

1. Conflict: Private incentives vs. Efficiency
    - Rational choice may lead to bad outcomes
2. Agreements beforehand do not matter, if players don't have incentives to follow agreement
3. Sometimes exist dominant strategies

Prisoners’ Dilemma 2

Prisoners' Dilemma 2

- Player 1 is a "moral person" (or altruist)
  - Utility = 20 - Σ#months
- Outcome matrix (months)

	Clam	Rat
Clam	1, 1	10, 0
Rat	0, 10	4, 4

- Payoff matrix

	Clam	Rat
Clam	18, 9	10, 10
Rat	10, 0	12, 6

Prisoners' Dilemma 2

	Clam	Rat
Clam	18, 9	10, 10
Rat	10, 0	12, 6

Q: Does player 1 have strictly dominated strategy?

Prisoners' Dilemma 2

	Clam	Rat
Clam	<u>18</u>, 9	10, 10
Rat	10, 0	<u>12</u>, 6

Q: Does player 1 have strictly dominated strategy?

A: No
- Better to clam if 2 clams
- Better to rat if 2 rats

Prisoners' Dilemma 2

	Clam	Rat
Clam	<u>18</u>, 9	10, 10
Rat	10, 0	<u>12</u>, 6

Q: What should player 1 do?

Prisoners' Dilemma 2

	Clam	Rat
Clam	<u>18</u>, 9	10, <u>10</u>
Rat	10, 0	<u>12</u>, <u>6</u>

A:
- Player 1 knows that player 2 will rat!
- Then better for 1 to also rat!

Prisoners' Dilemma 2

	Clam	Rat
Clam	<u>18</u>, 9	10, <u>10</u>
Rat	10, 0	<u>12</u>, <u>6</u>

Important insight
In a strategic situation, people need to put themselves into other peoples shoes

Prisoners' Dilemma 2

	Clam	Rat
Clam	<u>18</u>, 9	10, <u>10</u>
Rat	10, 0	<u>12</u>, <u>6</u>

Notice: if (rat, rat) would be played

- Player 1 plays a best reply against player 2's behavior
- Player 2 plays a best reply against player 1's behavior

Prisoners' Dilemma 2

We say (rat, rat) is an ***equilibrium***

Player 1 maximizes utility, given player 2's behavior

Player 2 maximizes utility, given player 1's behavior

Prisoners' Dilemma 2

- Q: Is any other outcome an equilibrium?
  - A: No!
  - E.g.: (clam, rat) => player 1 has incentive to change behavior

	Clam	Rat
Clam	9, 9	0, 10
Rat	10, 0	4, 4

Games in normal form

Normal Form

- Game in normal form
  - Players
  - Strategies
  - Payoffs (for all possible combinations of strategies)
- Prisoners Dilemma
  - Players: Prisoner 1, Prisoner 2
  - Strategies: rat, clam
  - Payoffs: u_1(clam, rat) = 10, and so on.

Normal Form

- Payoff matrix
  - Summarizes normal form (of 2-person game)
- Interpretation
  - Players choose simultaneously
  - Players know the game

Prisoners' Dilemma

- Definition: *Strategy profile*
  - A list of strategies, one for each player
- Example (Prisoners' Dilemma)
    - (rat, rat), (rat, clam), (clam, rat), (clam, clam)

Prisoners’ Dilemma

- Definition: *Nash equilibrium*
  - A strategy profile such that
    i. each player maximizes his utility,
    ii. given that all other players follow their strategies

Nash Equilibrium

- Formal definition for two-player game

Strategy profile $\left(s_1^*, s_2^*\right)$ is a Nash Equilibrium if :

$$u_1\left(s_1^*, s_2^*\right) \geq u_1\left(s_1, s_2^*\right) \quad \text{for all} \quad s_1 \quad \text{in} \quad S_1$$

$$u_2\left(s_1^*, s_2^*\right) \geq u_2\left(s_1^*, s_2\right) \quad \text{for all} \quad s_2 \quad \text{in} \quad S_2$$

Prisoners' Dilemma

- Why should we expect people to follow equilibrium?
  - Equilibrium behavior is by no means guaranteed,
  - but…

Prisoners' Dilemma

- Assume
    1. All people are ***rational***
       (= they maximize their utilities, given their expectations of what other people will do)
    2. All people ***know*** what will happen, before they make their choices
- Then
    - People must behave according to an equilibrium

Prisoners' Dilemma

- Argument: Assume the opposite
  - All people rational & All people know what will happen
  - Their behavior is not a NE (ex: Clam, Clam)
- Then
  - Then at least one person is supposed not to play best reply
  - Then at least this person will <u>deviate</u> from the prediction, since he is rational
  - Then, after all, people <u>didn't know</u> what was going to happen

Nash Equilibrium

- Formally

Rationality

$$u_1\left(s_1^*,\ E_1 s_2\right) \geq u_1\left(s_1,\ E_1 s_2\right) \quad \text{for all} \quad s_1 \quad \text{in} \quad S_1$$

Nash Equilibrium

- Formally

Rationality

$$u_1\left(s_1^*, E_1 s_2\right) \geq u_1\left(s_1, E_1 s_2\right) \quad \text{for all} \quad s_1 \quad \text{in} \quad S_1$$

Coordination

$$E_1 s_2 = s_2^*$$

Nash Equilibrium

- Formally

Rationality

$$u_1\left(s_1^*, E_1 s_2\right) \geq u_1\left(s_1, E_1 s_2\right) \quad \text{for all} \quad s_1 \quad \text{in} \quad S_1$$

Coordination

$$E_1 s_2 = s_2^*$$

Rationality & Coordination => Equilibrium

Nash Equilibrium

- Q: When should we use equilibrium analysis to predict behavior?
  - A: In situations where it is reasonable to assume that
    - People are rational
    - People for some reason understand what the outcome will be

Prisoners' Dilemma

- Exercise (for break)
  - Consider Prisoners' Dilemma Game with #months

	Clam	Rat
Clam	1, 1	10, 10 – r1
Rat	10 – r1, 10	10 – r2, 10 – r2

  - What "rebates" r1 and r2 do you need to give in order to:
    - Guarantee that (Rat, Rat) is an equilibrium?
    - Guarantee that (Rat, Rat) is the only equilibrium?

Prisoners' Dilemma

- Exercise (for break)
  - Consider Prisoners' Dilemma Game with #months

	Clam	Rat
Clam	1, 1	10, 10 – r1
Rat	10 – r1, 10	10 – r2, 10 – r2

  - What "rebates" r1 and r2 do you need to give in order to:
    - Guarantee that (Rat, Rat) is an equilibrium?
    - Guarantee that (Rat, Rat) is the only equilibrium?

Answers

Coordination Game

Coordination Game

- Situation
  - Cars meet on roads
  - If all keep to left (or right) they pass
  - Otherwise they crash
  - Sometimes choices are simultaneous
    - curves
    - top of hills

Coordination Game

- Lets try to represent such a situation as a game
- Lets make it as simple as possible

Coordination Game

- Represent situation as a game
  - Q: Three components of game?
    - Game = (Players, Strategies, Payoffs)
  - Q: Players?
    - Players = (driver 1, driver 2)
  - Q: Strategy sets?
    - Strategy set of driver i = (right, left)
  - Q: Payoff functions (and outcomes)?

Coordination Game

- Outcomes

	Left	Right
Left	Pass	Crash
Right	Crash	Pass

- Payoffs

	Left	Right
Left	1, 1	-1, -1
Right	-1, -1	1, 1

Coordination Game

- Q: What outcome should we predict?
  - A: Nash equilibrium
- Q: How do we find equilibrium?
  - A: Best reply analysis

Coordination Game

- Q: Best reply function for player 1?

	Left	Right
Left	1, 1	-1, -1
Right	-1, -1	1, 1

- A: "Do the same"

	Left	Right
Left	1, 1	-1, -1
Right	-1, -1	1, 1

Coordination Game

- Q: Best reply function for player 2

	Left	Right
Left	1, 1	-1, -1
Right	-1, -1	1, 1

- A: "Do the same"

	Left	Right
Left	1, $\underline{1}$	-1, -1
Right	-1, -1	1, $\underline{1}$

Coordination Game

- Q: What is the equilibrium strategy profile?
- A: (left, left) and (right, right)

	Left	Right
Left	<u>1</u>, <u>1</u>	-1, -1
Right	-1, -1	<u>1</u>, <u>1</u>

Coordination Game

- Multiple equilibria
  - In one and the same situation, there may exist several different outcomes that could be an equilibrium
  - But only one outcome will actually happen
- Which equilibrium will be played?
  - Requires some form of coordination
  - Somehow all players need to come to understand what will happen

Coordination Game

- How does coordination arise?
  - – Ordinary game theory has no answer

1. Dominance
   - Sometimes (e.g. prisoners' dilemma), but not here
2. Conventions
   - May be the result of learning
3. Pre-play communication
   - Anderson and Peterson specializing in comp. advantage
   - Self-enforcing agreement

Coordination Game

- Google:
  - Convention
  - Social norm

Chicken

Chicken

- Situation: Single-lane bridge
  - Drivers head for *single-lane* bridge from opposite directions
  - Sometimes two drivers arrive at same time
    - If both continue, they crash
    - If both stop, both are delayed
    - If one stops, he is delayed but the other can pass without delay

Coordination Game

- Represent situation as a game
  - Q: Three components of game?
    - Game = (Players, Strategies, Payoffs)
  - Q: Players?
    - Players = (driver 1, driver 2)
  - Q: Strategy sets?
    - Strategy set of driver i = (continue, stop)
  - Q: Payoff functions (and outcomes)?

Chicken

- Outcomes

	Stop	Continue
Stop	Delay, Delay	Delay, Pass
Continue	Pass, Delay	Crash, Crash

- Payoffs

	Stop	Continue
Stop	0, 0	0, 2
Continue	2, 0	-10, -10

Chicken

- Q: Find equilibrium

	Stop	Continue
Stop	0, 0	0, 2
Continue	2, 0	-10, -10

Chicken

- Two equilibria (Continue, Stop) and (Stop, Continue)

	Stop	Continue
Stop	0, 0	<u>0</u>, <u>2</u>
Continue	<u>2</u>, <u>0</u>	-10, -10

Chicken

- Both equilibria ***asymmetric***
  - Despite both players being in the "same situation"
  - They have to behave differently
  - They will receive different payoffs
  - Equilibrium (convention/norm) cannot be "fair"

Chicken

- Coordination
  - Pre-play communication difficult
    - But: with joint coin tossing, expected payoff =1.
  - Conventions/social norms
    - Young let old pass first

If time permits

Stag Hunt

Stag Hunt

- Situation: Two hunters are to meet in the forest
  - Two possibilities
    - Bring equipment for hunting stag (= collaboration)
    - Bring equipment for hunting hare (= not)
  - If both choose stag
    - Both get 10 kilos of meat
  - If both choose hare
    - One gets 2 kilos
    - Other gets nothing
    - Equal probabilities
  - If one chooses stag and the other hare
    - One with stag equipment gets nothing
    - One with hare equipment gets 2 kilos

Coordination Game

- Represent situation as a game
  - Q: Players?
    - Players = (hunter 1, hunter 2)
  - Q: Strategy sets?
    - Strategy set = (stag, hare)
  - Q: Payoff functions (and outcomes)?
    - Payoff = expected kilos of meat

Stag Hunt

- Payoff matrix

	Stag	Hare
Stag	10, 10	0, 2
Hare	2, 0	1, 1

Stag Hunt

- Q: Equilibria?

	Stag	Hare
Stag	10, 10	0, 2
Hare	2, 0	1, 1

- A: (stag, stag) & (hare, hare)

	Stag	Hare
Stag	<u>10</u>, <u>10</u>	0, 2
Hare	2, 0	<u>1</u>, <u>1</u>

Stag Hunt

- Q: Which should we believe in?

	Stag	Hare
Stag	<u>10</u>, <u>10</u>	0, 2
Hare	2, 0	<u>1</u>, <u>1</u>

  – Stag equilibrium - Pareto dominates

  – Hare equilibrium - less risky

Stag Hunt

- Q: Would pre-play communication work?

	Stag	Hare
Stag	10, 10	0, 2
Hare	2, 0	1, 1

- Not clear
  - Both would prefer stag-equilibrium
  - Player 1 may promise to bring stag equipment
  - But he would say so also if he plans to go for hare

Football Penalty Game

Football Penalties

- Situation
  - Two players: Shooter and Goal keeper
  - Shooter decides which side to shoot
  - Goalie decides which side to defend
  - Q: Simultaneous choices?

Football Penalties

- Outcomes

	Defend Left	Defend Right
Shoot Left	No goal	Goal
Shoot Right	Goal	No goal

- Payoffs

	Defend Left	Defend Right
Shoot Left	-1, 1	1, -1
Shoot Right	1, -1	-1, 1

Football Penalties

- Q: Find equilibria!

	Defend Left	Defend Right
Shoot Left	-1, 1	1, -1
Shoot Right	1, -1	-1, 1

Football Penalties

- Best-reply analysis

	Defend Left	Defend Right
Shoot Left	-1, <u>1</u>	<u>1</u>, -1
Shoot Right	<u>1</u>, -1	-1, <u>1</u>

- Conclusion
  – No equilibrium exists

Football Penalties

- Interpretation
  - Extreme competition: One player’s gain is the other player’s loss
  - Zero-sum game
  - Players don’t want to be predictable

Football Penalties

- What happens if goalie tosses a coin?
  - If shooter goes left => probability of goal = 50%
  - If shooter goes right => probability of goal = 50%
  - I.e. Probability of goal = 50%, independent of which side the shooter goes
  - Expected utility to both = 0, independent of which side the shooter goes

Football Penalties

- New game:

	Defend Left	Toss Coin	Defend Right
Shoot Left	-1, 1	0, 0	1, -1
Shoot Right	1, -1	0, 0	-1, 1

Football Penalties

- What happens if shooter tosses a coin?
  - Probability of goal = 50%,
    independent of which side the goalie goes
  - Expected utility to both = 0,
    independent of which side the goalie goes

Football Penalties

- New game

	Defend Left	Toss Coin	Defend Right
Shoot Left	-1, 1	0, 0	1, -1
Toss Coin	0, 0	0, 0	0, 0
Shoot Right	1, -1	0, 0	-1, 1

Football Penalties

- Best-reply analysis

	Defend Left	Toss Coin	Defend Right
Shoot Left	-1, $\underline{1}$	$\underline{0}$, 0	$\underline{1}$, -1
Toss Coin	0, $\underline{0}$	$\underline{0}$, $\underline{0}$	0, $\underline{0}$
Shoot Right	$\underline{1}$, -1	$\underline{0}$, 0	-1, $\underline{1}$

- Conclusion
  – Both tossing coin is equilibrium

Football Penalties

- Allowing players to toss coin restores equilibrium!
  - This is true in general…
  - …but we need to allow players to choose probabilities of different alternatives freely

Interpretation

- But, do people "toss coins"?
  - Not literarily…
  - …but in football penalty games the players sometimes go left and sometimes right
  - they try to be unpredictable
  - they behave *as if* they toss coins

Mixed Strategies and Existence of Equilibrium

Existence of Equilibrium

- If game has
  - Finitely many players
  - Each player has finitely many strategies
- Then, game has at least one Nash equilibrium
  - Possibly in mixed strategies

Illustration

Not included this year !

Existence of Equilibrium

- Example
  - 2 players
  - Player 1 has two pure strategies: Up
  - Player 2 has two pure strategies: Le
  - Player 1's Payoffs: B > A, C > D,
  - Player 2's Payoffs: a > c, d > b

Exercise:

Find the Nash equilibria

	Left	Right
Up	A, a	C, c
Down	B, b	D, d

Existence of Equilibrium

- Example
  - 2 players
  - Player 1 has two pure strategies: Up
  - Player 2 has two pure strategies: Le
  - Player 1's Payoffs: B > A, C > D,
  - Player 2's Payoffs: a > c, d > b

Solution:

No Nash equilibria

	Left	Right
Up	A, $\underline{a}$	$\underline{C}$, c
Down	$\underline{B}$, b	D, $\underline{d}$

Existence of Equilibrium

- Game in mixed strategies
  - Let us now define a ***new game***, which acknowledges that ***people may randomize*** their choices if they want to.
- Q: New game
  - Players: Same as before
  - Strategies: All possible probability distributions over "pure strategies"
  - Payoffs: Expected payoff

Existence of Equilibrium

- Mixed strategies
  - Player 2 selects Left with probability *p* (where $0 \leq p \leq 1$)
  - Player 1 selects Up with probability *q* (where $0 \leq q \leq 1$)

Existence of Equilibrium

- Expected utility

p*q = Prob (Up & Left)

$$U_1(q,p) = A \cdot \boxed{p \cdot q} + B \cdot p \cdot (1-q) + C \cdot (1-p) \cdot q + D \cdot (1-p) \cdot (1-q)$$

Where

$p = Prob\{Left\}$

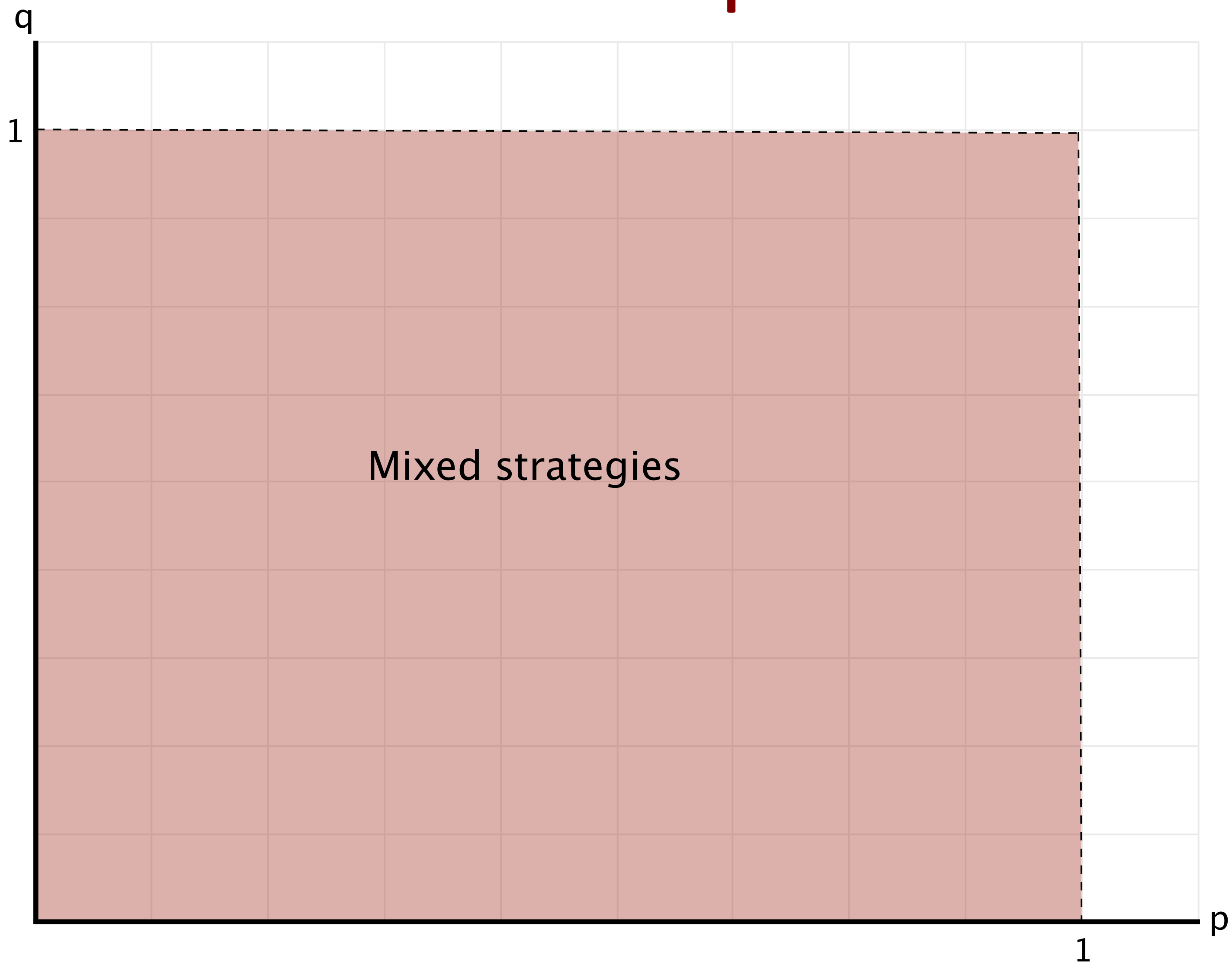
$q = Prob\{Up\}$

	Left	Right
Up	A, a	C, c
Down	B, b	D, d

Existence of Equilibrium

- Game in mixed strategies
  - Players: 1 and 2
  - Strategies: p in [0, 1] and q in [0, 1]
  - Payoffs: $U_1(p,q)$; $U_2(p,q)$

Existence of Equilibrium



Existence of Equilibrium

- Q: How do we make predictions?
  - Find Nash equilibria in the new game
- Q: What procedure to we use?
  - Derive best-reply functions

Existence of Equilibrium

- Notice: "the pure strategies are still there"
  - Player 2 going Right corresponds to $p = 0$
  - Player 2 going Left corresponds to $p = 1$
  - Player 1 going Down corresponds to $q = 0$
  - Player 1 going Up corresponds to $q = 1$

Existence of Equilibrium

- A useful “trick”
  - It turns out to be convenient to start out studying when the “pure strategies” are better than one another

Existence of Equilibrium

- Expected utility of pure strategies

$$U_1(p,1) = A \cdot p + C \cdot (1-p) \qquad q = 1 \Leftrightarrow \text{"}Up\text{"}$$

$$U_1(p,0) = B \cdot p + D \cdot (1-p) \qquad q = 0 \Leftrightarrow \text{"}Down\text{"}$$

$$p = Prob\{Left\}$$

	Left	Right
Up	A, a	C, c
Down	B, b	D, d

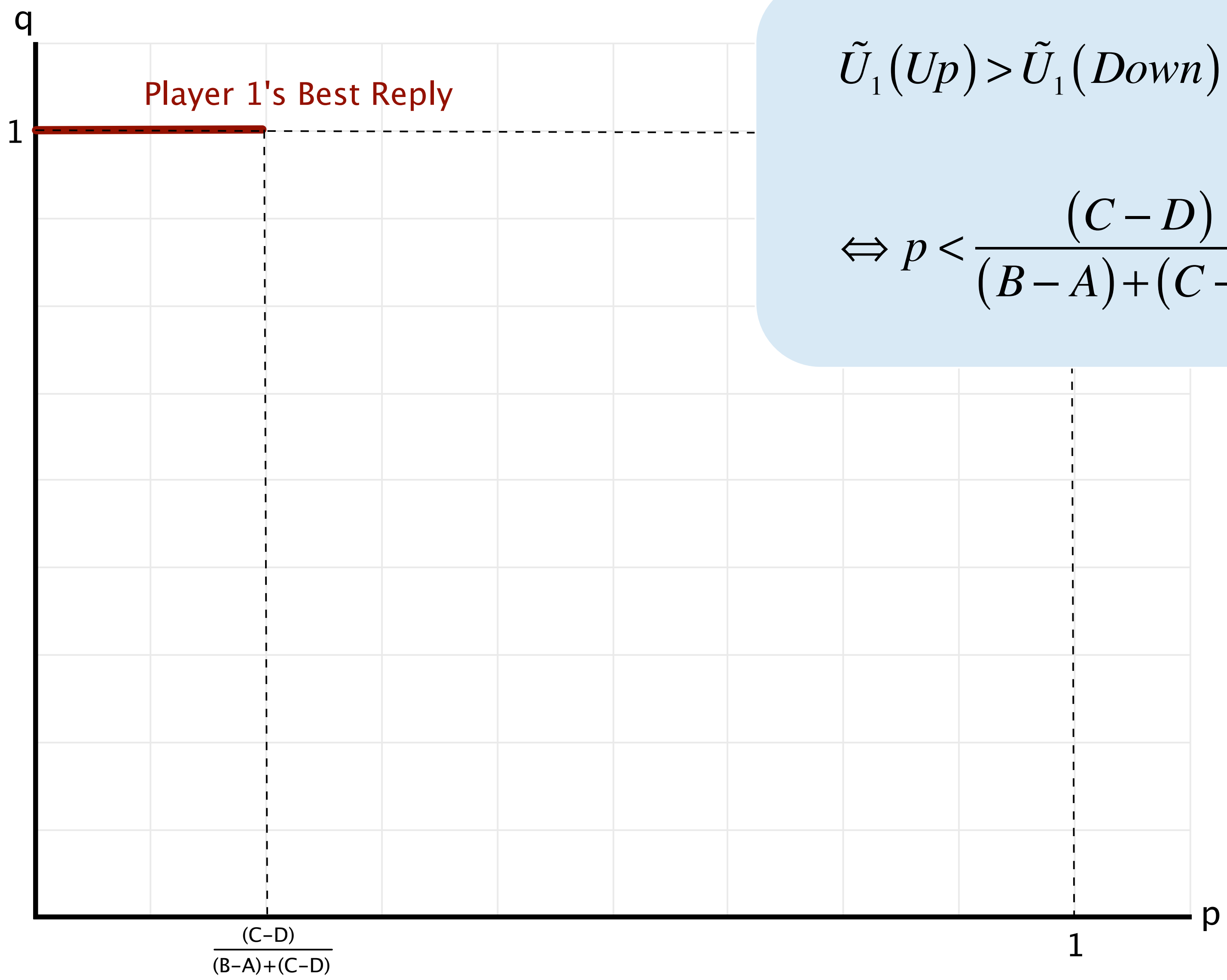
Existence of Equilibrium

- Player 1 prefers Up (ie q=1) if

$$\tilde{U}_1(Up) > \tilde{U}_1(Down)$$

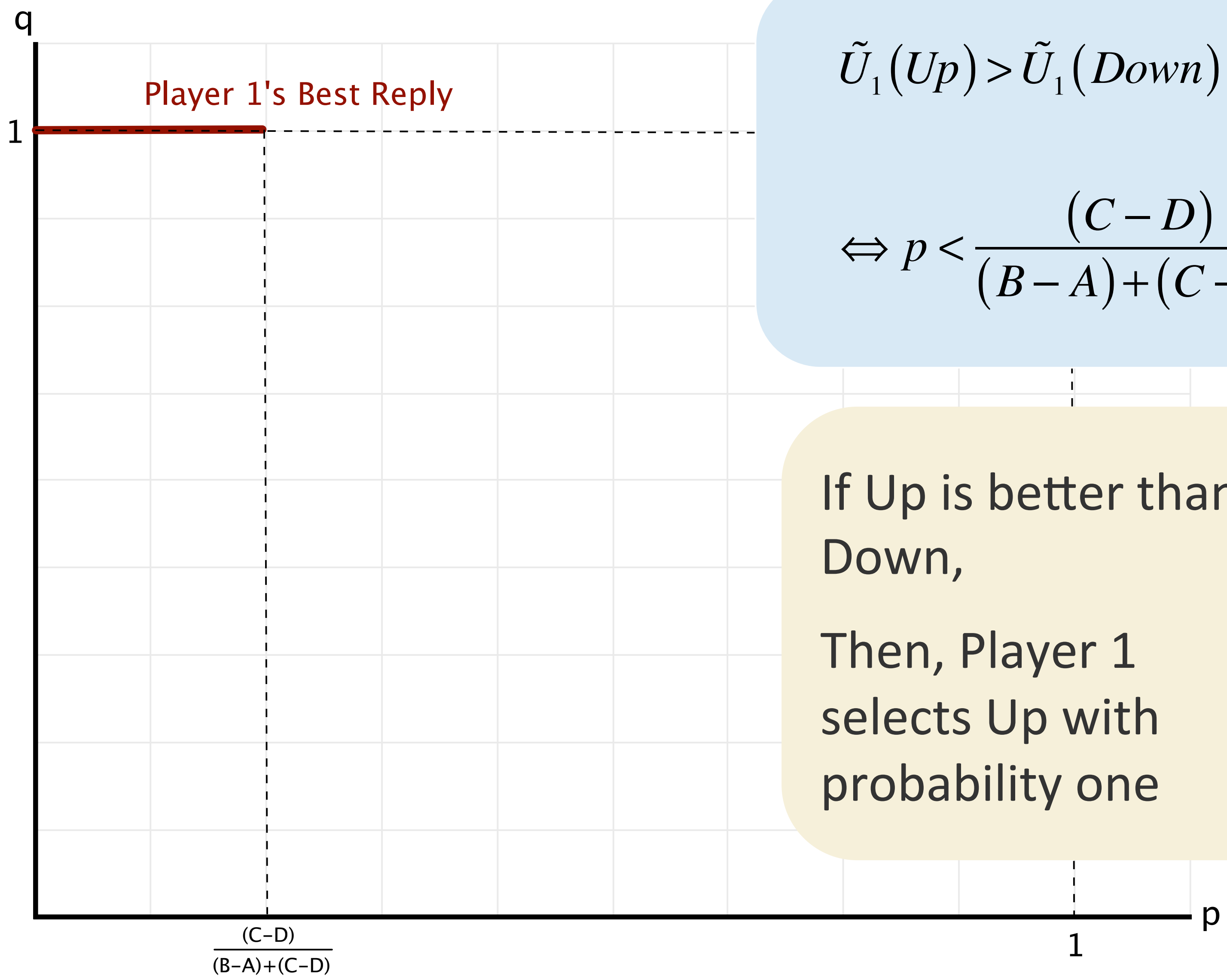
$$\Leftrightarrow A \cdot p + C \cdot (1 - p) > B \cdot p + D \cdot (1 - p)$$

$$\Leftrightarrow p < \frac{(C - D)}{(B - A) + (C - D)}$$



$$\tilde{U}_1(Up) > \tilde{U}_1(Down)$$

$$\Leftrightarrow p < \frac{(C-D)}{(B-A)+(C-D)}$$

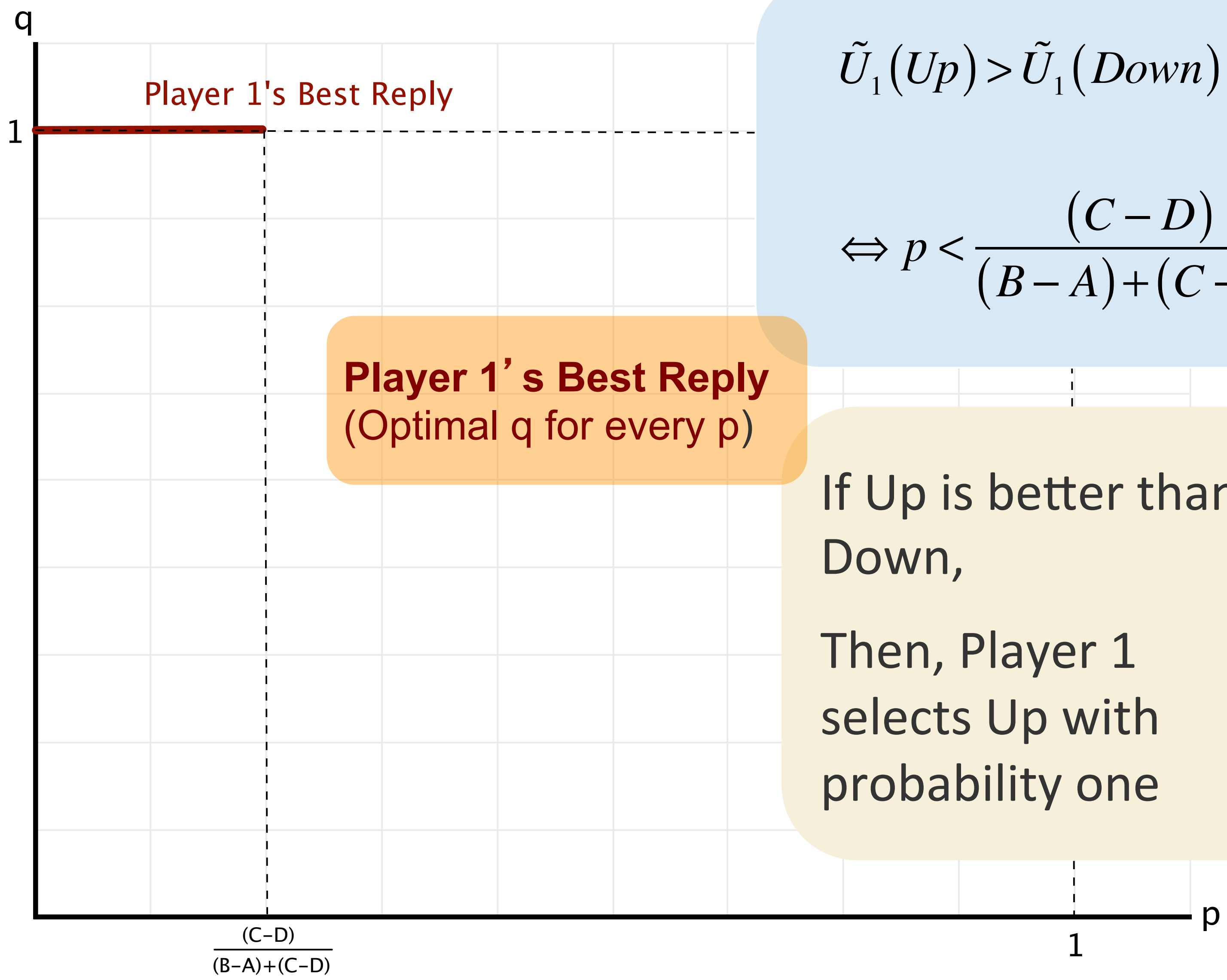


$$\tilde{U}_1(Up) > \tilde{U}_1(Down)$$

$$\Leftrightarrow p < \frac{(C-D)}{(B-A)+(C-D)}$$

If Up is better than Down,

Then, Player 1 selects Up with probability one



$$\tilde{U}_1(Up) > \tilde{U}_1(Down)$$

$$\Leftrightarrow p < \frac{(C-D)}{(B-A)+(C-D)}$$

Player 1's Best Reply
(Optimal q for every p)

If Up is better than Down,

Then, Player 1 selects Up with probability one

Existence of Equilibrium

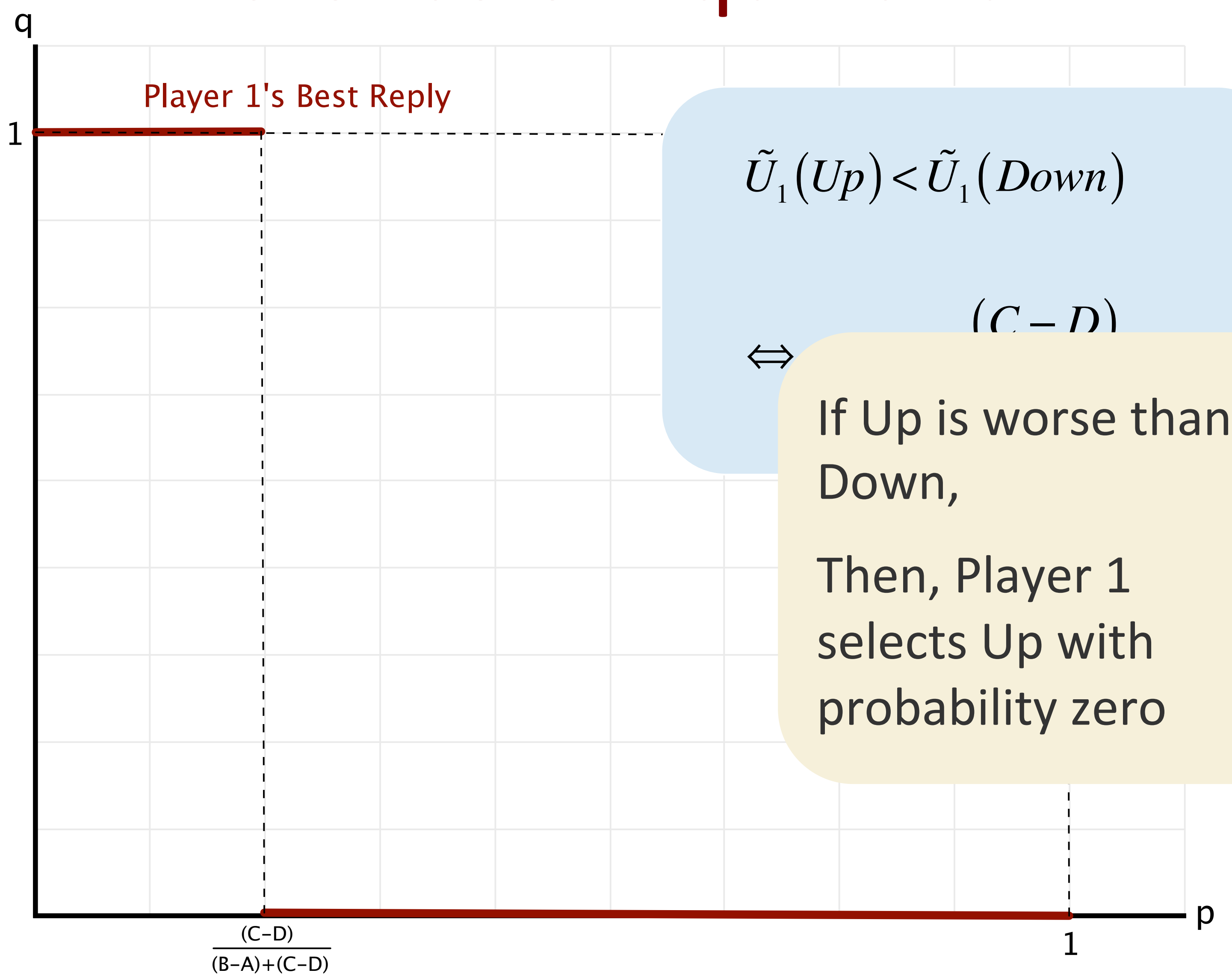
- Player 1 prefers Down (ie q=0) if

$$\tilde{U}_1(Up) < \tilde{U}_1(Down)$$

$$\Leftrightarrow A \cdot p + C \cdot (1 - p) < B \cdot p + D \cdot (1 - p)$$

$$\Leftrightarrow p > \frac{(C - D)}{(B - A) + (C - D)}$$

Existence of Equilibrium



$$\tilde{U}_1(Up) < \tilde{U}_1(Down)$$

$$\Leftrightarrow \quad (C-D)$$

If Up is worse than Down,

Then, Player 1 selects Up with probability zero

Existence of Equilibrium

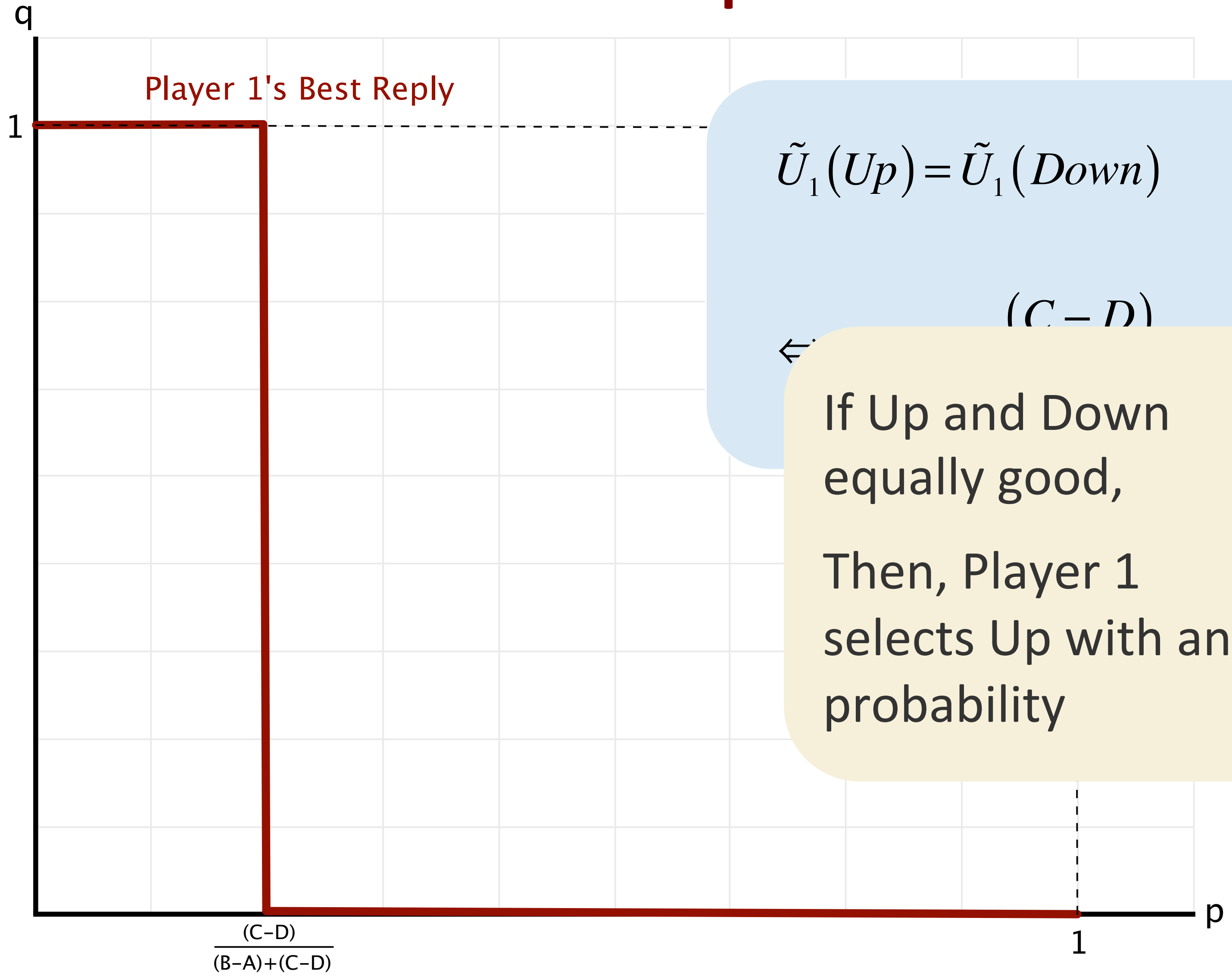
- Player 1 indifferent if

$$\tilde{U}_1(Up) = \tilde{U}_1(Down)$$

$$\Leftrightarrow A \cdot p + C \cdot (1 - p) = B \cdot p + D \cdot (1 - p)$$

$$\Leftrightarrow p = \frac{(C - D)}{(B - A) + (C - D)}$$

Existence of Equilibrium



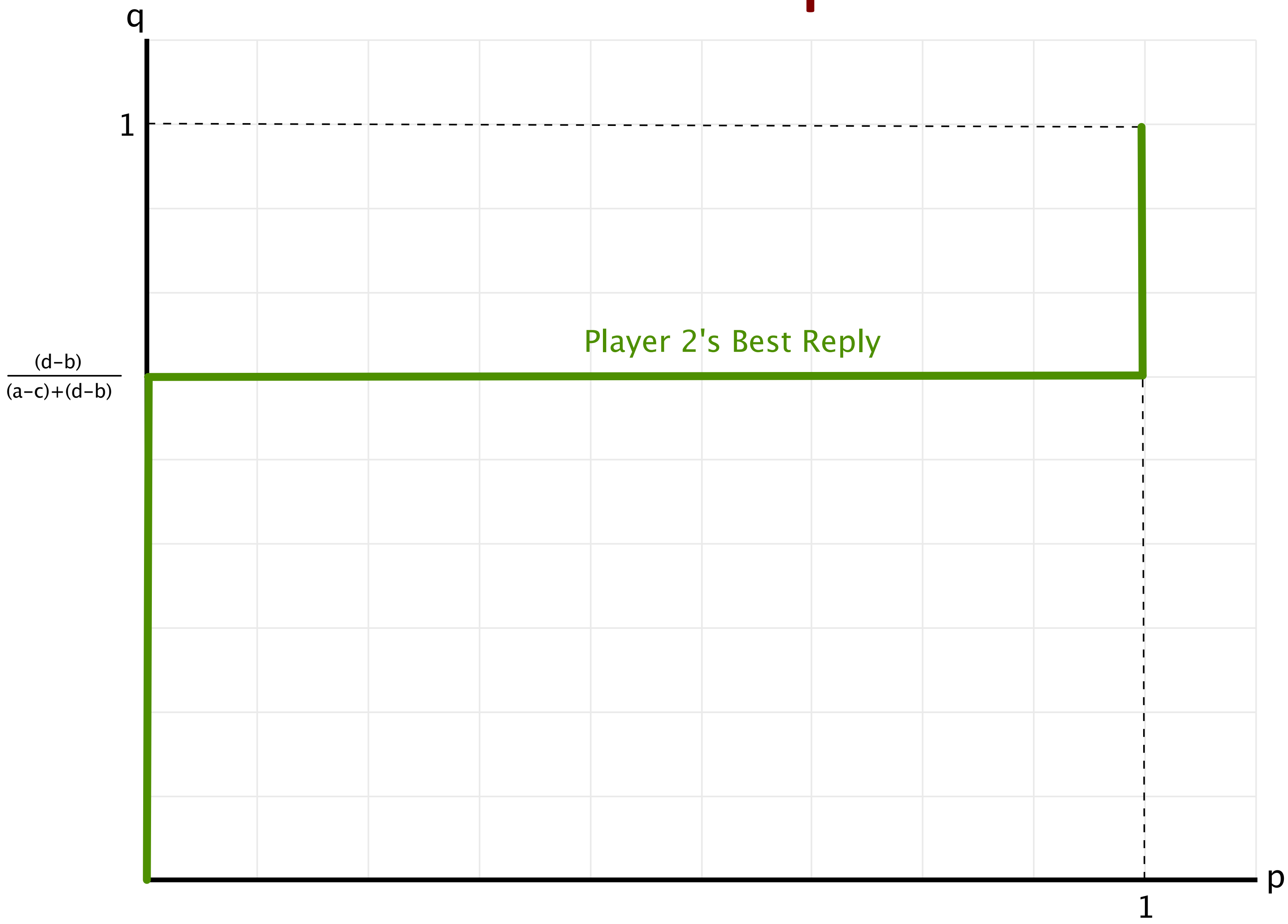
$$\tilde{U}_1(Up) = \tilde{U}_1(Down)$$

$$\Leftrightarrow \quad (C-D)$$

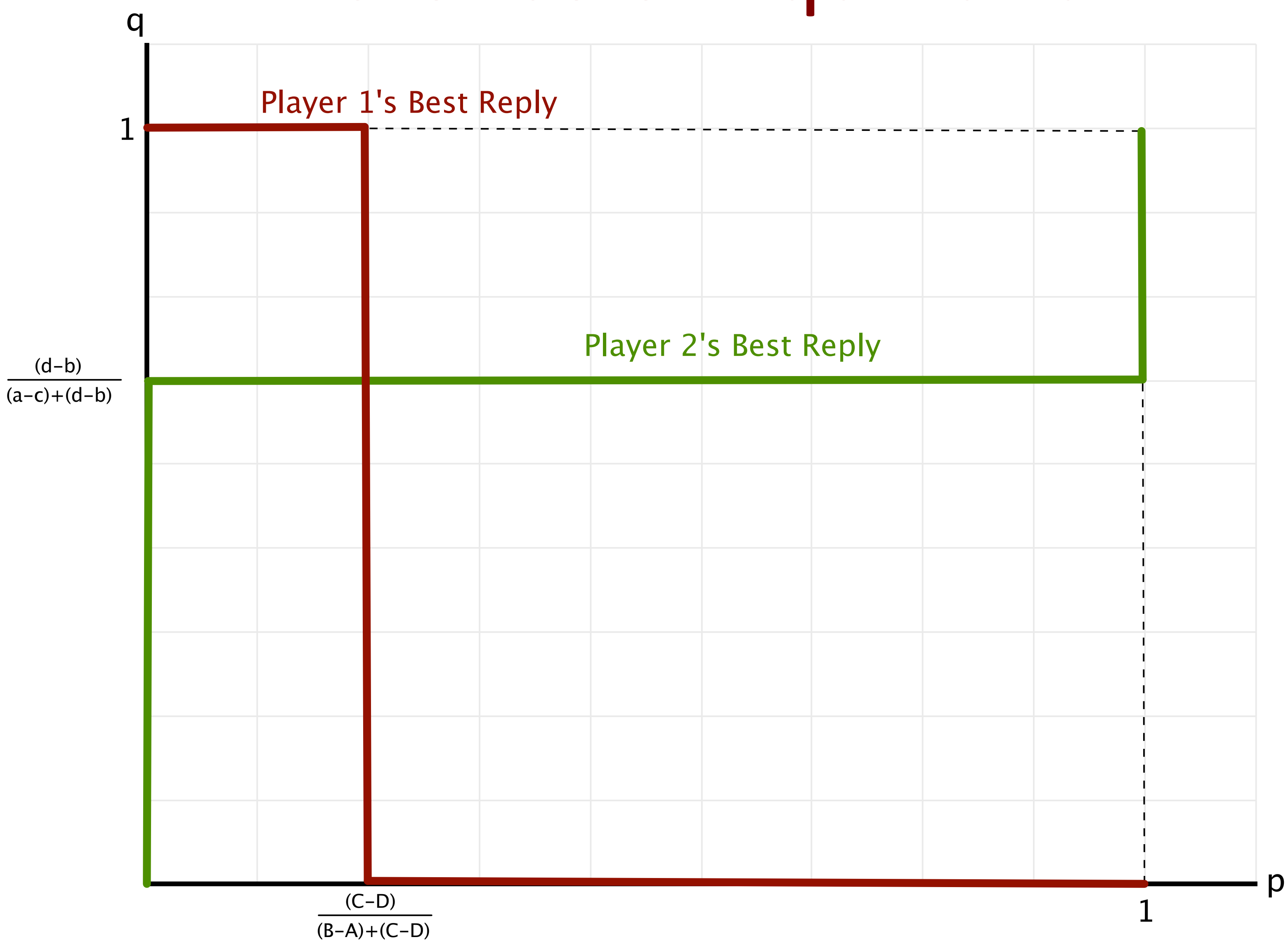
If Up and Down equally good,

Then, Player 1 selects Up with any probability

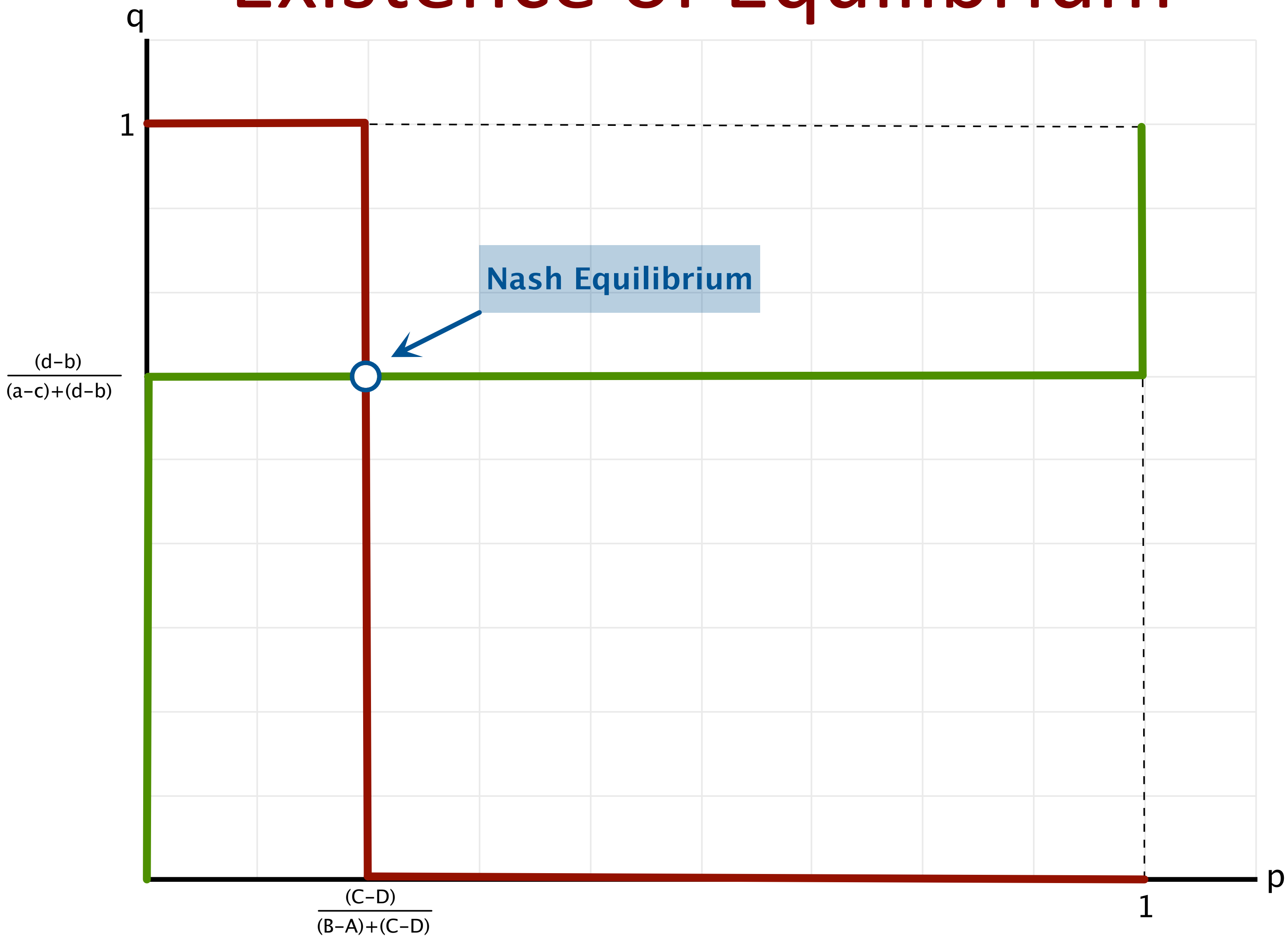
Existence of Equilibrium



Existence of Equilibrium



Existence of Equilibrium



Exercise

(mixed equilibrium)

Exercise

- Battle of the sexes
  - Two spouses want to go out, either to see a football game or a theater play
  - The man enjoys football (but not theater)
  - The woman enjoys theater (but not football)
  - They both enjoy each other’s company

Existence of Equilibrium

- Payoff matrix
  - Man is player one
  - v = value of preferred alternative (0 is value of other)
  - t = value of being together
  - Assume t > v.

	Football	Theater
Football	v+t, t	v, v
Theater	0, 0	t, v+t

Existence of Equilibrium

- To do
  - Define the game in mixed strategies
  - Find the man's best-reply function. Display in diagram
  - Same for woman
  - Find equilibria
  - Which is more plausible?

	Football	Theater
Football	v+t, t	v, v
Theater	0, 0	t, v+t