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Auctions

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Auctions

- Examples
  - Antiques, fine arts
  - Houses, apartments, land
  - Government bonds, bankrupt assets
  - Government contracts (roads)
  - Radio frequencies

Auctions

Why use auction?

- Seller's goal
  - Maximize revenues (you selling your apartment)
  - Efficient use (Government selling radio spectrum)
- Problem
  - Seller doesn't know what people are willing to pay
    - What is the highest valuation?
    - Who has it?
- Solution
  - Buyer *claiming* highest valuation gets the good
  - And will pay accordingly
- Auction = Mechanism to extract information

Auctions

But are auctions a good solution?

- Efficiency
  - IF: people really "tell the truth" = bid their valuations
  - THEN: good will be allocated correctly
- Revenues
  - IF: people really "tell the truth" = bid their valuations
  - THEN: price will be high (efficiency & extract WTP)
- Question: Do people "tell the truth"?
  - Need to study bidding behavior

Auctions

- Bidding behavior turns out to depend on:
  - Exact rules of the auction (Auction design)
  - How buyer’s valuations are related (Type of uncertainty)

Auctions

4 Designs

- Sealed bid, second price ("Vickrey")
  - Simultaneous
  - Winner pays second bid
- Sealed bid, first price
  - Simultaneous
  - Winner pays own bid
- English ("open cry")
  - Sequential + perf. info
  - Ascending bids
- Dutch
  - Sequential + perf. info
  - Descending offers

We will only study most common

Auctions

Types of Uncertainty

- Private value
  - Different buyers have different values
- Common value
  - Same value to all buyers
  - But different buyers have different information

We will only study private value

English Auction

English Auction

- Assume
  - One indivisible unit of the good
  - Two bidders
- Information
  - Bidders get to know own valuations, v_1 and v_2
  - Then the bidding game starts
- Bidding rules: a simple model
  - Players take turns bidding
  - Whenever one player does not bid at least €1 more, the good is sold to the current bid

English Auction

- Outcome
  - Winner = Highest bidder
  - Price = Highest bid

Second-Price Sealed-Bids

- Utility

$$u_i = \begin{cases} v_i - b_j & \text{if winning} \\ 0 & \text{otherwise} \end{cases}$$

English Auction

- Define: “marginal increases strategy” for i
  - If current bid < valuation, raise by €1
  - If current bid > valuation, stop bidding

- Formally
  - IF: $b_{jt-1} + 1 \leq v_i$, THEN: bid $b_{it} = b_{jt-1} + 1$
  - IF: $b_{jt-1} + 1 > v_i$, THEN: stop bidding

- Claim
  - This strategy is optimal (actually, dominant)

English Auction

- Sketch of proof
  - **If $b_{2t-1} < v_1$**
    - **Outbid**: Positive utility with (weakly) positive probability
    - **Withdraw**: $u_1 = 0$ for sure
    - No reason to raise by more than €1
  - **If $b_{2t-1} \geq v_1$**
    - **Withdraw**: $u_1 = 0$ for sure
    - **Outbid**: Negative utility with (weakly) positive probability
  - Note - dominance
    - Above strategy optimal
    - no matter how b_{2t-1} selected

English Auction

- Outcome
  - Q: "Truth telling"?
    - Sort of…
    - people keep raising the price until the bid is equal to their valuation (or nobody else continues to bid)
  - Q: Who gets the good?

English Auction

- Outcome
  - "Truth telling"
  - Efficiency
    - Bidder with highest valuation wins the good
  - Q: Who gets the surplus?

English Auction

- Outcome
  - "Truth telling"
  - Efficiency
    - Bidder with highest valuation wins the good
  - Surplus-sharing
    - p = SHV   (sometimes p = SHV + 1)

First-Price Sealed-Bids Auction

First-Price Sealed-Bids

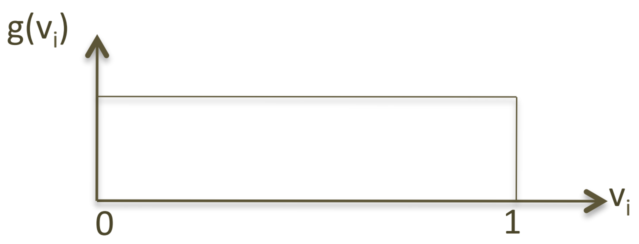
- Rules
  - – Simultaneous bids (= sealed bids)
  - – Winner pays his bid (= first price)

First-Price Sealed-Bids

- Trade-off
  - Higher bid → Higher probability of winning
  - Higher bid → Higher price

First-Price Sealed-Bids

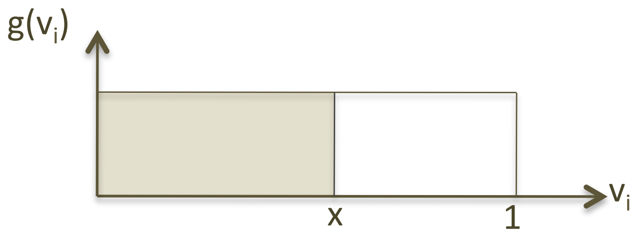
- Simplification
  - Two bidders: v_1, v_2
  - v_i uniformly distributed over [0, 1]



First-Price Sealed-Bids

- Q: Probability that $v_i < x$?

- A: $\text{Prob}(v_i < x) = x$



First-Price Sealed-Bids

- Payoff = expected utility
  - $E\pi_1(b_1) = (v_1 - b_1)\ \Pr(\text{win}) + 0\ \Pr(\text{loose})$
  - $E\pi_1(b_1) = (v_1 - b_1)\ \Pr(b_1 > b_2)$

Depends on

b_1 = own choice

b_2 = random variable

First-Price Sealed-Bids

- Payoff = expected utility
  - $E\pi_1(b_1) = (v_1 - b_1)\ Pr(win) + 0\ Pr(loose)$
  - $E\pi_1(b_1) = (v_1 - b_1)\ Pr(b_1 > b_2)$

We need to compute probability that $b_2 < b_1$

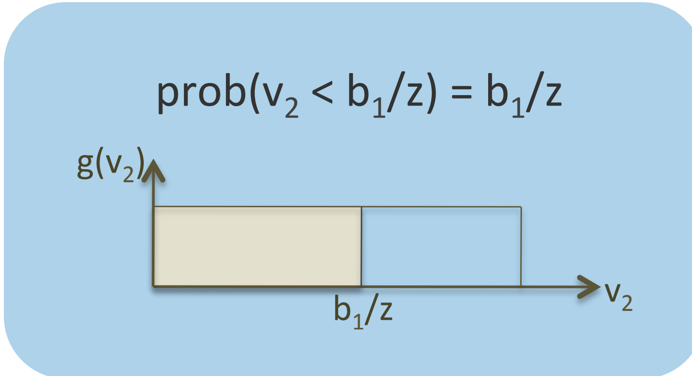
First-Price Sealed-Bids

- Payoff = expected utility
  - $E\pi_1(b_1) = (v_1 - b_1)\ Pr(win) + 0\ Pr(loose)$
  - $E\pi_1(b_1) = (v_1 - b_1)\ Pr(b_1 > b_2)$

Simplifying assumption:
$b_2 = z \cdot v_2$

First-Price Sealed-Bids

- Payoff = expected utility
  - $E\pi_1(b_1) = (v_1 - b_1)\ \Pr(\text{win}) + 0\ \Pr(\text{loose})$
  - $E\pi_1(b_1) = (v_1 - b_1)\ \Pr(b_1 > b_2)$
  - $E\pi_1(b_1) = (v_1 - b_1)\ \Pr(b_1 > z \cdot v_2)$
  - $E\pi_1(b_1) = (v_1 - b_1)\ \Pr(v_2 < b_1/z)$
  - $E\pi_1(b_1) = (v_1 - b_1)\ (b_1/z)$



First-Price Sealed-Bids

- Conclusion
  - IF: $b_2 = z \cdot v_2$
  - THEN: $E\pi_1(b_1) = (v_1 - b_1)\,(b_1/z)$
- Q: What is player 1's best reply?

First-Price Sealed-Bids

- What is 1's best reply?
  - $E\pi_1(b_1) = (v_1 - b_1)\,(b_1/z)$
  - FOC: $(-1)\,(b_1/z) + (v_1 - b_1)\,(1/z) = 0$

Utility if winning * Increased probability of winning

First-Price Sealed-Bids

- Assume
  - $B_2(v_2) = z\ v_2$
- What is 1's best reply?
  - $E\pi_1(b_1) = (v_1 - b_1)\ (b_1/z)$
  - FOC: $(-1)\ (b_1/z) + (v_1 - b_1)\ (1/z) = 0$

Decreased utility * probability of winning

First-Price Sealed-Bids

Proof

- Assume
  - $B_2(v_2) = z\, v_2$
- What is 1's best reply?
  - $E\pi_1(b_1) = (v_1 - b_1)\ (b_1/z)$
  - FOC: $-(b_1/z) + (v_1 - b_1)/z = 0$
  - Solve: $b_1 = ½ \cdot v_1$

First-Price Sealed-Bids

Proof

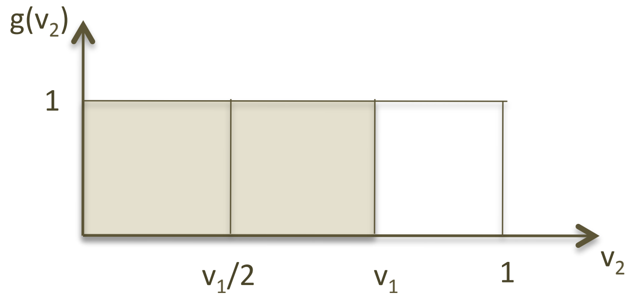
- Conclusion
  - IF: Bidder 2 uses a linear strategy: $B_2(v_2) = z \cdot v_2$
  - THEN: Best reply for bidder 1: $B_1(v_1) = ½ \cdot v_1$
- Note
  - Since $½ \cdot v_1$ is linear
  - Since players are symmetric
  - Both bidding $b_i = ½ \cdot v_i$ is a Nash equilibrium of a game where the strategy for each player is to choose some function $B_i(v_i)$.

First-Price Sealed-Bids

- Interpretation
  - Why bid ½ v ?
- Answer 1
  - Optimal balance between
    - probability of winning
    - price in case of winning

First-Price Sealed-Bids

- Interpretation
  - But why exactly ½ ?



- Answer 2
  - Assume you have highest valuation
  - Q: What is the expected second highest valuation?
  - Winner bids expected wtp of competitor
    => competitor no incentive to bid more

First-Price Sealed-Bids

- Remark
  - With more bidders, expected second highest wtp is closer to highest wtp
  - Bid larger share of wtp
  - As $n \rightarrow \infty$ $b \rightarrow$ wtp

First-Price Sealed-Bids

- Outcome
  - Q: Who gets the good?

First-Price Sealed-Bids

- Outcome
  - Efficiency
    - Bidder with highest valuation wins the good
  - Q: Who gets the surplus?

First-Price Sealed-Bids

- Outcome
  - Efficiency
    - Bidder with highest valuation wins the good
  - Surplus-sharing
    - p = ½ HV
  - Truth-telling?

First-Price Sealed-Bids

- Outcome
  - Efficiency
    - Bidder with highest valuation wins the good
  - Surplus-sharing
    - p = ½ HV
  - "Sort of truth-telling"
    - Players actually reveal their valuation

Game Theoretic "Details"

Auction = Game of Incomplete Information

Game of Incomplete Information

- Game with incomplete information
  - Buyers don't know each others' valuations
    - Ada is not able to predict Ben's bid exactly
    - It depends on Ben's valuation of the object
  - How should Ada and Ben analyze the situation?

Game of Incomplete Information

- Solution I: Change definition of strategy
  - Strategy = Function prescribing bid for every possible valuation a player may have
- Example of strategy
  - IF wtp = v_H THEN bid = b_H
  - IF wtp = v_L THEN bid = b_L
- Then, players able to
  - Predict rival's *strategy*, even if uncertainty about type and *bid* remains
  - Maximize expected payoff

Game of Incomplete Information

- But why are strategies functions?
  - Ada knows she has high valuation, v_H
  - Why should she choose strategy with instruction for v_L?
- Answer
  - Ben doesn't know Ada's valuation. Could be v_H or v_L
  - Ben must consider
    - What would Ada bid if v_H
    - What would Ada bid if v_L
  - To predict Ben's bid, Ada must also consider what she herself would have bid in case of v_L

Game of Incomplete Information

- Think of Ada's choice as two-step procedure

    1. Find optimal bid for all possible valuations: $b^{Ada}(v_H)$ and $b^{Ada}(v_L)$

    2. Select the relevant bid: $b^{Ada}(v_H)$

Game of Incomplete Information

- Solution II: Change definition of payoff
  - Payoff = expected utility

Game of Incomplete Information

Bayesian Nash Equilibrium

- Pair of strategies (b^{Ada}, b^{Ben}) such that
- b^{Ada} is a best reply to b^{Ben}
  - $b^{Ada}(v_H)$ maximizes Ada's expected utility
    - If Ada's valuation is v_H
    - Assuming Ben uses b^{Ben}
  - $b^{Ada}(v_L)$ maximizes Ada's expected utility
    - If Ada's valuation is v_L
    - Assuming Ben uses b^{Ben}
- b^{Ben} is a best reply to b^{Adam}

Most fundamental result of auction theory

Fundamental result

Note 1: No individual knows who has the highest valuation

Note 2: But if people play the auction game
$\Rightarrow$ person with highest valuation walks away with the good

No individual (even a dictator)
could have implemented the efficient allocation,
since nobody has sufficient information

But the <u>market mechanism</u> actually <u>solves</u> the <u>maximization problem</u>

May say the market <u>aggregates information</u>
* must use all the information to solve the max-problem
* despite the fact that it is scattered

Fundamental result

- Laboratory experiments
  - It works! (Vernon Smith)
  - Also double auctions
  - Even with "few" buyers and "few" sellers market quickly converges to competitive price
  - NB: must use laboratory to know people's valuations

Fundamental result

Sure, it is not perfect…

…there is also market failure…

- Coordination (mis-pricing; recessions)
- Double coincidence of wants (kidneys, apartments)
- Externalities (global warming; telecom)
- Public goods (R&D; legal system to enforce all contracts)
- Market power (medicines; district heating)
- Incomplete information (cars, insurance, labor, credit)

…and an uneven distribution of wealth

Fundamental result

- But even public policies to correct market failure use markets to aggregate information
  - Cap and trade
  - Public procurement

Comparison of Auction Designs (Revenues)

Comparison of Designs

Question 1

- Which auction gives the highest expected price?
  - FPSB (and Dutch): $p = \frac{1}{2}$ HV
  - English (and SPSB): $p =$ SHV

Recall: E(SHV) = ½ HV

Comparison of Designs

Answer 1

- Expected Revenue Equivalence Thm.
  (Vickrey, 1961)
  - All four auctions give the same *expected* price

Comparison of Designs

Question 2

- Is there any other way to sell the goods which would give a higher expected profit?
  - Lots of different possible ways
    - Bargaining
    - Other auction formats
    - Strange games

Comparison of Designs

Answer 2

- Generalization of Revenue Equivalence Thm
  - No!
  - This is example of "mechanism design" and uses the "revelation principle" (Leonid Hurwicz, Eric Maskin, Roger Myerson)